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Panel Vibro-Acoustic Response Prediction Based on Turbulent Boundary Layer Measurement

ZHAO Xiaojian[1], **AI Bangcheng**[1]
China Academy of Aerospace Aerodynamics, Beijing
100074 E-mail: **zhaox0714@126.com**

Abstract

Panel vibration due to TBL excitation is a simple model of simulating the noise generation process, and which can be used to formulate design criteria. In order to exactly simulate the TBL excitation, the data of measurement on aircraft is used to modify the theoretical results. The Corcos mode modified proposed in paper marches the measurement better than classical Corcos model. Transmission loss is defined with a new method, which can be used to the sensitivity analysis of TBL model. Three kinds typical TBL excitation encountered in engineering was investigated in the paper, including attachment flow, separation and wave oscillation. The investigation shows that the power of sound radiation is proportional to input power of TBL excitation in the linear range of structure response. The extension of peak value of TBL excitation plays a more important role for inducing structure vibration. The locality of peak value for TBL excitation is also a key factor determining the sound transmission. Different locality of peak value leads to different sound transmission loss. The work in paper has proven that the importance of core parameters, and which will help the future workers to develop a finer model for cabin noise study.

1 Introduction

The continual development of jet-powered, well-streamlined aircraft has driven an increasing number of studies concerning the sound and the vibration generation in the cabin for the airflow over the fuselage. More recently, the noise and vibration induced by TBL (turbulent boundary layer) is already a significant contributor to cabin sound levels as the other sources are further reduced. So it is necessary to develop a simple model of the noise generation process, which can be used to formulate design criteria. Although plate vibration induced by TBL has been widely investigated for long, the works still need to continue for some unsolved issue. A simply supported rectangular elastic plate driven by TBL excitation from wind tunnel test for aircraft is investigated in the paper. In order to simulate the TBL excitation more exactly, three kinds of TBL types including attachment flow, separation and wave oscillation are used for TBL modification. Except that the detailed analysis of plate response induced by different TBL condition is also the key point of investigation. Layout of text

2 Theory models of plate response

The model (in fig.1) used for validation is a thin, flat, rectangular and isotropic with no pre-stress. The plate is simply supported on all four edges and mounted in an infinite rigid plane baffle flush with the TBL.The plate lies in an x-y plane, and the flow is along the x-axis. Under the excitation of turbulent boundary layer the displacement cross-spectral density [1] between any arbitrary couple of points belonging to the plate, $A(x_A,y_A)$ and $B(x_A,y_A)$,can be expressed as:

$$X(x_A,y_A,x_B,y_B,\omega)=\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{\psi_j(x_A,y_A)\psi_n(x_B,y_B)}{Z_j^*(\omega)Z_n(\omega)}\right]\left[\frac{S_p(\omega)(ab)^2}{\gamma_j\gamma_n}\right]A_{Q_jQ_n}(\omega) \tag{1}$$

If point A and point B are the some point, the auto-spectral density of displacement at a point can be expressed as:

$$S_P(x_A,y_A,\omega)=\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{\psi_j(x_A,y_A)\psi_n(x_A,y_A)}{Z_j^*(\omega)Z_n(\omega)}\right]\left[\frac{S_p(\omega)(ab)^2}{\gamma_j\gamma_n}\right]A_{Q_jQ_n}(\omega) \tag{2}$$

Where

$$Z_j(\omega)=\rho h\left[\omega_j^2-\omega^2+i\eta\omega_j^2\right]$$

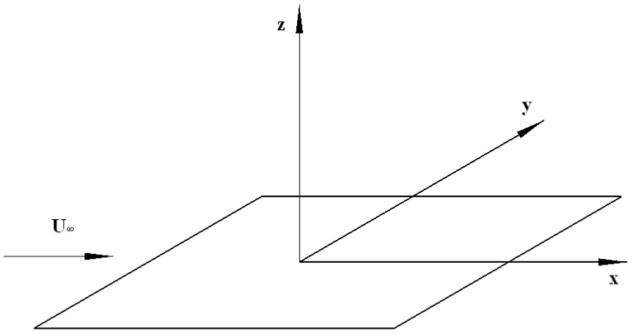


Fig.1 Sketch of the elastic plate under excitation of TBL

Normalized mode shapes:

$$\psi_j(\omega)=\sin\left(\frac{j_x\pi x}{a}\right)\sin\left(\frac{j_y\pi y}{b}\right)$$

