

SYNTHESIS OF CONTROL LAWS OF AEROSPACE SYSTEM BASED ON A SYNERGISTIC APPROACH

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Keywords: aerospace system, synthesis of control laws, invariant manifold, functional equation

Abstract

Synthesized control laws provide lift of aerospace system to the launch point, separating upper-stage rocket from launch aircraft and reaching hypervelocity acceleration.

1 Introduction

Delivery of heavy vehicles to the orbit is the main task of modern aerospace industry at the present stage of development. One of the effective solutions is using multistage aerospace system. Due to this, interest to the aerospace systems has increased significantly within recent years (MAKS, Hotel, Space Ship Two) [1–4]. Pilot errors are one of the significant causes leading to off-nominal situation. Therefore, for the sake of safety and reducing risk of uncontrolled development of off-nominal situation are developing and implementing automatic control systems. As a control object, aerospace system represents as multilevel multiply nonlinear dynamic system [5–6]. As a consequence, linear methods of classical control theory are not suitable for the synthesis of control laws of such object. Synergetic control theory gives a comprehensive approach for analysis and synthesis of control systems. This theory based on the principles of self-directed self-organizing in nonlinear dynamical systems [5]. These principles and approaches of synergetic control theory are forming method analytical design of aggregated regulators. On the principles and approaches of synergetic control theory is based method of aggregated-regulators analytical design method (ADAR). [6–7] ADAR method allows to synthesize

control laws for nonlinear dynamic objects and often used for the synthesis of control laws for objects with different nature. [5–8] The synthesis of the aircraft control laws using ADAR method.

2 Description of Aerospace system

Aerospace system consists of launch aircraft 1, upper-stage rocket 2 and spaceplane 3 (Fig. 1) [9].

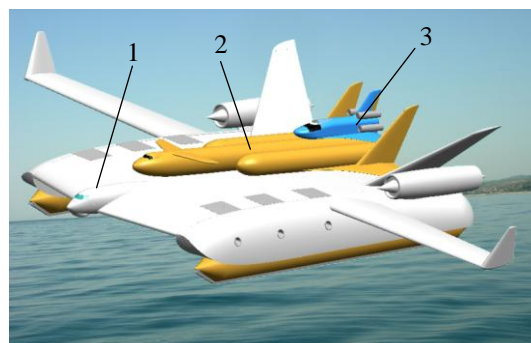


Fig. 1. Aerospace system

Aerospace system is designed to deliver into orbit heavy vehicles and cargoes. During “Air Start”, super-heavy amphibious aircraft is used as based platform for launching upper-stage rocket with spaceplane aboard.

Injection into Earth orbit consists from following stages. Launch aircraft with upper-stage rocket with spaceplane aboard lift at release altitude and reaching hypervelocity acceleration, then upper-stage rocket with spaceplane aboard are separating from launch aircraft. At this stage, upper-stage rocket with spaceplane aboard reaching hypervelocity acceleration and spaceplane is separating from upper-stage rocket. Solo flight of spaceplane with a predetermined orbital radius. There are considered the synthesis of control laws on

following stages: 1) lift of aerospace system to release altitude and reaching hypervelocity acceleration. 2) Upper-stage rocket with spaceplane aboard are separating from launch aircraft. 3) Upper-stage rocket with spaceplane aboard reaching hypervelocity acceleration.

3 Launching of aerospace system

3.1 Mathematical model

Autopilot control law is synthesized by using model of longitudinal movement of the aircraft [10]:

$$\begin{aligned}
 m\dot{V}(t) &= \sum_{j=1}^k P_j \cos \alpha - mg \sin(\vartheta - \alpha) - \\
 &- X - \sum_{i=1}^n N_i \sin(\vartheta - \alpha); \\
 \dot{H}(t) &= V \cdot \sin(\vartheta - \alpha); \\
 mV\dot{\alpha}(t) &= mV\omega_z - \sum_{j=1}^k P_j \sin \alpha + \\
 &+ mg \cos(\vartheta - \alpha) - Y + \sum_{i=1}^n N_i \cos(\vartheta - \alpha); \\
 \dot{\omega}_z(t) &= \frac{1}{I_z} M_{za}; \quad \dot{\vartheta}(t) = \omega_z; \\
 \dot{x}(t) &= V \cos(\vartheta - \alpha),
 \end{aligned} \tag{1}$$

