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DESIGNING OF AIRFRAMES HAVING MINIMUM MASS UNDER SEVERAL LOADINGS (NOT ONLY FULLY STRESSED)

Alexander I. Danilin
Samara State Aerospace University,
Russia, 443086, Samara, Moskovskoye Shosse, 34.
Mobile: +7-937-204-4950; Fax: +7-334-5683; E-mail: alexdan@ssau.ru
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Abstract

We discuss the features of optimal force transfer and use them to develop the method for discovering the distribution of material of structure for obtaining its minimal volume. The suggested method works with one or several external loadings and allows one to discover the optimal structure and to recognize the phenomenon of Razani in which a structure with unequal stresses due to several loadings is more light-weighted than fully stressed. With using the developed method we have solved the original Razani problem and apply it for wing optimization of the novel hypersonic airplane.

1 General Theory

The function of any structure is transfer (delivery) of forces from points of their application to points of their absorption (neutralization). From the analysis, we can use the analogy with thermodynamic system [1], which transfers heat from sources to sinks. From this, we can represent any structure having stationary loading as a stationary nonequilibrium system through which one can transfer the forces, and that the "quantity" of forces in structure is independent of time.

Hereafter, we will use the terminology from thermodynamics in application to mechanics of solids. The relations between used terms and classical notions of mechanics of solids can be discovered easily.

Internal energy in structure is the quantitative measure of external loading transfer and depends on internal forces in structural elements:

$$E = E\,(F_1,\, F_2,\, \ldots\, F_m). \tag{1}$$

Internal forces F_i, i=1,2,...m are derived from external loading and they define the effectiveness of external loading transfer and also the structure optimality, as the system is responsible for transfer.

Structure changing leads to changing of internal forces, and it follows that strain energy also will be changed. This energy change caused by structure changing can be defined as:

$$dE = U_1\, dF_1 + U_2\, dF_2 + \ldots + U_m\, dF_m\ ; \tag{2}$$

where

$$U_1 = \frac{\partial E}{\partial F_1};\ \ U_2 = \frac{\partial E}{\partial F_2};\ \ \ldots\ \ U_m = \frac{\partial E}{\partial F_m}. \tag{3}$$

We name magnitudes U_i, i = 1,2,...m as internal force potentials. Force potential difference defines the intensity of force transfer. Inasmuch as force transfer happens due to force potential difference, we can record:

$$dF_i = \sum_{r=1}^{m} K_{ir}\, dU_r\ ; \tag{4}$$

where

$$K_{ir} = \frac{\partial F_i}{\partial U_r}, \quad r = 1,2,\ldots m; \tag{5}$$

defines ***the conduction of the structural element*** r = 1,2,…m; ***concerning force*** $\boldsymbol{F_i}$ .The force potentials in turn are the functions of structure condition; therefore,

$$U_i = U_i\,(F_1,\, F_2,\, \ldots\, F_m). \tag{6}$$

Subsequently, change of i-th potential by structure changing can be expressed as:

$$dU_i = \sum_{r=1}^{m} C_{ir}\, dF_r\ ; \tag{7}$$

where values

$$C_{ir} = \frac{\partial U_i}{\partial F_r}, \quad r = 1,2,...m; \tag{8}$$

define the quantitative estimation of influence of the internal forces on potentials.

Comparing (5) and (8), we get:

$$C_{ri} = 1/K_{ir}\ ;\ i = 1,2,\ldots m;\ r = 1,2,\ldots m\ . \tag{9}$$

Magnitude, opposite to the conduction of the structural element $r = 1,2,\ldots m$ concerning force F_i, will be named ***resistance to transfer of the forceF$_i$***.

In contrast to classical understanding of strength of materials and structures, here we put another sense in concept of resistance to transfer of forces, namely, resistance to actual force transfer process. So, in a hole, the conduction of a structure concerning forces is zero, and resistance to force transfer is infinite; on the contrary, the absolute rigid body has zero resistance to force transfer and infinite conduction.

Optimality of a structure as a system for external loading transfer is directly connected with the concepts of force conduction and resistance to transfer of force.

Hypothesis 1. With a single loading, the optimal structure, made from a given material volume, is the structure with minimal resistance (maximal conduction) to external loading transfer.

Hypothesis 2. With several loadings, the optimal structure, made from a given material volume, will have minimal value of the sum of resistances to force transfer of each external loading.

If it is possible to express analytically the energy accumulated in a structure, as well as resistance to transfer external force, in the form of functions from applied forces then we can find material distribution on structural elements directly. Generally, it is impossible to define such an explicit relation; therefore, we will continue with problem arguing.

2 Linearly elastic system

As an universal method for computation of strain-stress distribution of elastic body, we will use the finite-element method, which is applied now very widely and is accessible in the form of commercial software packages.