Natural radian frequencies:

$$\omega_j=\sqrt{\frac{Et_p^2}{12\rho(1-\nu^2)}}\left[\left(\frac{j_x\pi}{a}\right)^2+\left(\frac{j_y\pi}{b}\right)^2\right]$$

Mode shape factor:

$$\gamma_j=\int_0^a\int_0^b\psi_j^2(x,y)dydx=\frac{ab}{4}$$

And:

$$A_{Q_jQ_n}(\omega)=\frac{1}{(ab)^2}\int_0^a\int_0^a\int_0^b\int_0^b\exp\left(\frac{-|\xi_x|\alpha_x\omega}{U_c}\right)\exp\left(\frac{-|\xi_y|\alpha_y\omega}{U_c}\right)\exp(-i\kappa_c\xi_1)\psi_j(x,y)\psi_n(x,y)dydy'dxdx'$$

The power spectral density with respect to the velocity of the plate is:

$$S_v(x_A,y_A,\omega)=\omega^2S_p(x_A,y_A,\omega) \tag{3}$$

The radiation pressure of panel can be determined from Rayleigh integral:

$$p(X)=\frac{i\omega\rho_0}{2\pi}\int_S\frac{\dot{w}(X')e^{-ikr}}{r}dX' \tag{4}$$

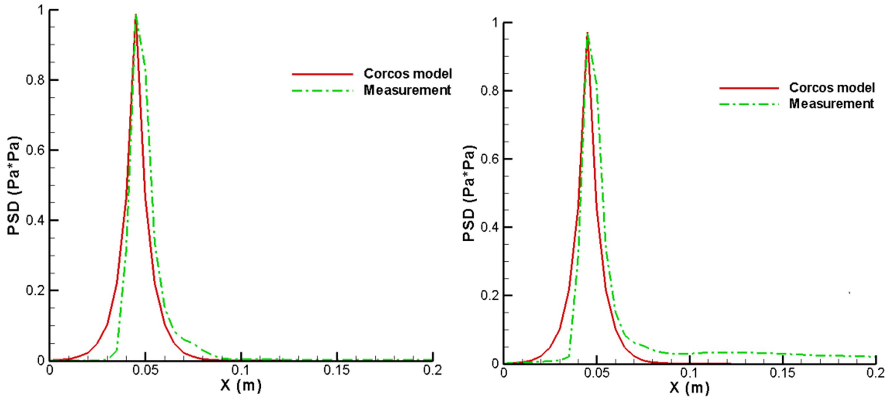
And, $k=\omega/c$ is the acoustic wavenumber, ω is frequency in radian, c is the speed of sound, ρ_0 is the density of air, and $r=|X-X'|=\sqrt{(x-x')^2+(y-y')^2}$, X' is the arbitrary point on the panel.

3 Wavenumber-Frequency Spectrum Models

The Corcos TBL Model proposed by Corcos during the last few decades has been widely used for many different types of problems. The study show that the Corcos is the most classical model ,and some other models are all developed for the elicitation of it more or less.Corcos assumes that the cross power spectral density at two different positions separated by the vector ξ can be expressed as [2,3]:

$$S_{pp}\left(\xi_1,\xi_2,\omega\right)=S_{pp}\left(\omega\right)e^{-\alpha_x\left|\frac{\omega\xi_x}{U_c}\right|-\alpha_y\left|\frac{\omega\xi_y}{U_c}\right|-j\frac{\omega\xi_x}{U_c}} \tag{5}$$

U_c indicates the convection velocity, usually, $0.5\cdot U_0\le U_c\le 0.7\cdot U_0$ is suggested, in this paper, $U_c=0.65U_0$, U_0 is the velocity of mean flow, ξ_x and ξ_y indicate the respective separation distances in the streamwise and spanwise directions ,α_x and α_y are the decay rates. Usually, α_x and α_y are chosen to yield the best agreement with experiment.



