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NEW DESIGN SCENARIO FOR FUTURE COMPOSITE LAUNCHER STRUCTURES

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Abstract

The Space industry demand for lighter and cheaper launcher transport systems. The finished EU project DESICOS (New Robust DESIgn Guideline for Imperfection Sensitive COmposite Launcher Structures, cf. [1]), which started in February 2012 and finished in July 2015, contributes to these aims achievements to a new design procedure for imperfection sensitive composite launcher structures, exploiting the worst imperfection approach efficiently by implementation of the Single Perturbation Load Approach [2]. Currently, imperfection sensitive shell structures prone to buckling are commonly designed according the NASA SP 8007 [3] guideline using the conservative lower bound curve. The guideline dates from 1968, and the structural behaviour of composite material is not considered appropriately, in particular since buckling load and imperfection sensitivity of shells made from such materials substantially depend on the lay-up design. This is not considered in the NASA SP 8007, which allows designing only so called "black metal" structures. Here is a high need for a new precise and efficient design approach for imperfection sensitive composite structures which allows significant reduction of structural weight and design cost. For most relevant architectures of cylindrical and conical launcher structures (monolithic, sandwich - without and with holes) DESICOS investigated a combined methodology from the Single Perturbation Load Approach and a Specific Stochastic Approach which guarantees an effective and robust design. Investigations demonstrated, that an axially loaded unstiffened cylinder, which is disturbed by a large enough single perturbation load, is leading directly to the design buckling load 45% higher compared with the respective NASA SP 8007 design [4]. Within DESICOS the new methods were further developed and validated by tests. The potential was investigated within different industrially driven use cases. This paper deals with the objectives of the DESICOS project, describes the new approach, and highlights selected results.

1 Imperfection sensitive structures

1.1 Introduction

Currently, imperfection sensitive shell structures prone to buckling are designed according the NASA SP-8007 guideline using the conservative lower bound curve (cf. Fig. 1) which was developed 1968 for metallic structures. There is a high need for a new precise and fast design approach for imperfection sensitive composite structures which allows significant reduction of structural weight and design cost. For that purpose a combined methodology from the Single Perturbation Load Approach (SPLA) and a specific stochastic approach is proposed which guarantees an effective and robust design. The SPLA is based on the observation, that a large enough disturbing load leads to the worst imperfection; it deals with the traditional (geometric and loading) imperfections [5]. The stochastic approach considers the non-traditional ones, e.g. variations of wall thickness and stiffness. Thus the combined approach copes with both types of imperfections. Developments demonstrate high potential [4].

This section presents in its first part the state-of-the-art in buckling of imperfection sensitive composite shells. The second part describes current investigations as to the SPLA, the stochastic approach and their combination. In a third part an outlook is given on further studies on this topic, which will be performed within the framework of the finished project DESICOS (New Robust DESIgn Guideline for Imperfection Sensitive COmposite Launcher Structures) funded by the European Commission; for most relevant architectures of cylindrical and conical launcher structures (monolithic, sandwich - without and with holes) the new methodology was further developed and validated by tests.

1.2 State of the art

1.2.1 Imperfection sensitivity

In Fig. 1 taken from [3], KDFs are shown for axially compressed cylindrical shells depending on the slenderness. The results are presented by dots and show the large scatter. The KDF decrease increasing the slenderness. The discrepancy between test and classical buckling theory has stimulated scientists and engineers on this subject during the past 50 years. The efforts focused on post-buckling, load-deflection behaviour of perfect shells, various boundary conditions and their effect on bifurcation buckling, empirically derived design formulas and initial geometric imperfections. Koiter was the first to develop a theory which provides the most rational explanation of the large discrepancy between test and theory for the buckling of axially compressed cylindrical shells. In his doctoral thesis published in 1945 Koiter revealed the extreme sensitivity of buckling loads to initial geometric imperfections. His work received little attention until the early 1960's, because the thesis was written in Dutch. An English translation by Riks was published 1967 [6].