Let there be a finite-element model of an investigated structure. The energy of transfer of ***external loading*** from points of their application to points of their absorption equal strain energy and in terms of finite-element method is defined as:

$$E = \{\mathbf{R}\}^T[\mathbf{C}]\{\mathbf{R}\}, \tag{10}$$

where $\{\mathbf{R}\}$ is the vector of an external loading and $[\mathbf{C}]$ is the flexibility matrix. In consideration of the process of external loading transfer in accordance with (2) - (3), and (8), we define resistance of structure to transfer the external loading as a second derivative from transfer energy on the acting loading:

$$\frac{d^2E}{d\{\mathbf{R}\}^2} = [\mathbf{C}]\ . \tag{11}$$

Equation (11) defines a flexibility matrix as the numerical characteristic of resistance to transfer external forces, but does not allow discovering the required values of its coefficients, which means the required values of structural element parameters for optimal transfer of real set of loadings.

Hence, although a structure should transfer external forces, its energy value and resistance to transfer is necessary to define as a function of internal forces, secondary on an external loading.

Let us turn to (11). Matrix magnitude in accordance with theory of matrix algebra in common case is evaluated as:

$$\|\,[\mathbf{C}]\,\| = \max_{\{\mathbf{x}\}\neq 0}\frac{\|\,[\mathbf{C}]\{\mathbf{x}\}\,\|}{\|\,\{\mathbf{x}\}\,\|}\,; \tag{12}$$

where $\|\ldots\|$ means some norm of matrix or vector. We need to minimize the resistance to transfer concrete loading; therefore, as a vector $\{\ \mathbf{x}\ \}$, we take the vector of external loading $\{\mathbf{R}\}$ and then

$$\|\,[\mathbf{C}]\,\| = \frac{\|\,[\mathbf{C}]\{\mathbf{R}\}\,\|}{\|\,\{\mathbf{R}\}\,\|} = \frac{\|\,\{\mathbf{d}\}\,\|}{\|\,\{\mathbf{R}\}\,\|}; \tag{13}$$

where $\{\mathbf{d}\}$ is the vector of elastic displacements (deformations) from loading $\{\mathbf{R}\}$.

Equation (13) gives an integral criterion of resistance to transfer external forces and expresses the dependence of resistance to transfer from both external forces **{R}** and material distribution among structural elements.

Due to several loadings, according to Hypothesis 2, the magnitude, which is subject to minimization for reaching an optimality of a structure, taking into account (13), will be defined as

$$C_\Sigma = \sum_{j=1}^{k} \frac{\| \{\mathbf{d}\}_j \|}{\| \{\mathbf{R}\}_j \|}; \qquad (14)$$

here j = 1,2,…k is the loading number.

Let us verify our theses and Hypothesis 2 on the Razani (1965) problem. Three-rod truss takes two loads R^I and R^{II}, which act at different times. Internal forces in the rods 1, 2, 3 from loadings R^I and R^{II} and other conditions are shown in Fig. 2. Here, S is the cross-section area of rods and E^*, ρ are the elasticity modulus and density of the rod's material.

3 General Algorithm for Linear-Elastic System Designing

Let there be a structure with arbitrary distribution of the material in which all elements work in proportional elastic limits by all applied loadings. Let also there be the finite-element model of this structure, allowing one to compute vectors of elastic displacements $\{\mathbf{d}\}_j$, where j = 1, 2,…k is the loading number. We accept as norms of vectors $\{\mathbf{d}\}_j$ the sum of absolute values of their components:

$$\| \{\mathbf{d}\}_j \| = \sum_{l=1}^{n} | d_l |_j \qquad (15)$$

Let us consider vector $\{\overline{\mathbf{R}}\}_j$, all components in which are equal (±1), and the sign of $\{\overline{\mathbf{R}}\}_j$ component coincides with the sign of the matching component of vector $\{\mathbf{d}\}_j$. Then, (15) can be written as:

$$\| \{\mathbf{d}\}_j \| = \sum_{l=1}^{n} | d_l |_j = \{\overline{\mathbf{R}}\}_j^T [\mathbf{C}] \{\mathbf{R}\}_j \, . \qquad (16)$$

From the analysis of (16), it follows that if vector $\{\overline{\mathbf{R}}\}_j$ is accepted as a unit load and for computing generalized displacement d_j^*, its value will be equal to accepted norm of vector $\{\mathbf{d}\}_j$. On the other hand, the generalized displacement can be computed using internal forces in structural elements with the help of Maxwell-Mohr integrals[1]. For the determinacy, we suppose that structural elements work in two-dimensional stress state and then:

$$\| \{\mathbf{d}\}_j \| = \sum_{l=1}^{n} | d_l |_j = \{\overline{\mathbf{R}}\}_j^T [\mathbf{C}] \{\mathbf{R}\}_j = \sum_{i=1}^{m} \frac{A_i \left[F_{ij}^* \right]}{\delta_i \, E_i^*} ; \qquad (17)$$