(a) f = 31.5Hz (b) f = 80.0Hz

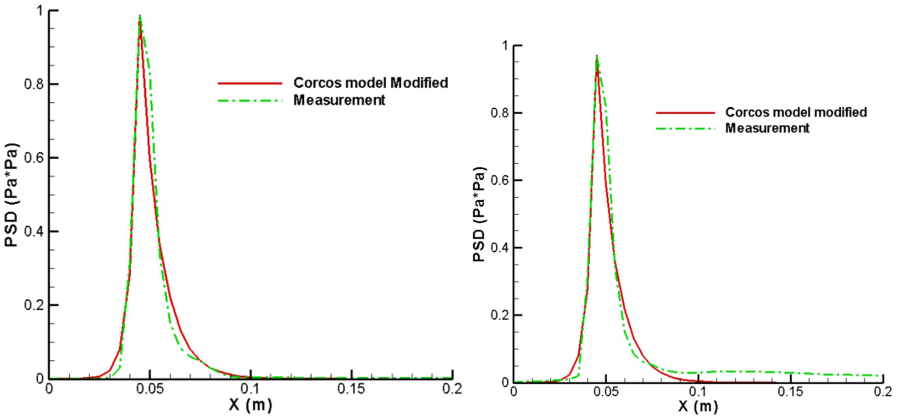
Fig.2 Excitation prediction with Corcos model and measurement

Some studies show that arbitrary TBL model have self-limitation. The TBL models need to be modified basing on test results before applied to the engineering. From the figure, it can be seen that the amplitude of TBL excitation decay both along the length streamwise and spanwise directions, but the phase of TBL excitation varies just along streamwise directions. The amplitude decay is determined by α_x and α_y, and the phase variation is determined by the term ($j\frac{\omega\xi_x}{U_c}$) in equation (8).So when test results of TBL are used to correct the Corcos model,$S_{pp}(\omega)$ and decay factors should be controlled to make the results from the theoretical method and measurement results marching. Fluctuating pressure test in wind tunnel is a most popular method used to measure the TBL excitation with scaled model of aircraft. Fig. 2 shows the TBL excitation with Corcos model and wind tunnel test respectively. But due the effect of incoming flow,

The decay speeds of $S_{pp}(\omega)$ for upstream and downstream are different. So the Corcos model is suggested to be expressed as follows to satisfy the measurement results:

$$S_{pp}\left(\xi_1,\xi_2,\omega\right)=S_{pp}\left(\omega\right)e^{-\alpha_{xup}\left|\frac{\omega\xi_x}{U_c}\right|-\alpha_{xdown}\left|\frac{\omega\xi_x}{U_c}\right|-\alpha_y\left|\frac{\omega\xi_y}{U_c}\right|-j\frac{\omega\xi_x}{U_c}}$$

And α_{xup} and α_{xdown} indicate the decay factors for upstream and downstream on the panelrespectively.Fig.3 Excitation prediction with Corcos mode modified and measurement. The Corcos mode modified marches the measurement better than classical Corcos model.



