NONLINEAR OPTIMAL CONFLICT RESOLUTION SOLVED WITH PARTICLE SWARM OPTIMIZATION

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Keywords: ***Air traffic management, conflict resolution, particle swarm optimization***

Abstract

A nonlinear conflict resolution problem which resolves conflicts by allowing aircraft to change heading angle and speed is studied. In general, the problem is hard to solve due to a nonlinearity introduced by the combined maneuver. However, by utilizing a particle swarm optimization, heading angle and speed changes required to resolve the conflicts can be obtained. Furthermore, the proposed algorithm requires a small computational load. The performance of the proposed algorithm is demonstrated by numerical simulation where an artificial traffic scenario is considered.

1 Introduction

One of the fundamental and challenging topics in air traffic management (ATM) is to ensure the safe separation of aircraft during flight. With regard to the safety issue, various conflict resolution algorithms have been proposed [1-3]. Pallottino et al. presented two conflict resolution models based on mixed integer linear programming (MILP) by allowing aircraft to perform either heading angle change (HAC problem) or speed change (SC problem) [1]. Christodoulou et al. developed a three-dimensional formulation for the SC problem [2], and Alonso-Ayuso et al. suggested a modified SC problem where combined speed and altitude changes are allowed to aircraft [3]. In these studies, conflict resolution was formulated as a linear programming problem because heading angle and speed changes are dealt with separately. Note that the linear problem can be solved with a standard optimization software such as CPLEX [4].

In the practical point of view, however, simultaneous heading angle and speed changes are required for aircraft maneuver to increase flexibility and efficiency. The combined HAC and SC problem is hard to solve due to a nonlinearity following from simultaneous heading angle and speed changes. Adan et al. attempted to solve the combined problem [5]. However, instead of utilizing optimization technique such as nonlinear programming to deal with the nonlinearity, linear approximation was used to formulate the problem as MILP. Although numerous efforts were made to obtain a reliable approximation, approximation errors were generated. Omer [6] proposed a space-discretized model for the combination of heading angle and speed changes. However, this approach has a problem of heavy computational load due to the large number of discrete variables.

In this study, a conflict resolution algorithm in which both heading angle and speed changes is considered. To handle the nonlinearity introduced by the simultaneous heading angle and speed changes, particle swarm optimization (PSO) is employed. PSO is a population-based stochastic optimization technique inspired from the social behavior and movement with communications of insects, birds and fish [7]. Each particle in the population travels in the search space to find a global minimum (or maximum), while adjusting its velocity combining its own experience with social experience. The capability of simple computation and rapid convergence of PSO have been proved by numerous recent studies [8-9].

The rest of this study is organized as follows. In Section 2, the non-conflict condition under geometrical considerations is introduced. In Section 3, the nonlinear conflict resolution problem is formulated, and PSO is implemented to solve the problem. In Section 4, numerical simulation is performed to demonstrate the performance of the proposed algorithm. In Section 5, conclusion and future research directions are presented.

2 Problem Description

Let us consider an arbitrary pair of aircraft (i,j) as shown in Fig. 1. Note that a conflict is defined as a situation where the safety disks of aircraft (i,j) overlap each other [1]. If one of the following conditions is satisfied, then there is no conflict between aircraft i and j.

$$\frac{v_i \sin\theta_i - v_j \sin\theta_j}{v_i \cos\theta_i - v_j \cos\theta_j} \geq \tan\alpha_{ij} \tag{1}$$

$$\frac{v_i \sin\theta_i - v_j \sin\theta_j}{v_i \cos\theta_i - v_j \cos\theta_j} \leq \tan\beta_{ij} \tag{2}$$

where

$$\theta_i = \theta_i^0 + \Delta\theta_i \tag{3}$$

$$v_i = v_i^0 + \Delta v_i \tag{4}$$

$$\alpha_{ij} = \psi_{ij} + \theta_s^{ij} \tag{5}$$

$$\alpha_{ij} = \psi_{ij} + \theta_s^{ij} \tag{6}$$

In Eqs. (1)-(4), θ_i is an optimal heading angle, θ_i^0 is an initial heading angle, and $\Delta\theta_i$ is a variation of heading angle of aircraft i. Also, v_i is an optimal speed, v_i^0 is an initial speed, and Δv_i is a variation of speed of aircraft i. In Eqs. (5) and (6), θ_s^{ij} is a minimum safety angle, which can be determined as follows

$$\theta_s^{ij} = \sin^{-1}\left(\frac{2r}{d_{ij}}\right) \tag{7}$$

where d_{ij} is a distance between aircraft i and j, and r is a radius of the safety disk of each aircraft. Depending on the sign of denominator,