where $\sum_{j=1}^k P_j$ – summary thrust of engines; j –

index number of engine; $\sum_{i=1}^n N_i$ – total force in

attachment point of upper-stage rocket to launch aircraft; i – index number of attachment point. Force of aerodynamic drag X ; ascensional force Y , pitching moment M_{za} are depending from aircraft configurations and position against apprise flow:

$$\begin{aligned}
 X &= c_x qS; \\
 Y &= c_y qS; \\
 M_{za} &= m_z qSb_a,
 \end{aligned}$$

where c_x, c_y, m_z – nondimensional aerodynamic force coefficient and pitching moment, S – wing area of launching aircraft, b_a – mean aerodynamic chord of launching aircraft, impact air pressure $q = \rho V^2 / 2$; ρ – atmospheric density. Considering design layout of launching aircraft (Fig. 1):

$$\begin{aligned}
 c_x &= c_x(\alpha) + c_x^{\delta_6} \delta_6; \\
 c_y &= c_y(\alpha) + c_y^{\delta_6} \delta_6; \\
 m_z &= m_z(\alpha) + m_z^{\bar{\omega}_z} \bar{\omega}_z + m_z^{\delta_6} \delta_6,
 \end{aligned} \tag{2}$$

where $c_x, c_y, m_z(\alpha)$ – dependence between aerodynamic coefficient and angle of attack α and taking into account influence of upper-stage rocket with spaceplane, $c_x^{\delta_6}, c_y^{\delta_6}, m_z^{\delta_6}$ – derivative with respect to deflection angle of elevating rudder, δ_6 ; $m_z^{\bar{\omega}_z}$ – derivative of pitching moment coefficient with respect to normalized angular velocity of pitch deflection $\bar{\omega}_z, \bar{\omega}_z = \frac{b_a \omega_z}{V_k}$.

Considering (2) system of equations (1) assumes an aspect:

$$\begin{aligned}
 \dot{V} &= \sum_{j=1}^k P_j \frac{\cos \alpha}{m} - g \sin(\vartheta - \alpha) - \\
 &- \frac{1}{m} \sum_{i=1}^n N_i \sin(\vartheta - \alpha) - \frac{qS}{m} (c_x(\alpha) + c_x^{\delta_6} \delta_6); \\
 \dot{H} &= V \cdot \sin(\vartheta - \alpha); \\
 \dot{\alpha} &= \omega_z - \sum_{j=1}^k P_j \frac{\sin \alpha}{mV} + \frac{g \cos(\vartheta - \alpha)}{V} + \\
 &+ \frac{\cos(\vartheta - \alpha)}{mV} \sum_{i=1}^n N_i - \frac{qS(c_y(\alpha) + c_y^{\delta_6} \delta_6)}{mV}; \\
 \dot{\omega}_z &= \frac{qSb_a}{I_z} \left[m_z(\alpha) + m_z^{\bar{\omega}_z} \frac{b_a \omega_z}{V} + m_z^{\delta_6} \delta_6 \right]; \\
 \dot{\vartheta} &= \omega_z; \quad \dot{x} = V \cos(\vartheta - \alpha),
 \end{aligned} \tag{3}$$

where m – mass; V – linear flight velocity; H – flight altitude; α – angle of attack in relation to longitudinal datum line of launching;

\mathcal{G} – pitch angle; ω_z – angular velocity of pitching; I_z – axial moment of inertia; g – acceleration of gravity; x – longitudinal motion.

Mathematical model (3) for the synthesized basic control laws considering the state variables assumes an aspect:

$$\begin{aligned} \dot{x}_1(t) &= \frac{u_2}{m} \cos(x_3) - g \sin(x_5 - x_3) - \\ &- \sum_{i=1}^n N_i \frac{\sin(x_5 - x_3)}{m} - \frac{qS}{m} (c_x(\alpha) + c_x^{\delta_e} u_1); \\ \dot{x}_2(t) &= x_1 \sin(x_5 - x_3); \\ \dot{x}_3(t) &= x_4 - \frac{u_2 \sin(x_3)}{mx_1} + \frac{g \cos(x_5 - x_3)}{x_1} + \\ &+ \sum_{i=1}^n N_i \frac{\cos(x_5 - x_3)}{mx_1} - \frac{qS(c_y(\alpha) + c_y^{\delta_e} u_1)}{mx_1}; \\ \dot{x}_4(t) &= \frac{qSb_a}{I_z} \left[m_z(\alpha) + m_z^{\bar{\omega}_z} \frac{b_a x_4}{x_1} + m_z^{\delta_e} u_1 \right]; \\ \dot{x}_5(t) &= x_4; \quad \dot{x}_6(t) = x_1 \cos(x_5 - x_3). \end{aligned} \quad (4)$$

where $x_1 = V$, $x_2 = H$, $x_3 = \alpha$, $x_4 = \omega_z$, $x_5 = \mathcal{G}$, $x_6 = x$ – state variables.

Control actions: $u_1 = \delta_e$; $u_2 = \sum_{j=1}^k P_j$.

Purpose objectives – launching aerospace system and achieving release point with predetermined flight altitude and flight velocity:

$$H = H^*, V = V^*.$$

Setting up a problem. It is required to find a control vector in analytical form \bar{u} , as coordinate function of state system (4), enabling performance of lifting aerospace system to predetermined flight altitude H^* (where upper-stage rocket with spaceplane aboard are separating from launch aircraft), also movement of aerospace with predetermined flight velocity V^* at this flight altitude.

3.2 Synthesis of control laws

According ADAR method, introducing macrovariables:

$$\begin{aligned} \psi_1 &= x_1 - x_1^*; \\ \psi_2 &= x_4 - \varphi_1, \end{aligned} \quad (5)$$

where $x_1^* = V^*$ – technological invariant; φ_1 – internal control.

Macrovariables (5) shall satisfy a solution of the system of functional equations [5]:

$$\begin{aligned} T_1 \cdot \dot{\psi}_1 + \psi_1 &= 0; \\ T_2 \cdot \dot{\psi}_2 + \psi_2 &= 0, \end{aligned} \quad (6)$$

where T_1, T_2 – time constant, which affect to the dynamic processes in the system «object – regulator». Condition of asymptotic stability of the equation (6) while $\psi_1 = 0, \psi_2 = 0$ is given by: $T_1 > 0, T_2 > 0$. On the intersection $\psi_{12} = 0$ of invariant manifolds $\psi_1 = 0, \psi_2 = 0$ be observed effect of dynamic "compression" of the phase space. Decompose the system takes the form:

$$\begin{aligned} \dot{x}_2(t) &= x_1^* \sin(x_5 - x_3); \\ \dot{x}_5(t) &= \varphi_1; \end{aligned} \quad (7)$$

For the system (7) is introduced macro variable ψ_3

$$\psi_3 = x_1^* \sin(x_5 - x_3) + x_2 - x_2^*, \quad (8)$$

which must satisfy the solution of the functional equation:

$$T_3 \cdot \dot{\psi}_3 + \psi_3 = 0, \quad (9)$$

where $x_2^* = H^*$ – technological invariants; T_3 – time constant.

From the joint solution of equations (7), (8), (9) is obtained expression for the "internal" controls φ_1 :

$$\begin{aligned} \varphi_1 &= -(1 + \frac{1}{T_3})tg(x_5 - x_3) - \\ &- \frac{x_2 - x_2^*}{T_3 x_1^* \cos(x_5 - x_3)}. \end{aligned} \quad (10)$$

External controls u_1, u_2 are found from the joint solution of (6) together with expression φ_1 , equations of the object model (4) and macrovariables (5). The expressions for the control actions (deflection angle of elevating rudder δ_ϵ and engine thrust $\sum_{j=1}^k P_j$) are external controls. They depend on the state variables of the system.