$$\left[F_{ij}^* \right] = \overline{F}_{ijx} \, (F_{ijx} - \mu_i F_{ijy}) + \overline{F}_{ijy} \, (F_{ijy} - \mu_i F_{ijx}) + 2 \, (1 + \mu_i) \, \overline{F}_{ijxy} \, F_{ijxy} \; ;$$

where $\overline{F}_{ij}$ is the line force in i-th structural element from unit loading $\{\overline{\mathbf{R}}\}_j$, corresponding to elastic displacement $\{\mathbf{d}\}_j$; F_{ij} is the line force in i-th structural element from design loading $\{\mathbf{R}\}_j$; A_i is the element plan area; δ_i is the thickness of i-th element; E_i^*, μ_i are the material elasticity modulus and Poisson's ratio; and m is the quantity of elements. Line force is computed as $F_i = \delta_i \, \sigma_i$.

Thus, the magnitude of resistance to transfer external loading $\{\mathbf{R}\}_j$ is defined by internal forces in structural elements and (13) taking into account (17) will become:

$$\| [\mathbf{C}] \| = \frac{\| \{\mathbf{d}\}_j \|}{\| \{\mathbf{R}\}_j \|} = \frac{1}{\| \{\mathbf{R}\}_j \|} \sum_{i=1}^{m} \frac{A_i \left[F_{ij}^* \right]}{\delta_i \, E_i^*} \, . \qquad (18)$$

Due to several loadings, (14) defined the total resistance of the structure in accordance with Hypothesis 2 and taking this into account (17), will be converted to the form:

$$C_\Sigma = \sum_{j=1}^{k} \frac{\| \{\mathbf{d}\}_j \|}{\| \{\mathbf{R}\}_j \|} = \sum_{j=1}^{k} \frac{1}{\| \{\mathbf{R}\}_j \|} \sum_{i=1}^{m} \frac{A_i \left[F_{ij}^* \right]}{\delta_i \, E_i^*} \, . \qquad (19)$$

As the result, we have the following problem of constrained minimization: to find such material distribution of the given mass M_0 among structural elements, which provides

$$C_\Sigma \Rightarrow \min \; ; \qquad (20)$$

by

$$\sum_{i=1}^{m} \rho_i \, A_i \, \delta_i - M_0 = 0 \, . \qquad (21)$$

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[1] This is also referred to as the unit load or dummy load method.

On the step of developing recurrent relationships, we will suppose that internal force in structural elements does not depend on material distribution. We take into account this dependence later by development of algorithm for searching the structure with minimum of resistance to transfer external loadings.

We will solve the minimization problem using the Lagrange multiplier method. Here, the Lagrange function is

$$Lg = C_\Sigma + \lambda \left(\sum_{i=1}^{m} \rho_i \, A_i \, \delta_i - M_0 \right) ; \qquad (22)$$

where λ is the Lagrange multiplier. Conditions of its minimum have the form:

$$\begin{cases} \dfrac{\partial Lg}{\partial \delta_i} = -\dfrac{1}{\delta_i^2 E_i^*} \sum_{j=1}^{k} \dfrac{\left[F_{ij}^* \right]}{\| \{\mathbf{R}\}_\mathbf{j} \|} + \lambda \rho_i = 0 \; ; \\ \qquad\qquad i = 1,2,\ldots m \; ; \\ \dfrac{\partial Lg}{\partial \lambda} = \sum_{i=1}^{m} \rho_i \, A_i \, \delta_i - M_0 = 0. \end{cases} \qquad (23)$$

The first m equations of system (23) define the conditions of minimum of a structure resistance to transfer of external loadings $\{R\}_j$, j=1,2,…k. These conditions are considered as the optimality criteria of a structure:

$$\frac{1}{\rho_i \; \delta_i^2 \; E_i^*} \sum_{j=1}^{k} \frac{\left[F_{ij}^* \right]}{\| \{\mathbf{R}\}_\mathbf{j} \|} = \lambda = const \; ; \qquad i = 1,2,\ldots m \; ; \qquad (24)$$

Expressing δ_i from (24) and substituting it in the last equation of system (23), we find the λ value:

$$\lambda = \frac{\left(\sum_{l=1}^{m} A_l \sqrt{\dfrac{\rho_l}{E_l^*} \sum_{j=1}^{k} \dfrac{\left[F_{lj}^* \right]}{\| \{\mathbf{R}\}_\mathbf{j} \|}} \right)^2}{M_0^2} \; . \qquad (25)$$

We put λ in (24) and find the required thicknesses of structural elements δ_i, which by accepted assumption give the function C_Σ a minimal value:

$$\delta_i = \frac{M_0}{\sum_{l=1}^{m} A_l \sqrt{\dfrac{\rho_l}{E_l^*} \sum_{j=1}^{k} \dfrac{\left[F_{lj}^* \right]}{\| \{\mathbf{R}\}_\mathbf{j} \|}}} \left[\frac{1}{\sqrt{\rho_i E_i^*}} \sqrt{\sum_{j=1}^{k} \frac{\left[F_{ij}^* \right]}{\| \{\mathbf{R}\}_\mathbf{j} \|}} \right] , \qquad i = 1,2,\ldots m. \qquad (26)$$

From (26), we can see that new thickness of structural element δ_i is necessary to take proportionally to square root from sum of normalized to loadings subintegral functions of Maxwell-Mohr integrals, divided on the product of density by elasticity modulus of structural element. Do all loadings in an equal measure define material distribution in structure?