Based on a number of experimental tests in the 1950s and 60s the determination of lower bounds led to design regulations like NASA SP-8007, but the given KDFs are very conservative. To improve the ratio of weight and stiffness and to reduce time and cost, numerical simulations could be used during the design process. The consideration of imperfections in the numerical simulation is essential for safe constructions. Usually, these imperfections are unknown in the design phase, thus pattern and amplitude have to be assumed.

In general, one can distinguish between loading imperfections and geometric imperfections. Both kinds of imperfections have a significant influence on the buckling behaviour.

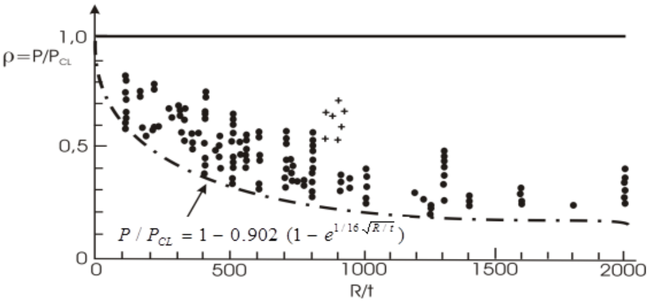


Fig 1. Distribution of test data for cylinders Subjected to axial compression [3]

Loading imperfections mean any deviations from perfect uniformly distributed loading, independent of the reason of the perturbation. Geier et al. tested composite cylindrical shells with different laminate designs [7], and they applied thin metal plates locally between test shell and supporting structure to perturb the applied loads and performed the so-called shim tests [8]. Later, numerical investigations were performed and compared to the test results; the importance was verified [9]. The need to investigate loading imperfections for practical use was shown for instance by Albus et al. [10] by the example of Ariane 5.

Geometric imperfections mean any deviations from the ideal shape of the shell structure. They are often regarded the main source for the differences between computed and tested buckling loads. Winterstetter et al. [11] suggest three approaches for the numerical simulation of geometrically imperfect shell structures: “realistic”, “worst” and “stimulating” geometric imperfections. Stimulating geometric imperfections like welded seams are local perturbations which “stimulate” the characteristic physical shell buckling behaviour [12]. “Worst” geometric imperfections have a mathematically determined worst possible imperfection pattern like the single buckle [13]. “Realistic” geometric imperfections are determined by measurement after fabrication and installation. This concept of measured imperfections is initiated and intensively promoted by Arbocz [14]; a large number of test data is needed, which has to be classified and analysed in an imperfection data bank. Within the study presented in this paper, real geometric imperfections measured at test shells are taken into account.

Hühne et al. [2] showed that for both, loading imperfections and geometric imperfections the loss of stability is initiated by a local single buckle. Therefore unification of imperfection sensitivity is allowed; systems sensitive to geometric imperfections are also sensitive to loading imperfections. Single buckles are realistic, stimulating and worst geometric imperfections.

Using laminated composites, the structural behaviour can be tailored by variation of fibre orientations, layer thicknesses and stacking sequence. Fixing the layer thicknesses and the number of layers, Zimmermann [15] demonstrated numerically and experimentally that variation of fibre orientations affects the buckling load remarkably. The tests showed that fibre orientations can also significantly influence the sensitivity of cylindrical shells to imperfections. Meyer-Piening et al. [16] reported about testing of composite cylinders, including combined axial and torsion loading, and compared the results with computations.

Hühne [2] selected some of the tests described in [15] and [16] performed additional studies. Within a DLR-ESA study one of these cylinder designs, which is highly imperfection sensitive, was manufactured 10 times and tested. It allowed a comparison with already available results and enlarged the data base [4].

1.2.2 Single-Perturbation-Load Approach

Hühne [2] proposed an approach based on a single buckle as the worst imperfection mode leading directly to the load carrying capacity of a cylinder. Figure 2 shows the lateral perturbation load SPL used to disturb the otherwise unloaded shell, and the axial compression load F is applied until buckling. This is repeated with a series of different perturbation loads, starting with the undisturbed shell and the respective buckling load (approximately 190 kN in Figure 2). The buckling load depends on the perturbation load which is applied. It is shown that the buckling load using a perturbation load larger than the threshold value identified as “P1” is almost constant. The buckling load at this level is called “F1”. A further increase of the perturbation load has no significant change on “F1”, and this is considered to be the design buckling load using this approach.