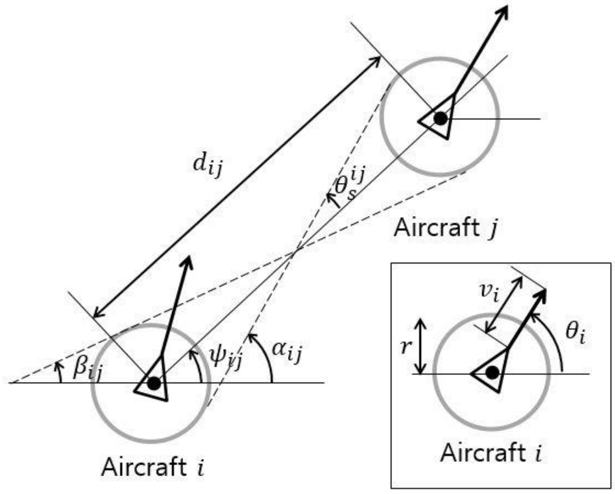

Fig. 1. Geometry of conflict resolution for arbitrary pair of aircraft *i* and *j*.

Eqs. (1) and (2) can be expressed as following four linear inequality equations.

$$v_i \sin\theta_i - v_j \sin\theta_j \geq \tan\alpha_{ij}\left(v_i \cos\theta_i - v_j \cos\theta_j\right) \tag{8}$$

$$v_i \sin\theta_i - v_j \sin\theta_j \leq \tan\alpha_{ij}\left(v_i \cos\theta_i - v_j \cos\theta_j\right) \tag{9}$$

$$v_i \sin\theta_i - v_j \sin\theta_j \leq \tan\beta_{ij}\left(v_i \cos\theta_i - v_j \cos\theta_j\right) \tag{10}$$

$$v_i \sin\theta_i - v_j \sin\theta_j \geq \tan\beta_{ij}\left(v_i \cos\theta_i - v_j \cos\theta_j\right) \tag{11}$$

3 Optimal Conflict Resolution

3.1 Nonlinear Conflict Resolution Problem

Based on the geometry of arbitrary pair of aircraft i and j as discussed in the previous section, conflict resolution can be formulated as a following nonlinear optimization problem.

$$\min \; J_1 = \sum_{i=1}^{N}\left(\rho_\theta \Delta\theta_i + \rho_v \Delta v_i\right) \tag{12}$$

subject to

one of four conditions Eqs. (8)-(11)

where N is a number of aircraft, and ρ_θ and ρ_v are penalty constants for heading angle change and speed change, respectively. Parameters ρ_θ and ρ_v can be determined by a heuristic manner as follows

$$\rho_\theta = \frac{1}{30}[1/\deg] \tag{13}$$

$$\rho_v = \frac{1}{v_{\max} - v_{\min}}[\min/\mathrm{nm}] \tag{14}$$

where $v_{\min}$ and $v_{\max}$ are minimum and maximum speed, which are set to 6.96 nm/min and 10.44 nm/min, respectively.

3.2 The Proposed Approach

In this study, all constraints should be included in a performance index. The proposed conflict resolution problem can be formulated as the following optimization problem.

$$\begin{aligned} \min\ J_2 = &\sum_{i=1}^{N}\left(\rho_\theta \Delta\theta_i + \rho_v \Delta v_i\right) \\ &+ C\sum_{i=1}^{N}\sum_{j=i+1}^{N}\max\left(0, P_{ij} - Q_{ij}\tan\alpha_{ij}\right) \end{aligned} \tag{15}$$

where

$$\left(P_{ij}, Q_{ij}\right) = \begin{cases} \left(p_{ij}, q_{ij}\right) & \text{if Eqs.(9) or (11) holds} \\ \left(-p_{ij}, -q_{ij}\right) & \text{if Eqs.(8) or (10) holds} \end{cases}$$

$$p_{ij} = v_i \sin\theta_i - v_j \sin\theta_j$$

$$q_{ij} = v_i \cos\theta_i - v_j \cos\theta_j$$

In Eq. (15), the second term is a penalty function to satisfy the conflict resolution condition, and C is a weighting parameter.