Definition. We will name dimensioning such loadings, whereby at least one structural element has stress state exceeding stress state in this element from other loadings.

Because the internal force distribution in structural elements in common case depends on material distribution, the set of dimensioning loadings also depend on material distribution and can be changed by iterations.

Now, we have the complete set of theoretical prerequisites for development of algorithm for designing a linear-elastic system. As the method for stress and strain state calculation, we will use the finite-element method; therefore, all described operations are relative to finite-element model of the structure.

1. Let there be given the material of the structural elements and its some initial mass M_0. The external loadings also are given and they are stationary and do not depend on material distribution.

2. Let us assign some initial material distribution among structural elements δ_{0i}. We suppose that a structure consists of membrane structural elements having two-dimensional state of stress and inside the element each kind of stress is the same at all points of the element.

3. For each loading, to compute the elastic displacements $\{\mathbf{d}\}_\mathbf{j}$, j = 1,2,…k, the stresses in structural elements include equivalent stress. For membrane elements, we have:

$$\sigma_{ij}^{equ} = \sqrt{\sigma_{ijx}^2 + \sigma_{ijy}^2 - 2 \, \mu_i \; \sigma_{ijx} \sigma_{ijy} + 2 \, (1 + \mu_i) \; \sigma_{ijxy}} \; ; \qquad (27)$$

4. Using σ_{ij}^{equ} to select t dimensioning loadings in accordance with Definition, $t \le k$.

5. For dimensioning loadings to form the unit loading vectors $\{\overline{\mathbf{R}}\}_\mathbf{j}$, j=1,2,…k and computing

subintegral functions $[F_{ij}^*]$ of Maxwell-Mohr integrals, see (17).

6. To compute by (24) optimality criterions λ_i for all structural elements i=1,2,…m, select their maximal and minimal values. If

$$\left(\frac{\lambda_{max}}{\lambda_{min}} - 1\right) \le \varepsilon \, , \qquad (28)$$

where ε is the small prescribed value, then go to point 9.

7. To calculate the new thicknesses δ_{1i} by (26) for t selected loadings.

8. To assign the thicknesses δ_{1i} as initial δ_{0i}, go to point 3.

9. Exit from iterations. Here, it is necessary to modify the obtained material distribution in accordance with strength requirements for structural elements. For it, we find the multiplier of thickness changing

$$K_\sigma = \max_{i=1,\dots m}\left[\max_{j=1,\dots t}\left(\frac{\sigma_{ij}^{equ}}{[\sigma]_i}\right)\right] ; \qquad (29)$$

where $[\sigma]_i$ are the allowable strengths and define their new values:

$$\delta_i^\sigma = K_\sigma \, \delta_i \, ; \qquad (30)$$

where δ_i are thicknesses, found in iterations 3-8. Because we proportionally change the thickness of all structural elements, saving their proportions, the relative force distribution in structure will not vary. Here, we can define the mass of such structures:

$$M^\sigma = \sum_{i=1}^{m} \rho_i \, A_i \, K_\sigma \, \delta_i = K_\sigma \, M_0 \, . \qquad (31)$$

4 Testing of Method: Razani Problem [2]

Let us verify our theses and Hypothesis 2 on the Razani [2] problem. Three-rod truss takes two loads $\boldsymbol{R}^I$ and $\boldsymbol{R}^{II}$, which act at different times, see Fig. 1.

Razani this problem solved by the following initial data: elasticity modulus of the rod's material $\boldsymbol{E}^*$ = 1; density ρ = 1; external loads $\boldsymbol{R}^I$ = $\boldsymbol{R}^{II}$ = 1; allowable stresses [σ] = 1; length of the rod No. 2 is $\boldsymbol{L}$ = 1.

Structure of uniform strength by $\boldsymbol{S_2} = \boldsymbol{0}$ and $S_1 = S_3 = 1$ has mass $\boldsymbol{M}$ = **2.828** units while optimal structure has the following parameters: $S_2 = 0.408$; $S_1 = S_3 = 0.789$; $S_1/S_2 = 1.93$; mass of rods $\boldsymbol{M_{opt}}$ = **2.638** units. At that, strength requirements are NOT violated, but equal stress in all rods is not achieved; central rod No. 2 by both loadings has the stress, which is less than allowable.