Figure 2 also shows how the load shortening curve looks like before and after “P1”. For a PL lower than “P1” only one instability load is verified, which is the global buckling load. When using a perturbation load higher than “P1” a first instability can be seen before the global buckling. The typical displacement patterns of these two instability points is shown in Figure 3.

This concept promises to improve the KDF and allows designing any CFRP cylinder by means of one calculation under axial compression and a single-perturbation load. Within a DLR-ESA study, this approach was confirmed analytically and experimentally, cf. [4]. However, there is still the need for a multitude of further studies.

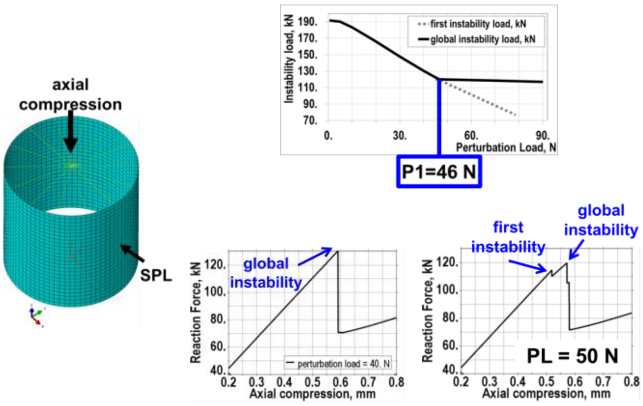


Fig. 2: Single perturbation load approach (SPLA)

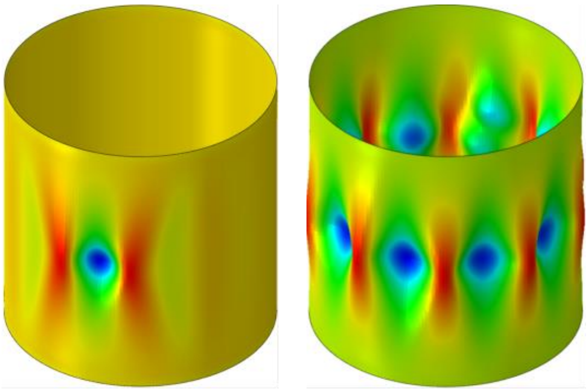

a) first instability b) global instability

Fig. 3: Displacement patterns of the first and global instabilities

1.2.3 Probabilistic research

In general, tests or analysis results are sensitive to certain parameters as boundary conditions or imperfections. Probabilistic methods are a possibility to assess the quality of results. The stochastic simulation with Monte Carlo (e.g. [17]) allows the statistical description of the sensitivity of the structural behaviour. It starts with a nominal model and makes copies of it whereas certain parameters are varied randomly. The random numbers, however, follow a given statistical distribution. Each generated model is slightly different, as in reality.

Recently, probabilistic simulations found the way into all industrial fields. In automotive engineering they are successfully applied in crash or safety (e.g. [18]). Klein et al. [19] applied the probabilistic approach to structural factors of safety in aerospace. Sickinger and Herbeck [20] investigated the deployable CFRP booms for a solar propelled sail of a spacecraft using the Monte Carlo method.

Velds [21] performed deterministic and probabilistic investigations on isotropic cylindrical shells applying finite element buckling analyses and showed the possibility to improve the KDF. However, setting-up of a probabilistic design approach still suffers by a lack of knowledge due to the incomplete base of material properties, geometric deviations, etc..

Arbocz and Hilburger [22] published a probability-based analysis method for predicting buckling loads of axially compressed composite cylinders. This method, which is based on the Monte Carlo method and first-order second-moment method, can be used to form the basis for a design approach and shell analysis that includes the effects of initial geometric imperfections on the buckling load of the shell. This promising approach yields less conservative KDFs than those used presently by industry.

1.2.4 Specific Stochastic Approach

Figure 4 shows the variation (gray shaded band) of the buckling load resulting from its sensitivity to the scatter of the non-traditional imperfections (e.g. thickness variations). It demonstrates the need to cover this by the development of an additional KDF ρ_2 in combination to the KDF ρ_1 from SPLA.