To determine the variations of heading angle $\Delta\theta_i$ and speed Δv_i for $i = 1, \cdots, N$, which are the solutions of the optimization problem minimizing Eq. (15), PSO is utilized in this study. In PSO, the velocity $u_s \in \mathbb{R}^{2N}$ and position $x_s \in \mathbb{R}^{2N}$ of particle s are updated in each iteration step as follows

$$\begin{aligned} u_s = K\{&u_s + c_1 r_1 (pBest_s - x_s) \\ &+ c_2 r_2 (gBest - x_s)\} \end{aligned} \tag{16}$$

$$x_s = x_s + u_s \tag{17}$$

where $pBest_s \in \mathbb{R}^{2N}$ is the best previous position of the particle s, and $gBest \in \mathbb{R}^{2N}$ is a position of the best particle among all the particles. Also, c_1 and c_2 are acceleration constants, r_1 and r_2 are uniform random values between 0 and 1, and K is the constriction factor to ensure the convergence of PSO, which is typically set to 0.7298 [9]. Note that, in the proposed algorithm, the position x_s of the particle s is considered as follows

$$x_s = [\Delta\theta_1, \cdots, \Delta\theta_N, \Delta v_1, \cdots, \Delta v_N]^T \tag{18}$$

After initialization, each particle evaluates its current position x_s by computing J_2 in Eq. (15). If a particle realizes that the current J_2 is better than the previous one, then it updates its personal best $pBest_s$. After all particles compute and update their J_2, the values are compared with each other and their global best $gBest$ is determined. In the following step, each particle updates its position x_s and velocity u_s using Eqs. (16) and (17). Through iterations, the performance index value converges to the minimum value of J_2, in which the penalty term is equal to zero (i.e., non-conflict condition is achieved), and the variations of heading angle and speed is minimized.

4 Numerical Simulation

Numerical simulation is performed to demonstrate the performance of the proposed algorithm using a desktop PC with an Intel Core (2.80GHz) processor. The number of population and the maximum number of iteration in PSO are set to 100 and 200, respectively. A separation standard of 5 nm is considered.

Figure 2 shows an arbitrary air traffic situation involving four aircraft in the square airspace. The small triangle in Fig. 2 represents the initial position and orientation of the aircraft. The black and red arrows represent initial and final velocity vectors, respectively. Figure 3 shows the time histories of the relative distance between two aircraft without conflict resolution maneuvers. And Fig. 4 shows the time histories of the relative distance between two aircraft when the conflicts are resolved by the proposed algorithm. As shown in Fig. 3, a relative distance between aircraft 1 and 3 is 0.46 nm, which is less than 5 nm, if conflict resolution maneuver is not performed. In other words, aircraft 1 conflicts

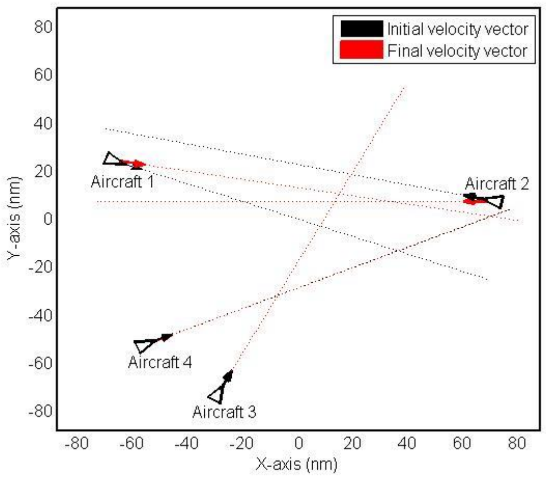

Fig. 2. Heading angle and speed changes by the proposed approach.