Analytical form of autopilot control law of elevating rudder is assuming the form:

$$u_1 = - \frac{A_1 \cos(x_5 - x_3) \cdot \sin(x_5 - x_3)}{A_5 \cos(x_5 - x_3)^2} + \frac{A_2 \sin(x_5 - x_3) + A_3 \cos(x_5 - x_3)}{A_5 \cos(x_5 - x_3)^2} - \frac{A_4}{A_5}, \quad (11)$$

where $A_1 \div A_5$ can be described by expressions:

$$\begin{aligned} A_1 &= I_z x_1 (T_2 x_1 + x_1^* + T_3); \\ A_2 &= T_2 I_z x_1 x_4 (x_2 - x_2^*); \\ A_3 &= I_z x_1 (x_2 - x_2^*); \\ A_4 &= T_2 T_3 q S b_a x_1^* (b_a m_z^{\delta_\epsilon} x_4 + m_z(\alpha) \cdot x_1) \\ &\quad + T_3 I_z x_1 x_1^* x_4; \\ A_5 &= T_2 T_3 q S b_a m_z^{\delta_\epsilon} x_1 x_1^*. \end{aligned} \quad (12)$$

As a result of transformation, control law of the engine thrust assuming the form:

$$u_2 = - \frac{(B_1 + B_2) \sin(x_5 - x_3) + B_3}{B_8 \cos(x_3)} - \frac{(B_4 \cos(x_5 - x_3) + B_5) \sin(x_5 - x_3)}{B_8 \cos(x_3) \cos(x_5 - x_3)^2} + \frac{B_6 \cos(x_5 - x_3) + B_7}{B_8 \cos(x_3) \cos(x_5 - x_3)^2}, \quad (13)$$

where $B_1 \div B_8$ are equality:

$$\begin{aligned} B_1 &= T_1 T_2 T_3 q S b_a x_1^* (-c_x(\alpha) \cdot m_z^{\delta_\epsilon} x_1 + c_x^{\delta_\epsilon} b_a m_z^{\delta_\epsilon} x_4 + c_x^{\delta_\epsilon} m_z(\alpha) \cdot x_1); \\ B_2 &= T_1 T_3 I_z c_x^{\delta_\epsilon} x_1 x_1^* x_4 + T_2 T_3 m b_a m_z^{\delta_\epsilon} (x_1^2 x_1^* - (x_1^*)^2 x_1); \\ B_3 &= -T_1 T_2 T_3 b_a m_z^{\delta_\epsilon} x_1 x_1^* (m_{CH} g + \sum_{i=1}^n N_i); \\ B_4 &= T_1 I_z c_x^{\delta_\epsilon} x_1 (T_2 x_1 + T_3 x_1^* + x_1^*); \\ B_5 &= T_2 x_4 B_5; \quad B_6 = T_1 I_z c_x^{\delta_\epsilon} x_1 (x_2 - x_2^*); \\ B_7 &= T_1 T_2 I_z c_x^{\delta_\epsilon} x_1 x_1^* x_4 (T_3 + 1); \\ B_8 &= T_1 T_2 T_3 b_a m_z^{\delta_\epsilon} x_1 x_1^*. \end{aligned} \quad (14)$$

Place the expressions (11–14) into mathematical model of the object (4) and obtain closed-loop system. It depends on state variables. For simulation, set regulator parameters and choose invariants.

3.3 Simulation

Initial data for modeling. Invariants: $H^* = 10000$ m –release point altitude; $V^* = 800$ км/ч – flight speed at release point. Initial condition: $V(0) = 100$ m/s; $H(0) = 400$ m; $\alpha(0) = 6$ deg; $\omega_z(0) = 0$ deg/s; $\mathcal{G}(0) = 10$ deg; $x(0) = 15000$ m. Regulator parameters: $T_1 = 8$ s; $T_2 = T_3 = |(x_2^* - x_2(0)) / 20|$ s.

Transient process. The closed-loop nonlinear system of differential equations (4), (11), (13) is numerically solved in software «Maple» by using Runge-Kutta method with 4th order. The results are shown in Fig. 2 – 9. Figures 2 – 6 shows the dependence of the phase coordinates from time, and Fig. 7 – 8 – control actions. Aerospace system achieves predetermined flight altitude value (Fig. 2) and flight altitude (Fig. 3), angular velocity of pitching is fading out (Fig. 7), phase portrait (Fig. 9) shows, that movement of closed-loop system is asymptotically stable at whole region of phase space for various combinations of initial conditions. Exceptions is the point at which the mathematical model of the object is not defined (at an angle of attack $x_3 = \pi / 2$).