An efficient design is feasible, if knowledge about possibly occurring imperfections exists and if this knowledge is used within the design process. Whereas the traditional imperfections are dealt with the SPLA, the non-traditional ones are taken into account by probabilistic methods, which enable the prediction of a stochastic distribution of buckling loads. Once the distribution of buckling loads is known, a lower bound can be defined by choosing a level of reliability. Degenhardt et al. [4] found less conservative KDFs than through the NASA SP-8007 lower bound, by executing probabilistic analyses with non-traditional imperfections.

The work for the stochastic approach consists in checking which structural parameters substantially influence the buckling load and defining realistic limits for their deviations from the nominal values, in varying them within the limits and performing buckling load computations for these variations. The results are evaluated stochastically in order to define a guideline for the lower limits of the buckling loads within a certain given reliability. From these limits a KDF is derived.

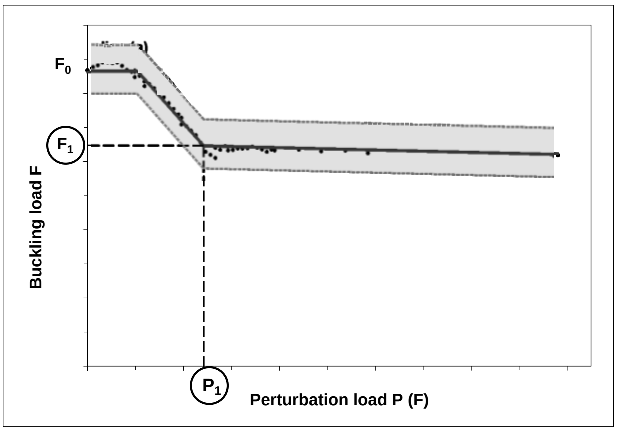


Fig. 4: Scatter of buckling load due to the scatter of non-traditional imperfections

1.2.5 Summary

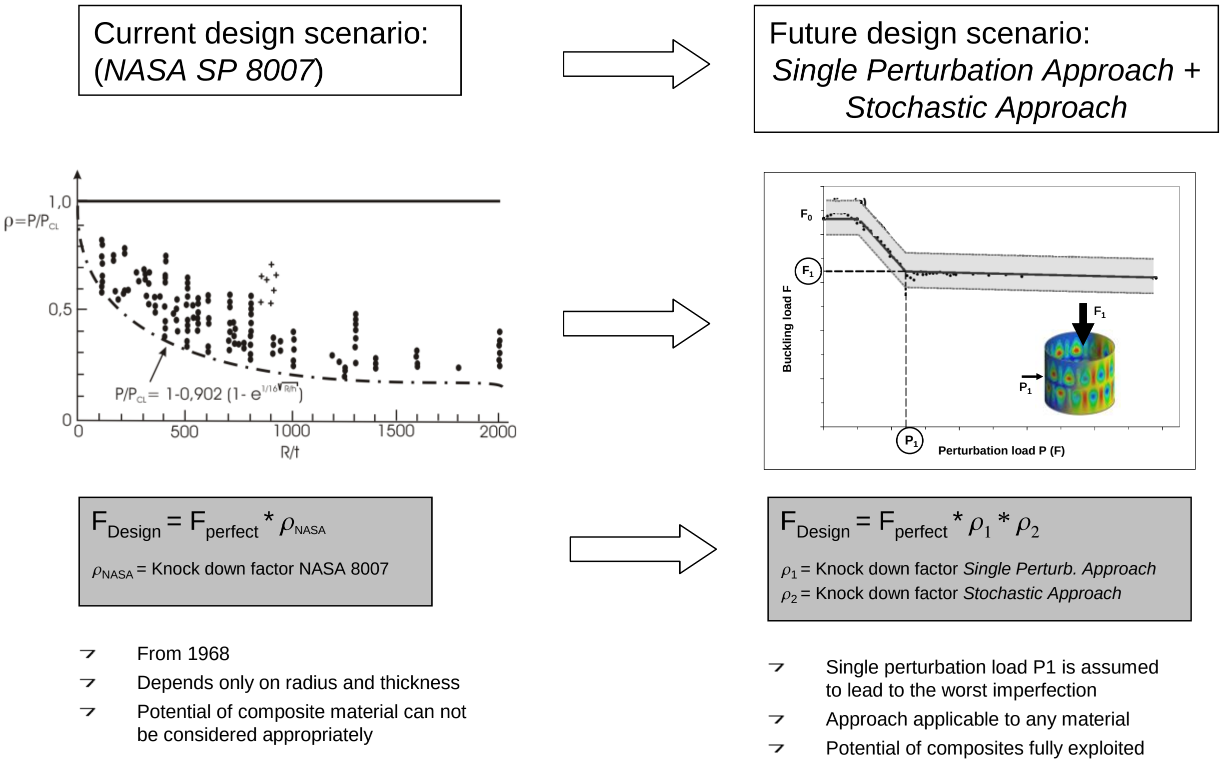
From all this it becomes obvious that a great deal of knowledge is accumulated concerning the buckling of cylindrical shells under axial compression. However, the NASA SP-8007 guideline for the KDFs from 1968 is still in use, and there are no appropriate guidelines for unstiffened cylindrical CFRP shells. To define a lower bound of the buckling load of CFRP structures a new guideline is needed which takes the lay-up and the imperfections into account. This can be for instance a probabilistic approach or the Single-Perturbation-Load approach, combined with a specific stochastic approach. In the following the second one is considered in more detail. Independent of the approach dozens of additional tests are necessary, in order to account for statistical scatter as well as for software and guideline validation.