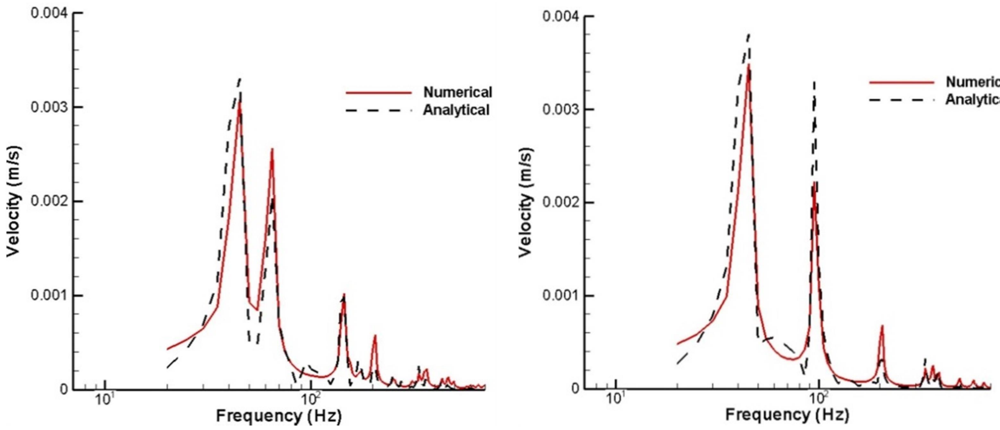
(a) f = 31.5Hz (b) f = 80.0Hz

Fig.3 Excitation prediction with Corcos mode modified and measurement

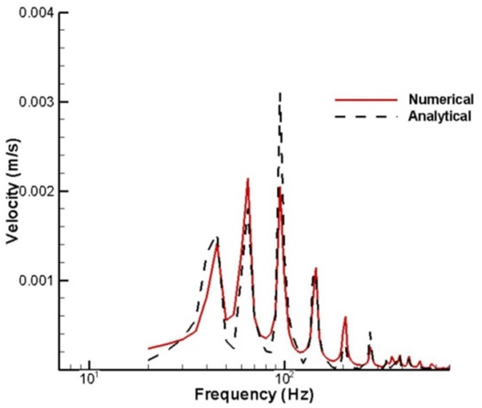
Length(a)	$0.768m$
Width(b)	$0.328m$
Thickness(t_p)	0.0016
Young's modulus(E)	$7.0\times10^{10}\ N/m^2$
Poisson's ratio(v)	0.33
Density(ρ)	$2.7\times10^3\ kg/m^3$
Damping loss factor(η)	0.02

Table .1 Panel dimensions and material properties

For comparison, the FEM (Finite Element Method) solver in VA One was applied validate the analytical method above. The panel dimensions and material properties refer to table.1.The results with two kinds of method are show in fig.5.From the fig.4, it can be seen that there is a good match in PSD of velocity obtained using the FEM and the analytical modal. Because the calculation of sound radiation with the Rayleigh integral is time-consuming, the validation of sound radiation were not performed.Nevertheless,the comparison of PSD of velocity still could adequately validate the analytical method.



(a) . x=0.5059m, y=0.1462m (b) . x=0.384m, y=0.164m



(c) . x=0.1m, y=0.164m

Fig.4 PSD of velocity of a simply supported panel

4 Results and Discussion

The finally target of research is to sever the aircraft design, so the direction of study should aim at the engineering demand. For aircraft in flight, the typical TBL character flows include attachment flow, separation and wave oscillation [2].Previous wind tunnel studies have shown that the pressure fluctuating in attachment flow is weak, so the excitation induced by TBL for aircraft is also weak too. Flow separation is usually generated for sharp change of out contour of aircraft, and the fluctuation pressure around the separation point is strong, which will induce more serious TBL excitation. Usually the separation point is fixed for expansion airflow. The wave oscillation is another important flow which will induce more serious pressure fluctuation on the surface of aircraft. Generally both the locality of wave oscillation and the extension affected on the surface of aircraft are varied as the change of inflow speed.