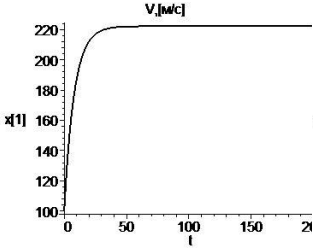


Fig. 2. Flight velocity of aerospace system, m/s

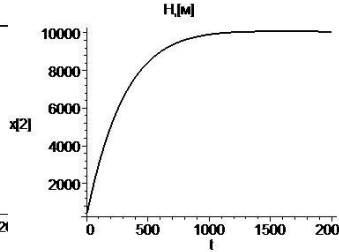


Fig. 3. Flight altitude, m

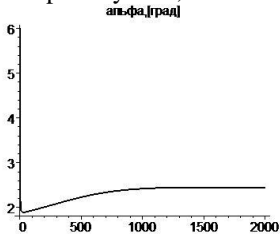


Fig. 4. Angle of attack, deg.

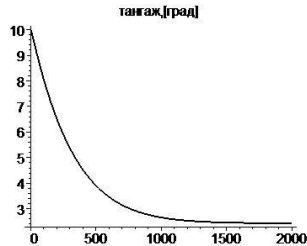


Fig. 5. Pitch angle, deg

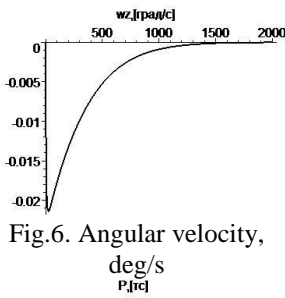


Fig. 6. Angular velocity, deg/s

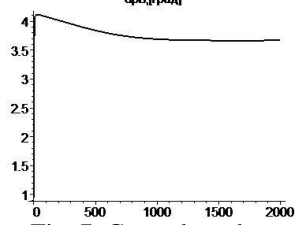


Fig. 7. Control u_1 , deg.

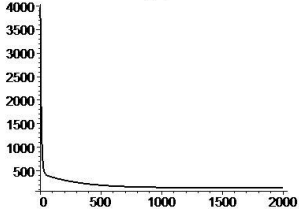


Fig. 8. Control u_2 , t*s

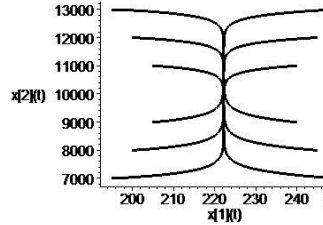


Fig. 9. Phase portrait,
 $x_1^* = 222$ m/s; $x_2^* = 10000$ m

Supplement mathematical model (4) with integrating element and write an extended model:

$$\begin{aligned} \dot{x}_1(t) &= \frac{u_2}{m} \cos(x_3) - g \sin(x_5 - x_3) - \\ &- \sum_{i=1}^n N_i \frac{\sin(x_5 - x_3)}{m} - \frac{qS}{m} (c_x(\alpha) + c_x^{\delta_e} u_1); \\ \dot{x}_2(t) &= x_1 \sin(x_5 - x_3) + z_1; \\ \dot{x}_3(t) &= x_4 - \frac{u_2 \sin(x_3)}{m x_1} + \frac{g \cos(x_5 - x_3)}{x_1} + \\ &+ \sum_{i=1}^n N_i \frac{\cos(x_5 - x_3)}{m x_1} - \frac{qS(c_y(\alpha) + c_y^{\delta_e} u_1)}{m x_1}; \\ \dot{x}_4(t) &= \frac{qS b_a}{I_z} \left[m_z(\alpha) + m_z^{\delta_e} \frac{b_a x_4}{x_1} + m_z^{\delta_e} u_1 \right]; \\ \dot{x}_5(t) &= x_4; \quad \dot{x}_6(t) = x_1 \cos(x_5 - x_3); \\ \dot{z}_1(t) &= k(x_2 - x_2^*), \end{aligned} \quad (15)$$

where z_1 – dynamic variable, estimation of external non-measurable disturbance, k – constant coefficient. The last equation in the system (15) is a dynamic model of non-measurable disturbance, acting on the system. In drawing up the disturbance model takes into account the requirement of technological invariant. Mathematical model is taking into account requirement of technological invariant $H = H^*$. Synthesis procedure is similar to procedure, shown in section 3. Write invariant manifolds:

$$\begin{aligned} \psi_1 &= x_1 - x_1^* = 0; \\ \psi_2 &= x_4 - \varphi_1 = 0. \end{aligned} \quad (16)$$