2 SPLA combined with specific stochastic approach

2.1 The procedure and first results

Figure 11 summarises the future design scenario for imperfection sensitive composite structures in comparison to the current design scenario. Currently, the buckling load of the perfect structure F_{Perfect} has to be multiplied by the KDF ρ_{NASA} from the NASA SP-8007 guideline. This approach was developed for metallic structures in 1968 and does not at all allow exploiting the capacities of composite structures. Accordingly, with the new design scenario F_{Perfect} is multiplied by ρ_1 which results from SPLA and ρ_2 which comes from the specific stochastic approach.

First studies (cf. [4]) demonstrated the high potential of this combined approach which is summarized in Figure 5. In this example a composite cylinder (R/t=500) with 4 layers was designed according the current and the future design scenarios. The classical buckling load was calculated and utilized as reference (scaled buckling load ρ=1.0, marked by a star). The buckling load calculated by the SPLA was at ρ=0.58 (marked by a star). All experimentally extracted results revealed first buckling beyond the one calculated by the SPLA (safe design). The KDF from the SPLA was found to be 0.58 (times 0.8 from stochastic), whereas the one from NASA SP was 0.32. The result was that the load carrying capacity could be increased by 45%. It corresponds to approximately 20% weight reduction for the same load. In [4] the results were validated by tests on 10 nominally identical structures.



Current design scenario: (*NASA SP 8007*)