In order to study the effect of TBL excitation for different flow structure to the aircraft, three kinds of typical TBL excitation were simulated with Corcos model as follows:

$$S_{pp}(\xi_1,\xi_2,\omega)=S_{pp}(\omega)e^{-\alpha_x\left|\frac{\omega\xi_x}{U_c}\right|-\alpha_y\left|\frac{\omega\xi_y}{U_c}\right|-j\frac{\omega\xi_x}{U_c}} \tag{6}$$

Different $S_{pp}(\omega)$ can be used to indicate the PSD of the boundary excitation for different flow type. The locality of $S_{pp}(\omega)$ can be fixed at the flow separation point or where wave oscillation happened. The decay factors is usually used to describe the width of peak value (see fig.7) for turbulent boundary layer excitation, the smaller the decay factors, the wider the extension of peak value. Because the TBL excitation is even for attachment flow, the decay factors $\alpha_x=\alpha_y=0$ suggested. The input power of TBL excitation:

$$P_{in}=\int_S\frac{S_{pp}(\xi_1,\xi_2,\omega)}{\rho_0c_0}dX' \tag{7}$$

ρ_0 , c_0 are fluid density and sound speed in fluid respectively. The input power is not only determined by the value of $S_{pp}(\omega)$,but the locality of $S_{pp}(\omega)$ is also an important factor. So the input power in this paper is a finer definition than the expression in ref. [4].

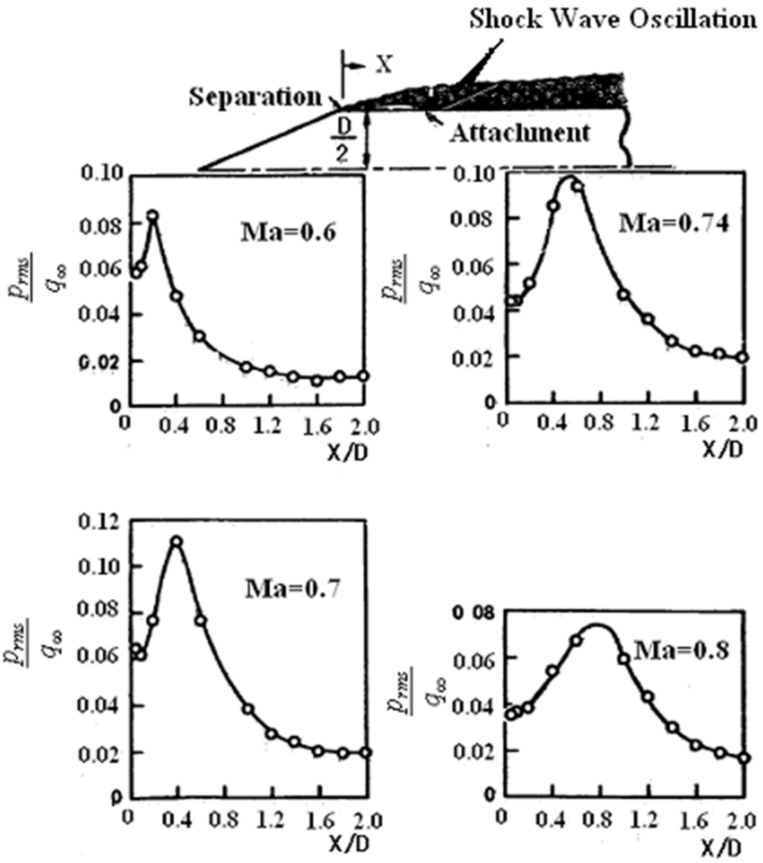


Fig.6 Typical flow structure and fluctuation pressure distribution character

The sound radiation power of plate can be expressed as:

$$P_{out} = \int_S \frac{1}{2}\rho_0 c_0 \left(\dot{w}_a\left(X'\right)\right)^2 dX' \tag{8}$$

$\dot{w}_a\left(X'\right)$ Indicates the amplitude of velocity for arbitrary point on the panel.

The power transmission loss is defined as[4]:

$$IL = 10Log10\left(\frac{P_{in}}{P_{out}}\right) \tag{9}$$

The power transmission is a statistic which can be used to scale the power loss for different panel structure. It is helpful to precede parameters sensitivity analysis of TBL model. Parameters sensitivity analysis is implemented in the following section with analytical method introduced above.