They must satisfy the system of functional equations:

$$\begin{aligned} T_1 \cdot \dot{\psi}_1 + \psi_1 &= 0; \\ T_2 \cdot \dot{\psi}_2 + \psi_2 &= 0, \end{aligned} \quad (17)$$

where: T_1, T_2 – time constant. Decomposed system is assuming an aspect:

4 Separating stage

While separating stage, connections are uncoupling and total force in attachment point of upper-stage rocket to launch aircraft is equal to zero. Due to this, launching aircraft is influenced by piecewise constant.

Setting up a problem. It is required to synthesize a control vector in analytical form, providing following invariants $V = V^*$, $H = H^*$ and parrying indeterminate external disturbances.

$$\begin{aligned} \dot{x}_2(t) &= x_1^* \sin(x_3 - x_3) + z_1; \\ \dot{x}_5(t) &= \varphi_1; \\ \dot{z}_1(t) &= k(x_2 - x_2^*). \end{aligned} \quad (18)$$

For the system (18) introduce invariant manifold ψ_3 :

$$\psi_3 = x_1^* \sin(x_3 - x_3) + z_1 + x_2 - x_2^* = 0 \quad (19)$$

φ_1 finding from the equation:

$$T_3 \cdot \dot{\psi}_3 + \psi_3 = 0, \quad (20)$$

Internal control

$$\begin{aligned} \varphi_1 &= (1 + \frac{1}{T_3}) \operatorname{tg}(x_3 - x_5) - \\ &- \frac{(T_3 + 1)z_1 + (T_3 k + 1)(x_2 - x_2^*)}{T_3 x_1^* \cos(x_3 - x_5)}. \end{aligned} \quad (21)$$

Equations (16), (19), functional equation (17), (20) and model equation (15) are resolved jointly, by using ADAR method. Obtaining expressions for the control actions, which depend on variables of system state.

$$\begin{aligned} u_1 &= f(x_1 \div x_5, z_1); \\ u_2 &= f(x_1 \div x_5, z_1). \end{aligned} \quad (22)$$

Place the expressions (22) into mathematical model of the object (15) and obtain closed-loop system for the simulation. For simulation, instead estimation of disturbance z_1 in the right side of the equation controlled variable x_2 set piecewise constant disturbance: $\Delta H = 10m$.

Invariants: $H^* = 9950m$, $V^* = 215m/c$ – altitude and flight speed of launching aircraft after separations stage. Initial condition: $V(0) = 222$ m/s ; $H(0) = 10000$ m; $\alpha(0) = 2.4$ deg; $\omega_z(0) = 0$ deg/s; $\mathcal{G}(0) = 2.4$ deg; $x(0) = 400$ km. Regulator parameters: $T_2 = T_3 = |(x_2^* - x_2(0)) / 20|$ s.

Transient processes are presented at Fig. 10–15.

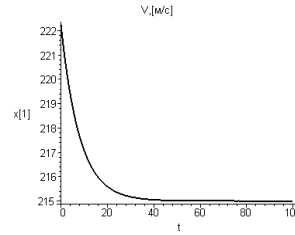


Fig. 10. Flight speed of launching aircraft, m/s

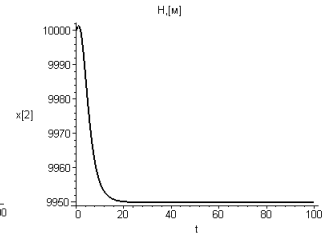


Fig. 11. flight altitude of launching aircraft, m

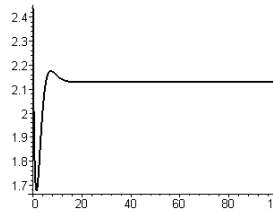


Fig. 12. Angle of attack, deg.