Future design scenario: *Single Perturbation Approach + Stochastic Approach*

$F_{Design} = F_{perfect} * \rho_{NASA}$

ρ_{NASA} = Knock down factor NASA 8007

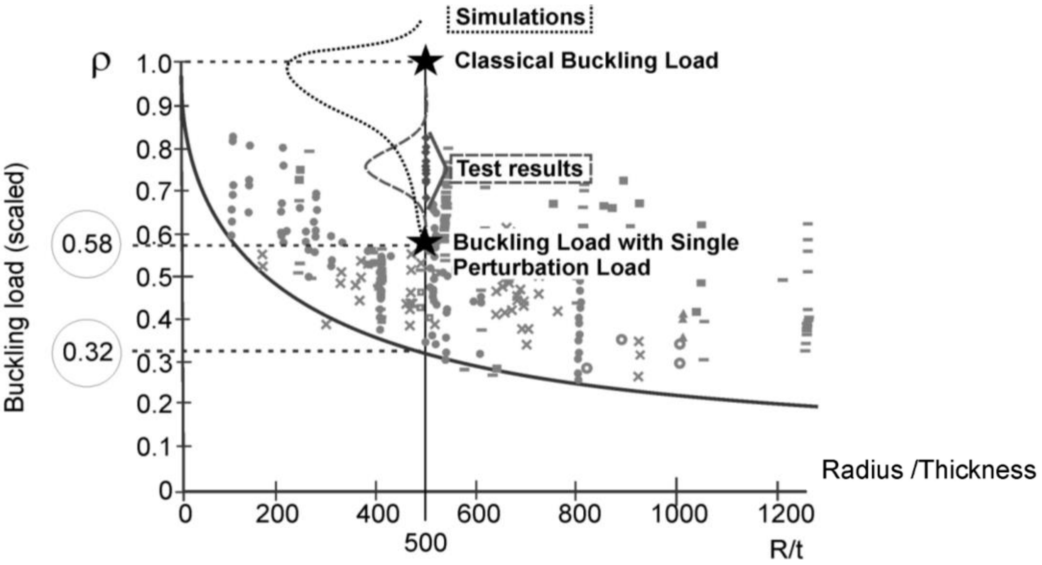
$F_{Design} = F_{perfect} * \rho_1 * \rho_2$

ρ_1 = Knock down factor *Single Perturb. Approach*
ρ_2 = Knock down factor *Stochastic Approach*

- From 1968
- Depends only on radius and thickness
- Potential of composite material can not be considered appropriately

- Single perturbation load P1 is assumed to lead to the worst imperfection
- Approach applicable to any material
- Potential of composites fully exploited

Fig. 5: Future design scenario for composite unstiffened structures



ρ_{NASA} = 0.32

ρ_1 = 0.58

ρ_2 = 0.80 (Conservative estimation)

$F_{Design} = F_{perfect} * \rho_{NASA}$
F_{Design} = 32 * 0.32 = **10.2 kN**

$F_{Design} = F_{perfect} * \rho_1 * \rho_2$
F_{Design} = 32 * 0.58 * 0.8 = **14.8 kN**

Improvement: 45 %

Fig. 6: Potential of the future design scenario [4], Example: CFRP cylindrical shell (R/t=500), 4 plies, Fperfect = 32 kN

3 DESICOS project

3.1 Main objective

The main objective of DESICOS was to establish an approach on how to handle imperfection sensitivity in space structures endangered by buckling, in particular for those made from fiber composite materials. It aims to substitute the NASA SP-8007, which is extremely conservative and not really applicable for composite structures, cf. Figure 5.

The DESICOS consortium merges knowledge from 2 large industrial partners (Airbus D&S from France Germany), one enterprise belonging to the category of SME (GRIPHUS from Israel), 2 research establishments (DLR from Germany and CRC-ACS from Australia) and 7 universities (Politecnico di Milano from Italy, RWTH Aachen, Leibniz University and the Private University of Applied Sciences Göttingen from Germany, TECHNION from Israel, TU-Delft from Netherlands and Technical University of Riga from Latvia). The large industrial enterprises and the SME bring in their specific experience with designing and manufacturing of space structures as well as their long grown manufacturing philosophies for high quality stiffened composite structures. The academic partners and the research organisations provide their special knowledge in methods and tool development as well as testing. This consortium composition assures the expected rapid and extensive industrial application of the DESICOS results.

3.2 Main results

To reach the main objective, improved design methods, experimental data bases as well as design guidelines for imperfection sensitive structures are needed. The experimental data bases are indispensable for validation of the analytically developed methods. Reliable fast methods will allow for an economic design process. Industry brings in experience with the design and manufacture of real shells; research contributes knowledge on testing and on development of design methods. Design guidelines are defined in common, and the developed methods are validated by industry.

The main DESICOS results can be summarised as following:

1) Benchmarking results:
    a. Collection of all worldwide existing papers to buckling experiments
    b. Imperfection data base with existing measurements
    c. ABQUS plug-in for improved modelling and evaluation of cylindrical and conical structures with different loads, boundary conditions, cut-outs, imperfections, …
2) Experimental data base on:
    a. Material properties of different materials used in the project
    b. Manufacturing of structures
    c. Buckling experiments
3) New design approaches:
    a. Modelling and analyses
    b. New design approaches
    c. Validation and application of the design approaches
4) Design recommendations

The main results were published in 30 peer-reviewed papers (see www.desicos.eu). The main outcome was presented at the 3rd Int. Conference on Buckling and Postbuckling Behaviour of Composite Laminated Shell Structures, Braunschweig, Germany, 25-27 March, 2015. One can summarize that the application of analysis based design methods, using different approaches to represent the imperfections, seems to lead to less conservative KDFs than those obtained by the NASA SP. However, additional studies are needed to collect imperfection data of the real structures, and how these imperfections should be represented in an efficient way.