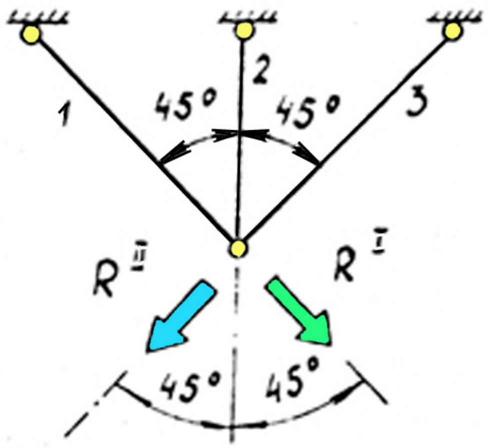

Fig. 1. Razani truss.

Our algorithm has solved this problem very easy and completely repeats Razani solution [2].

Now methods of optimization are used in aircraft industry not only for a parametric optimization, for example for determination of optimum distribution of a material among structural elements within the given framework, but also for structural optimization when it is necessary to define the framework of airframe. The usual plan of action at discovering the framework consists of the following.

Into geometrical area where the future airframe should be disposed, we place the continual model, which contains all possible frameworks. As usual, it is a finite-element model. To the given model, we apply all spectrums of loadings and model the supporting conditions. We assign some initial material distribution and start the iteration algorithm of this or that optimization method for discovering material distribution. During the job of the algorithm, the material in model is being redistributed: some areas increase the rigidity and others are degenerated. As a result, we discover the framework of an airframe, which shows paths and force kind (extension-compression, shift) by which the exterior loadings are transmitted from application points to points of their absorption (neutralization). On the basis of the analysis of this information, the designer suggests the framework of an airframe and implements its parametric optimization.

Truss structures in accordance with their nature are already the frameworks of airframes as force can go only along rods and if the designer has not made the path, that is, has not specified a rod, then for force this possible path is closed. Continual models with the elements, which have two-dimensional or three-dimensional stress state, have more degrees of freedom. Such models give for external loadings more possibilities in path selection from points of their application to points of their absorption (neutralization). Therefore, no wonder that the question appeared: whether applied methods of synthesis of frameworks out of continual models can distinguish the phenomenon of Razani [2] and, at its presence, to discover framework with unequal stresses when several loadings act. Let us compare the results of the offered theory and algorithm of searching the framework on a problem shown in Fig. 2.

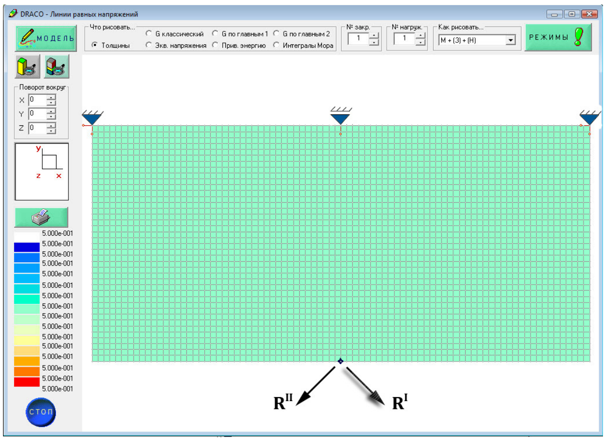

Fig. 2. Razani problem

Optimization was implemented using the author's package "DRACO," realizing finite-element method and various algorithms of optimization, based on optimality criteria.

Results of optimization are shown in Fig. 3 and Fig. 4.

It is visible that the developed algorithm allows one to discover effective frameworks and to recognize the phenomenon of Razani [2] that a structure with unequal stress by several loadings is easier than the structure with equal maximal stresses in elements. In the considered example, the distribution of a material in Fig. 4 on 5.046% is easier than the “fully stressed” distribution of a material shown in Fig. 3.

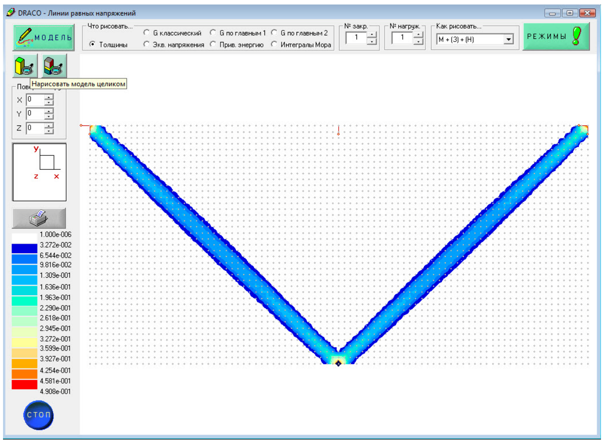

Fig. 3.“Fully stressed” material distribution.