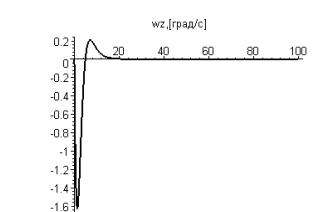


Fig. 13. Angular velocity, %c

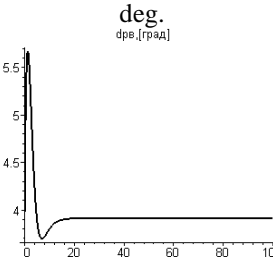


Fig. 14. Control u_1 , deg.

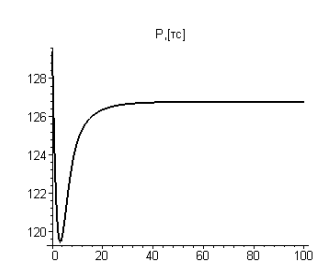


Fig. 15. Control u_2 , t's

The simulation results shows, that the synthesized astatic regulator ensures the asymptotic stability of the closed system relative to the invariants $V = V^*$, $H = H^*$, as well as invariant affects to external unmeasured piecewise constant disturbance.

5 Reaching hypervelocity

At this stage, upper-stage rocket with spaceplane aboard are reaching hypervelocity $M=5$, by using reactive thrust. Reactive thrust is a result of separation particles with elementary mass $|dmI|$ from the control object. Mass of the control object m is a variable value. Its motion is described by the vector equation of Meshcherskiy [11, 12]:

$$m \frac{d\vec{V}}{dt} = \vec{P}_r + \vec{F}, \quad (23)$$

Mass m is time-varying function:

$$m(t) = m_0 + m_1(t), \quad (24)$$

where $m_0 = \text{const}$ – unvarying part of mass;

$m_1(t) \geq 0$ – reaction mass;

P_r – reactive force:

$$P_r = a_r \cdot \frac{dm_1}{dt}, \quad (25)$$

where a_r – relative velocity of separating particles. Equation of Meshcherskiy (23) is projected to the wind-body axes coordinate system and using differential equation system (4). Supplement differential equation system with mass variation equation, as a result of separation particles Write mathematical model of jet propulsion of upper-stage rocket with spaceplane aboard:

$$\begin{aligned} \dot{x}_1(t) &= -g \sin(x_5 - x_3) + \frac{2a_r u_2}{m_{t0} - 2x_7} \times \\ &\times \cos(x_3) - \frac{qS}{m_{t0} - 2x_7} (c_x(\alpha) + c_x^{\delta_e} u_1); \\ \dot{x}_2(t) &= x_1 \sin(x_5 - x_3); \\ \dot{x}_3(t) &= x_4 + g \frac{\cos(x_5 - x_3)}{x_1} - \\ &- \frac{2a_r u_2}{(m_{t0} - 2x_7)x_1} \sin(x_3) - \\ &- \frac{qS}{(m_{t0} - 2x_7)x_1} (c_y(\alpha) + c_y^{\delta_e} u_1); \\ \dot{x}_4(t) &= \frac{qS b_a}{I_z} \left[m_z(\alpha) + m_z^{\bar{\omega}_z} \frac{b_a x_4}{x_1} + m_z^{\delta_e} u_1 \right]; \\ \dot{x}_5(t) &= x_4; \quad \dot{x}_6(t) = x_1 \cos(x_5 - x_3); \\ \dot{x}_7(t) &= u_2, \end{aligned} \quad (26)$$

where $x_1 = V$, $x_2 = H$, $x_3 = \alpha$, $x_4 = \omega_z$,
 $x_5 = \mathcal{G}$, $x_6 = x$, $x_7 = m_1$ – state variables;
 m_{t0} – mass of the system at initial;
 $u_1 = \delta_e$; $u_2 = \dot{m}_1$ – controls.

Control target – acceleration of upper-stage rocket with spaceplane aboard to a hypersonic speed at predetermined altitude:

$$H = H^*, V = V^*.$$

Setting up a problem. It is required to find a control vector in analytical form, as coordinate function of state system (26), providing

achievement of upper-stage rocket desirable hypervelocity V^* with predetermined flight altitude H^* . Synthesis procedure is similar to procedure, shown in section 3. Control actions u_1, u_2 depend on state variables:

$$\begin{aligned} u_1 &= f(x_1 \div x_7); \\ u_2 &= f(x_1 \div x_7). \end{aligned} \quad (27)$$

Initial condition: $H^* = 10000$ m; $V^* = 1498$ m/s;
 $V(0) = 222.2$ m/s; $H(0) = 9980$ m; $\alpha(0) = 3$ deg;
 $\omega_z(0) = 0$ deg/s; $\mathcal{G}(0) = 3$ deg; $x(0) = 400$ km;
 $T_1 = 4$ s; $T_2 = T_3 = 1$ s.