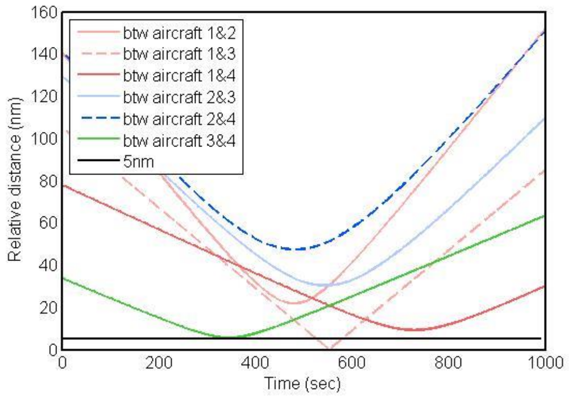

Fig. 3. Time histories of relative distance between any two aircraft before conflict resolution.

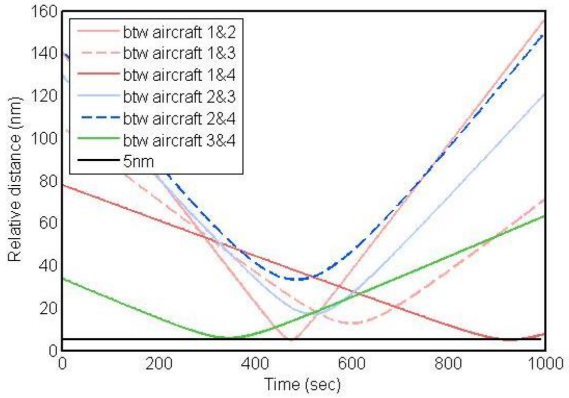

Fig. 4. Time histories of relative distance between any two aircraft after conflict resolution.

Table 1. Initial heading angle and speed of aircraft

Aircraft	Heading angle	Speed
1	-20.00 deg	8.70 nm/min
2	168.00 deg	8.70 nm/min
3	62.00 deg	8.70 nm/min
4	23.00 deg	8.70 nm/min

Table 2. Final heading angle and speed of aircraft when the conflicts are resolved by the proposed algorithm

Aircraft	Heading angle	Speed
1	-9.66 deg	9.17 nm/min
2	180.00 deg	8.70 nm/min
3	62.00 deg	8.70 nm/min
4	23.00 deg	8.67 nm/min

with aircraft 3. To resolve the conflict by the proposed method, aircraft 1, 2 and 4 change their heading angles and speeds. Aircraft 1 changes its heading angle from -20.00 deg to -9.66 deg and simultaneously speeds up to 9.17 nm/min from 8.70 nm/min. Aircraft 2 changes its heading angle from 168.00 deg to 180.00 deg, and aircraft 3 slows down to 8.67 nm/min from 8.70 nm/min. As shown in Fig. 4, the minimum relative distance is 5 nm, which is greater than 5 nm by applying the proposed conflict resolution algorithm. The overall simulation results are summarized in Tables 1 and 2. Note that the proposed algorithm takes 6.17 seconds of CPU time, and therefore it can be used as a real time conflict resolution method.

5 Conclusion

In this study, a nonlinear optimal conflict resolution scheme was proposed. The key strength of this study is that aircraft is allowed to change both its heading angle and speed to resolve the conflicts in contrast to the previous air traffic management algorithms. To deal with a nonlinearity induced by the combined maneuver, particle swarm optimization, which is one of the population-based algorithms, is adopted. Through numerical simulation considering an artificial air traffic scenario, the performance of the proposed algorithm was demonstrated.

For future work, a more practical performance index for the conflict resolution problem will be investigated. The current performance index is concerned with the conflict events only. However, the conflict resolution maneuvers may result in a deviation of the pre-planned flight path (or time). Therefore, by taking a broader perspective, the performance index should be defined considering aircraft safe separation as well as overall traffic flow.

Acknowledgement

This work was supported by the Development of Integrated Departure/Arrival Management Technologies Project funded by the Ministry of Land, Infrastructure and Transport.

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