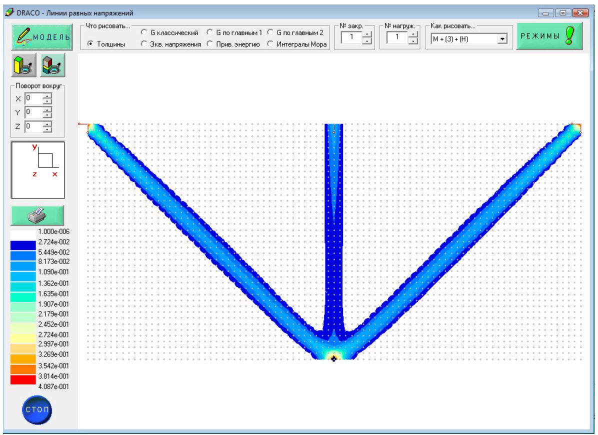

Fig. 4. Achieved material distribution

5 Hypersonic Airplane Preliminary Design

Commercial aviation development always links with reduction of time of transportation onto the large distances. The modern park of passenger airplanes consists from the subsonic aircrafts flying at altitudes of 10-12 km with velocities of 800-950 km\h. Thus, flights on distance of 6000-8000 km take 8-12 hours, that in the modern dynamical world of business and mobility any more do not satisfy the necessities of passengers.

New passenger airplanes, which appear each 4-5 years are more effective, however they do not have essential new, breakthrough technologies. Aerodynamic properties of wing and performance of engines are improved; the noise levels and ecological contaminations are reduced; the seat of the passenger now looks like the individual center of entertainments, but at the same time, there is no revolutionary transition onto a new technological level of passenger transportations.

If problems of supersonic flight with velocities of 2000-2500 km\h are solved successfully enough, then increasing a velocity to 4000-10000 km\h comes across the new physical phenomena that demand new ideas and technological researches, and not ordinary designer solutions. But only by this way, it is possible to achieve ***breakthrough*** in technology of hypersonic airline traffic.

Let's consider the problem to design exterior forms equally effective on take-off and landing and on cruiser hypersonic velocity. The large quantities of researches are directed to aerodynamics of a hypersonic cruising flight, but nobody consider a problem of reaching of this hypersonic velocity. Researchers suggest optimal aerodynamic forms for various diapasons of hypersonic velocities, however the airplane begins moving from a zero speed, and suggested aerodynamic forms on subsonic speeds simply will not ensure the necessary ascentional force.

We suggest the new scheme of airplane which can satisfy different requirements for wing performances on different flight modes, see Fig. 5, 6.

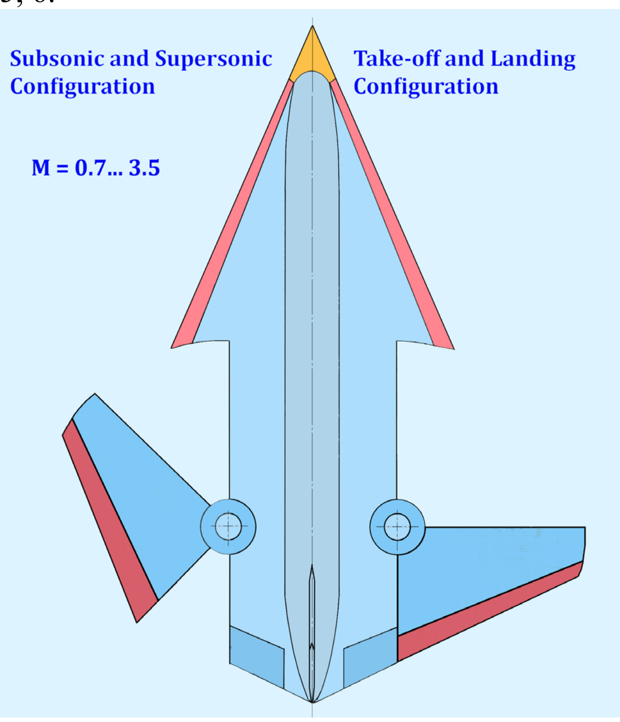

Fig. 5. Configuration for initial stages of flight

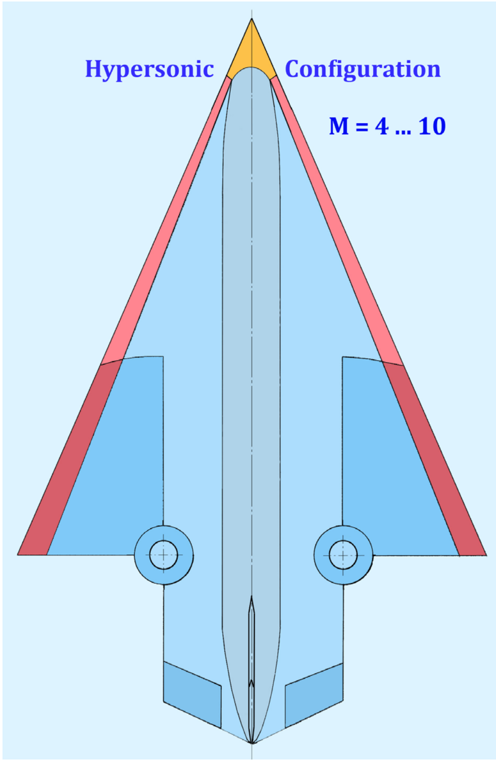

Fig. 6. Hypersonic cruising configuration

The offered construction has following singularities.