Simulation results are presented at Fig. 16-25. Figures 24 and 25 shows the mass variation of entire system and consumed fuel weight.

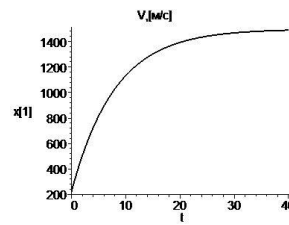


Fig. 16. Flight speed of upper-stage rocket, m/s

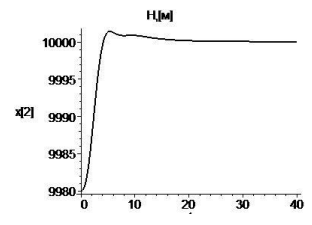


Fig. 17. Flight altitude of upper-stage rocket, m

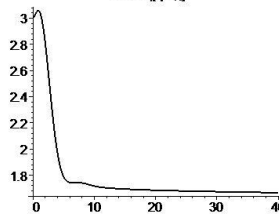


Fig. 18. Pitch angle, deg.

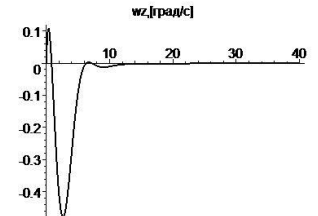


Fig. 19. Angular velocity °/c

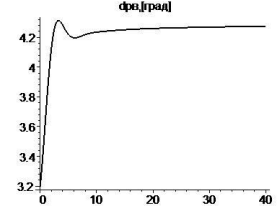


Fig. 20. Control u_1 , deg.

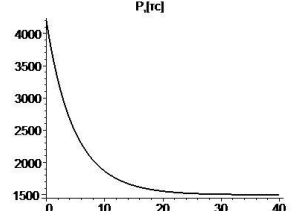


Fig. 21. Control u_2 , t/s

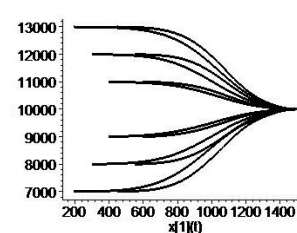


Fig. 22. Phase portrait,
 $x_1^* = 1498$ m/s; $x_2^* = 10$ km

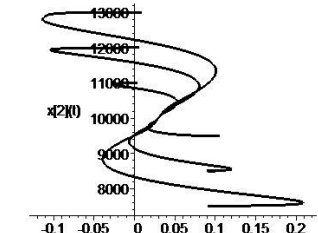


Fig. 23. Phase portrait,
 $x_2^* = 10000$ m

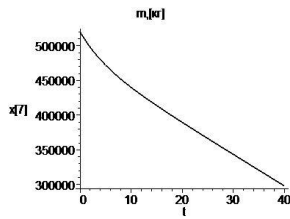


Fig. 24. Mass variation of entire system, kg

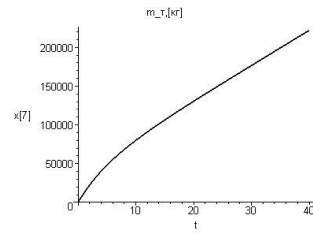


Fig. 25. Fuel consumption, kg

6 Conclusion

Procedure of analytical synthesis of vector controls strategy of aerospace system was developed, by using nonlinear model of longitudinal movement of the launching aircraft and jet propulsion model of upper-stage rocket. Control laws provide asymptotic stability of closed-loop nonlinear system at each stage of flight and implement predetermined control target:

- lift of aerospace system at release point with predetermined flight altitude H^* and flight speed V^* ;
- restraining the predetermined flight altitude and speed at separating stage of upper-stage rocket with spaceplane aboard from launching aircraft and parrying indeterminate external disturbances;
- reaching hypervelocity of the upper-stage rocket with spaceplane aboard.

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