The $S_{pp}(\omega)$ indicates the peak value of TBL excitation which has been referred above. Different flow character generates different $S_{pp}(\omega)$.Fig.5 shows the sound transmission for different $S_{pp}(\omega)$, and the parameters used in computation are referred to table.1 and table.2. Sp in fig.8 indicates the amplitude of $S_{pp}(\omega)$ at each frequency. From the fig.8, the sound transmission for different frequency has irrelevance with the value of $S_{pp}(\omega)$ at each frequency. Considering the main reason, the power of sound radiation is proportional to input power of TBL excitation in the linear range of structure response.

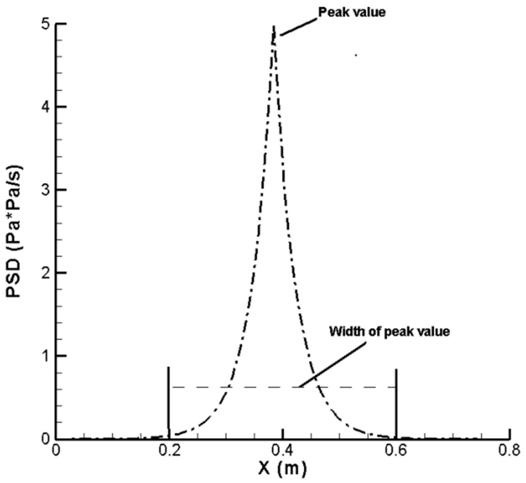


Fig.7 the sketch map of TBL excitation

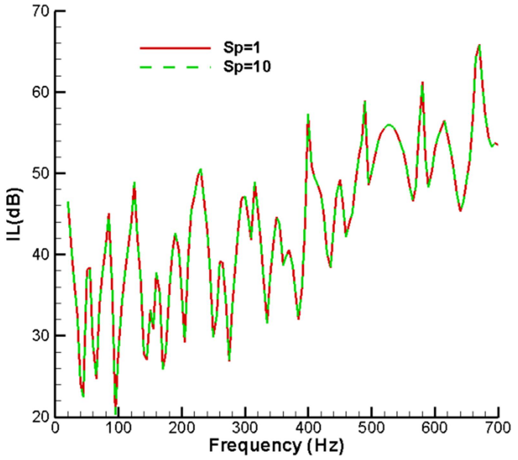


Fig .8 Sound transmission loss for different $S_{pp}(\omega)$

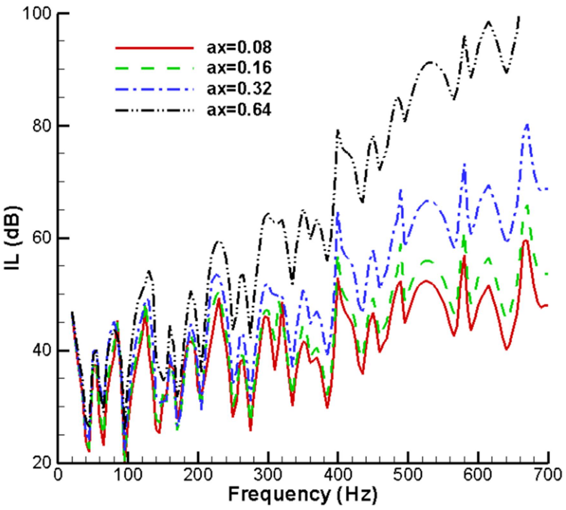


Fig .9 Sound transmission loss for different α_x at (0.384m, 0.164m)

Fig.9 and fig.10 show the relationship between decay factors and the sound transmission. From the figures, the higher decay factors would lead to higher sound transmission, and the slope of power loss speed for high frequency is quicker than low frequency. From the above paper, higher decay factors means narrower extension of peak value of TBL excitation. So it is easy to get a conclusion that the extension of peak value of TBL excitation plays a more important role for inducing structure vibration, and the high frequency excitation is more difficult to drive the structure vibration.