1. Elevons in a take-off/landing configuration turn to a deflected nose in a hypersonic configuration for control of a shock wave adjoining at various Mach numbers.
2. In subsonic and transonic configuration all loadings on the console are taken by a swiveling block. In a take-off/landing and hypersonic configuration the console is in addition docked to a motionless part of a wing in four points by a wing collet fixture as it is displayed on model, see Fig.7, 8.
3. As in a swivel section spars and ribs in various configurations change places, we offer cobweb skeleton, displayed on Fig. 7.

Structural analysis in three various configurations has discovered, that the strain-stress distribution of take-off and landing and subsonic configurations practically coincide, as the swiveling block has incomparably higher stiffness factor in comparison with collet fixture and agglomerates on itself all external loading. Therefore for optimization we will consider two loading cases: take-off/landing and hypersonic.

The airfoil section in classical sense here is not present, the wing represents as a plate, therefore aerodynamic pressure on a wing we will take as uniform.

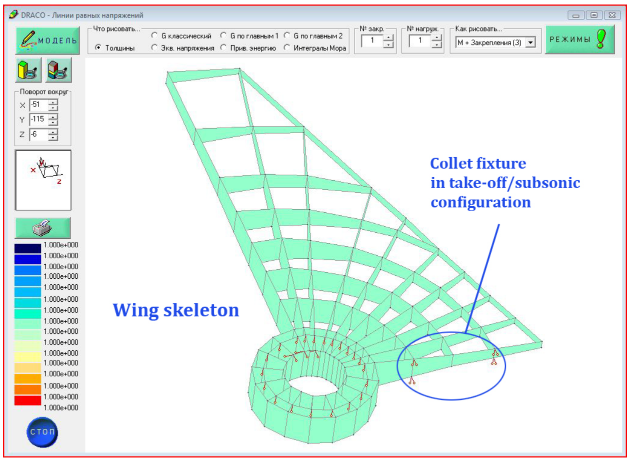

Fig. 7. Wing skeleton

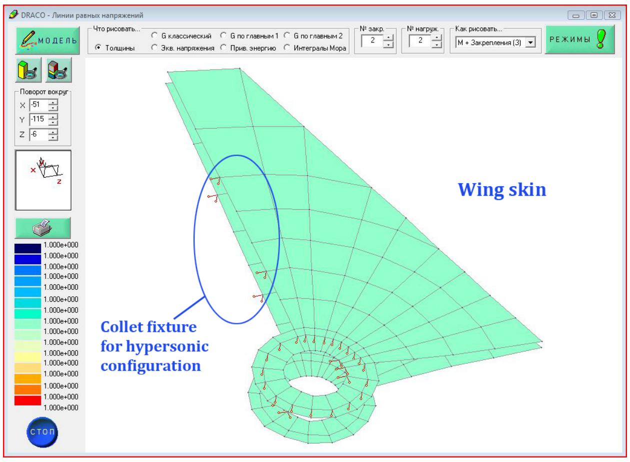

Fig. 8. Wing skin

Span of outer wing (from wing stab to wing end) in take-off/landing configuration equal 21m, and in hypersonic configuration equal 11.5m.

Operational G-force in take-off/landing mode we will take n_y = 1.5, and in hypersonic mode, n_y = 3. Part of the take-off mass of airplane supporting in air by the outer wing is 20 378 kg. Allowable stress [σ] = 400 MPa.

Let's execute optimization of a structure of a swivel section of a wing with usage of a developed method and compare the results with classical full stressed material distribution [4].

Optimization results for skeleton are displayed on the Fig. 9, 10 and for skin on Fig. 11, 12.

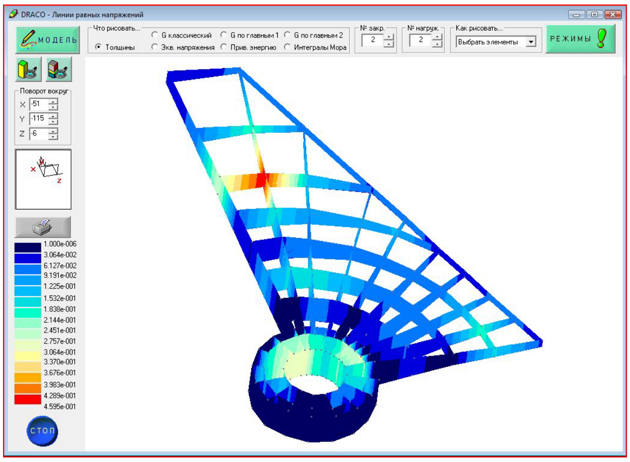

Fig. 9. "Fully stressed" material distribution on walls of skeleton: δ_{max} = 4.6 mm.

