

**Balanced Allocation on Arrival Air Traffic Flow**

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**Keywords:** air traffic flow; nonlinear integer programming model; average allocation method

**Abstract:** For arrival flight in an airport terminal airspace, the traditional first come first service caused unfair allocation of air traffic flow. A nonlinear integer programming model (NIPM) with constraints was proposed. Compared with the average allocation method, it is far better than the latter.

**Summarization**

Terminal airspace is regarded as a hub airspace, which always has a high incidence of traffic congestion. As is shown in Figure 1:

![Terminal Airspace Diagram](image)

**Figure 1** a terminal airspace

Typically, when the controllers make an allocation schedule, they generally depend on their work experience which is very subjective, the most common method which they use is the average allocation method (Hereinafter referred to as AAM) [3], as the airspace increases, AAM will lead to obvious delays in each leg. Due to flight are integers, to solve the issue of uneven allocation in each leg which is caused by AAM, a nonlinear integer programming model (Hereinafter referred to as NIPM) will be proposed [4, 5].

The advantages of NIPM are as below: the method of mathematical modeling will objectively and quantitatively give a rational allocation for the approach flights, the flow in each leg will be tend to be equilibrium, and will also reduce flight delays [6], provide supplementary allocation decisions for the controllers. The most commonly used method to solve NIPM in Operations Research is Branch and Bound method [7, 8, 9].

According to the defined parameters, with Figure 1, the flight delays of each leg in the $k$-level airspace is:

$$d^{(k)}_\eta = a^{(\delta)}_\eta - X^{(\delta)}_\eta,$$

$$(k \in \mathcal{N}, \eta \in \{1,2,3,...,m\}).$$  \hspace{1cm} (1)

The total flight delays of the $k$-level airspace can be expressed as:

$$D^{(\delta)} = \sum_{\eta=1}^{m} d^{(\delta)}_\eta, \quad (k \in \mathcal{N})$$ \hspace{1cm} (2)

The variance of the flight delays of each leg in the $k$-level airspace is:
\[
\sigma^2(k) = \frac{1}{n_k-1} \sum_{q=1}^{n_k} (d_{q}^{(k)} - D^{(k)} / n_k)^2 ,
\]

\((k \in \mathcal{N})\) \hspace{1cm} (3)

In order to minimize the sum of variance between actual allocation and the number of predicted flights in each leg, the objective function is:

\[
\min \left\{ \sum_{k=1}^{K} \sigma^2(k) \right\}
\]

(4)

The following constraints should be satisfied:

Constraint (1): The total number of each leg in the \(k\)-level airspace can not respectively exceed the total number of predicted flights.

Constraint (2): The actual flow of each leg can not exceed the predicted flow in the \(k\)-level airspace.

Constraint (3): The total number of allocated flights of each leg in the \(k+1\)-level airspace which is adjacent to the \(k\)-level airspace can not exceed the allocated aircraft in the \(k\)-level.

Constraint (4): actual flight sorties in each leg should be non-negative integers.

Based on the airspace structure, airspace information (10:00 to 11:00), the results solved by LINGO \(^{10}\), and a comparison of two methods are listed below.

**Table 1 results solved by LINGO**

<table>
<thead>
<tr>
<th>Code</th>
<th>Origin</th>
<th>Destination</th>
<th>Connected route</th>
<th>Predicted flow (sorties)</th>
<th>Actual allocated flow (sorties)</th>
<th>Flight delays of AAM</th>
<th>Flight delays of NIPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HO</td>
<td>XIY</td>
<td>H14</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>YIJ</td>
<td>XIY</td>
<td>G212</td>
<td>9</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>SHX</td>
<td>LXZS</td>
<td>H14</td>
<td>10</td>
<td>8</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>NSH</td>
<td>LXZS</td>
<td>G212</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1-1</td>
<td>JIG</td>
<td>HO</td>
<td>H14</td>
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<td>3</td>
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<td>G212</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

As is shown in Table 1, compared with NIPM, the results solved by AAM will lead to serious flight delays in each leg in each airspace, the predicted flow in first-level leg is relatively more than others, there are five flight can not enter next-level airspace during 10:00 to 11:00. With further spread of air traffic congestion, leg 3-1 and 3-2 have had more flight delays, especially leg P53-SHX, there will be six flight delays in this leg in an hour.