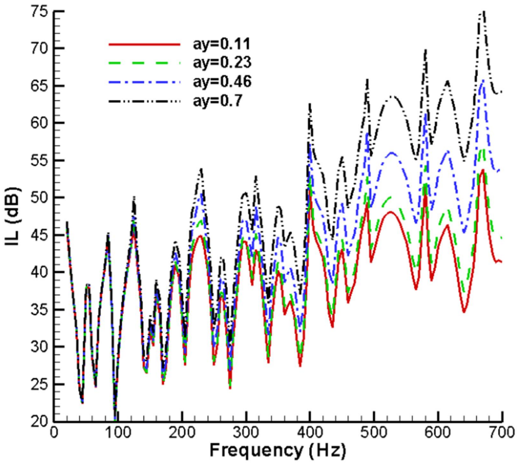


Fig .10 Sound transmission loss for different α_y at (0.384m, 0.164m)

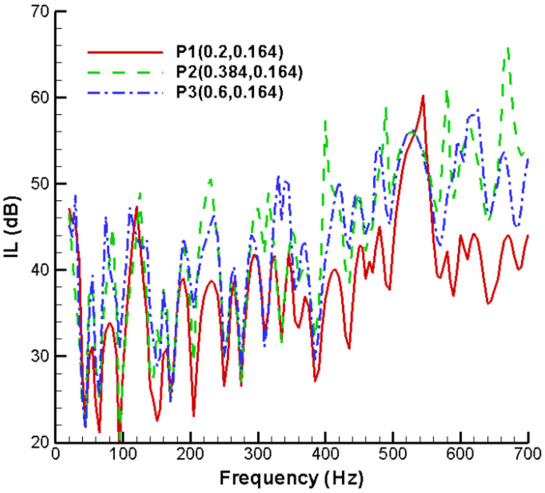


Fig .11 Sound transmission loss for different locality of $S_{pp}(\omega)$

The locality of $S_{pp}(\omega)$ for TBL excitation is also a key factor determining the sound transmission. It can be seen from fig.11, different locality of $S_{pp}(\omega)$ leads to different sound transmission loss. The change of $S_{pp}(\omega)$ locality has little effect on the disciplinarian of structure vibration, but leads to different sound radiation. So the importance of exactly predicting the locality of peak for TBL excitation is certain when evaluating the cabin noise level in aircraft.

6. Conclusion

Although the model basing on the plate vibration induced by TBL has been widely investigated for long, the works still need to continue for some unsolved issue. The more study in detail should be proceeded to find the most appropriate model for cabin noise evaluation. Through the investigation, the conclusions of the paper are listed as follows:

1) Some studies show that arbitrary TBL model have self-limitation. So the TBL models need to be modified basing on measurement before applied to the engineering if the measurement results of TBL excitation have been obtained. The Corcos mode modified was proposed in the paper. The results show that the Corcos mode modified marches the measurement better than classical Corcos model.

2) The power transmission proposed in the paper is a statistic which can be used scale the power loss for different panel structure. It is helpful to precede parameters sensitivity analysis of TBL model. Three kinds typical TBL excitation encountered in engineering was investigated in the paper, and the different TBL excitation could be simulated by modifying the character parameters of Corcos model, including peak value of TBL excitation, the extension and the locality peak value.

3) According to results, the power of sound radiation is proportional to input power of TBL excitation in the linear range of structure response. Higher decay factors means narrower extension of peak value of TBL excitation. So the extension of peak value of TBL excitation plays a more important role for inducing structure vibration. The locality of $S_{pp}(\omega)$ for TBL excitation is also a key factor determining the sound transmission. Different locality of $S_{pp}(\omega)$ leads to different sound transmission loss.

The investigation above gave a suggestion that the finally aim of all the theoretical study is to satisfy the engineering need, the model applied should be justified on the basis of the typical structural and boundary layer parameters of the problem. So for any model, especially the TBL model, should agree well with the real turbulent boundary layer. The work in paper has given a show that the importance of core parameter, and which will help the future works to develop a finer model for cabin noise study.

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