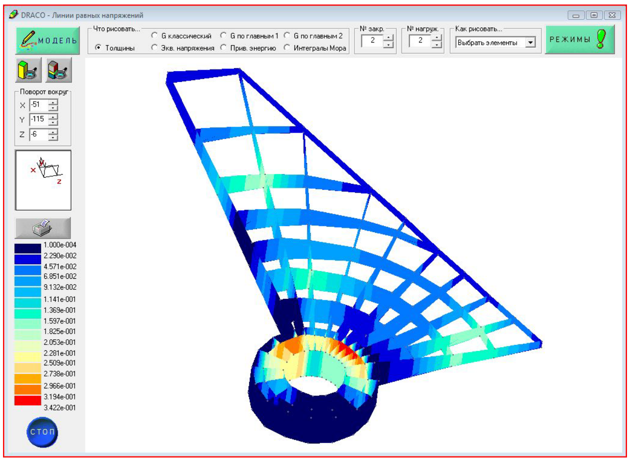

Fig. 10. Material distribution on walls of skeleton by our theory: δ_{max} = 3.4 mm.

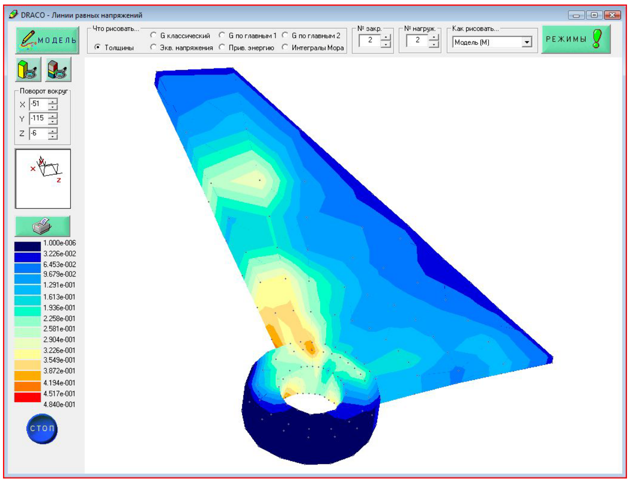

Fig. 11. "Fully stressed" material distribution on skin: δ_{max} = 4.8 mm.

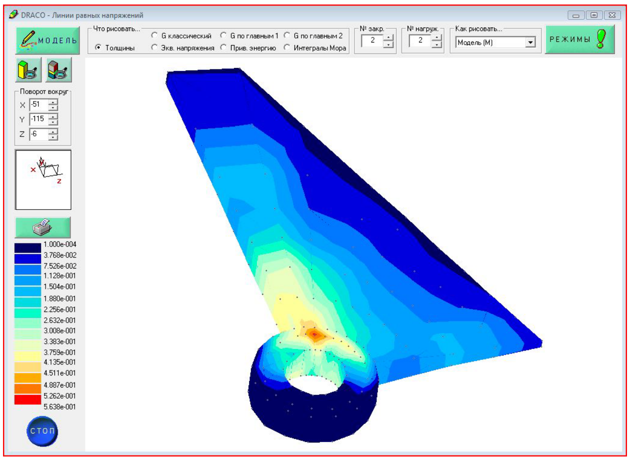

Fig. 12. Material distribution on skin by our theory: δ_{max} = 5.6 mm.

Under such conditions the mass of "fully stressed" material distribution equal 2677 kg, the while mass of discovered by our theory structure equal 2442 kg. Our structure is easier than "fully stressed" one. ***Mass benefit is 8.78%.*** The proof strength k = σ_{max} / [σ] of the both structures approximately equal: for "fully stressed" structure k = 1.00528; for our structure k = 0.999964.

Comparison Fig. 9, 10 and Fig. 11, 12 show, that our material distribution essentially another than in "fully stressed" structure.

Detail analysis of results show that in our structure most part of elements works in both loadings, the while in "fully stressed' structure we can find many elements, which work in one case of loading but do not work in another one. Therefore "fully stressed" structure under several loadings not always is optimal.

6 Conclusion

We suggest the new method, which discover an optimal structure under acting of several loading. We have tested it on Razani problem and have solved the model optimization problem of hypothetic wing for hypersonic airliner. We have demonstrated that suggested method has some advantage in achievement the minimal mass structure and give to designer the new tool for structural optimization on early stage of airframe designing.

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Contact Author Email Address

Alexander Danilin, mailto: alexdan@ssau.ru

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