3.3 Summary

The current design process according the NASA SP-8007 is shown and its limitations to design structures made of composites are explained.

The main outcome of the DESICOS-project is summarized in the following. This is mainly for the two “new” methods, the SPLA method, the stochastic method, or combination of both.

I) About the “new” methods

SPLA:

- SPLA proved in the project to represent only geometrical imperfections.
- SPLA is representing the global buckling which is not the worst imperfection as for instance local buckling may occur first. It has to be find out in the future activities if representing the global buckling by SPLA is sufficient for the design.
- Loading imperfections are not covered by SPLA.
- For sandwich structures:
    - SPLA was studied on small test structures. The numerical studies led to clear results. But it could not be confirmed experimentally on cylindrical sandwich specimens, because the structures failed too early by material damage after the first buckling test without applying SPLA.
    - Numerical study performed for industrial Use case is not so clear.
    - Thus, the application of SPLA for real sandwich structures is therefore not clear enough.

Stochastic method:

- This method seems more powerful to represent the physics.
- This method usually requires sufficient computational effort, higher than other methods.
- Also, the difficult issue is to know (or to choose a priori) the stochastic values for the different parameters.
- To simplify this issue, as the DESICOS study has shown that the geometric imperfections are mostly the dominant parameter for imperfection sensitive structures, even when composite structures, there could be a proposal to limit to treat as stochastic the geometric parameter, and to apply an additional Knock Down Factor for taking account all other parameters. For that, DESICOS gives results to help to choose some adequate additional KDF for the application in the projects.
- However, despite such possible simplification, the resulting KDF would still depend on the choice of the ratio a / t (where “a” is the amplitude of the imperfections, and “t” the thickness). That means the same difficulty than for the methods using modal or axisymmetric imperfections. So the advantage to use complex stochastic method, compared to more simple ones method is not obvious. Current more simple methods are nonlinear analysis with modal shape pattern for geometric imperfections, or axisymmetric pattern when more appropriate (in particular if well-defined thickness transition or singularity along the meridian).
- In fact, to establish a reliable design of a sensitive structure without too much conservatism, it should be known the pattern and amplitude of the most significant geometric imperfections induced by the manufacturing process of this particular structure. This has to be measured on a batch including an enough number of real manufactured specimens.

II) About the possible improvement of the NASA KDF, for less conservative design

- The application of analysis based design methods, using different approaches to represent the imperfections, seems to lead to less conservative KDFs than those obtained by the NASA SP.
- Additional studies are needed to collect imperfection data of the real structures, and how these imperfections should be represented in an efficient way.

III) Design recommendations

This could appear common to say that, and not always possible to apply in the projects, but good design for aeronautic and space structures should ovoid structures which are too much imperfection sensitive, because they are particularly difficult to size with the good margin for being safe without being not too conservative.

Several means are known to let the structures less imperfection sensitive, as to add longitudinal stiffeners, circular frames, and the combination of both, to pressurize enough the structural tank.

More details with all publications can be found at www.desicos.de.

4. Conclusions

This paper demonstrates that the potential of composite light weight structures, which are prone to buckling, is currently in the field of aircraft and space applications not fully exploited as appropriate guidelines do not exist. It will be distinguished between imperfection tolerant structures (e.g. stiffened panels) as used in aerospace applications and imperfection sensitive structures (e.g. unstiffened cylinders) more used in space applications. Both types of structures show a complex buckling behaviour which is combination of different local and global buckling modes. This paper deals with the state-of-the-art, shows the advances and challenges related to stability analysis of composite structures.

5. Acknowledgments

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6. Copyright Statement

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See www.desicos.eu