REAL TIME PATH PLANNING METHOD OF AIRCRAFT FORMATIONS

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Abstract

In this paper, a new, real time method to organise aircraft formations and their paths is proposed. In order to make most use of the drag reduction effect of formation flying, the proposed method deploys two envelopes to select efficient topologies. Numerical calculations show that the proposed method can successfully select efficient topologies, with more than 60% of the topologies having energy parameters below 5%.

1 Introduction

Formation flight has attracted a lot of attention in recent years. This is because formation flight offers aircraft a great induced drag reduction of up to 70% (Fig. 1) which enables aircraft to go up to 80% farther. Also, for unmanned aerial vehicles (UAVs, e.g. Fig. 2), formation flight has two more advantages by having multiple units airborne. They are, namely,

1. Payload can be distributed between units and more complex performed.
2. Robustness to unit failures can be increased by having other units take over failed unit's role.

Most of previous researches have concentrated their focus on aerodynamic aspects and control of aircraft formations. For example, the aerodynamic advantages of formation flight are stated in detail in Lissaman [7], and several control approaches have been proposed in Fowlwer [2], Wolfe [10] and Hino [4]. Also, research has already been on task assignment and reassignment for multiple UAVs. Shima [9] used genetic algorithm to solve the task assignment problem and Healey [3] proposed a task reassignment method in case of unit failures. However, when it comes to planning efficient paths for aircraft formations, not much research has been done, with Ribichini [8] the only research the author has came across that takes the aerodynamic advantage of formation flight into account. Ribichini [8] organised formations for trans-ocean flights. Therefore the problem was constrained to problems with two degrees of freedom (time and position along airway).

In this paper, we will propose a formations formation organisation method that is applicable to various problems, such as

1. problems with different degree of freedom
   (a) two degrees of freedom (e.g. airway) – distance along airway and time
   (b) three degrees of freedom (e.g. free flying airspace) – planar coordinates and time
2. problems with different number of departures, destinations
   (a) point-to-multipoint (P2MP) problems – problems with one departure and multiple destinations, or vice versa.

Table 1 Specification of MARS07AF.

Item	Value	Unit
Total Weight	2.00	kg
Wingspan	1.75	m
Wing area	0.392	m^2
Aspect ratio	7.7	
Cruise speed	15.0 - 30.0	m/s
Endurance	30.0	min
Payload	0.50	kg

   (b) multipoint-to-multipoint (MP2MP) problems – problems with multiple departures and multiple destinations.

3. problems with various constraints
   (a) time of arrival constraints
   (b) airspeed limitations
   (c) energy constraints

P2MP problems and MP2MP problems are similar to Steiner minimal tree (SMT) problems (Hwang [6]) and Directed Steiner Forest (DSF) problem (Feldman [1]), respectively, but greatly differ from them as the weights on each path (or path segment) changes dynamically with the number of aircraft flying along it. The author has attempted to apply knowledge from SMT problems to P2MP problems with and without energy constraints (Hino [5]) with great success. The optimal topology was selected for more than 80% of the cases tested. However, we found it difficult to apply it to MP2MP problems or problems with various constraints. In this paper, we will work on another idea proposed in Hino [5], and develop a method that method that can be applied to all of the problems noted above.

This paper is organised as follows. In the following section, the proposed formation organisation method will be described in detail. In section 3, the features of the proposed method will be stated. Then, results of some test cases will be shown in section 4. Finally, the paper will be concluded in section 5.

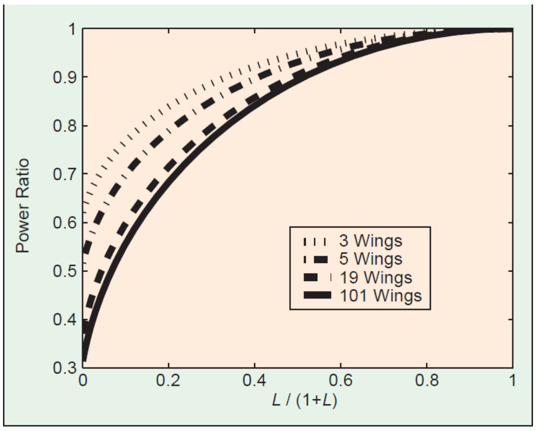


Fig. 1 Induced drag reduction by formation flight（Fowlwer [2]）. L is the distance between wingtips normalised by wingspan

Fig. 2 Example of small UAV - Mitsubishi MARS07AF

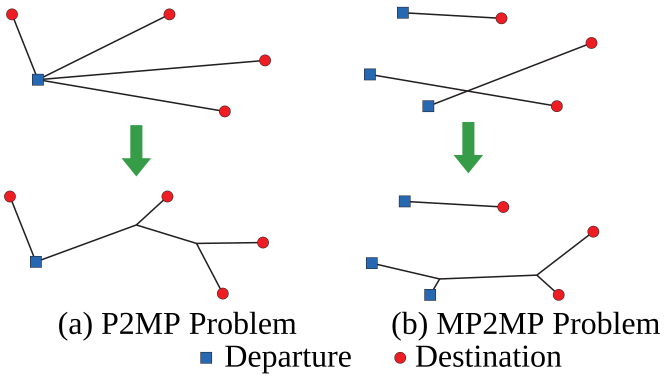

(a) P2MP Problem (b) MP2MP Problem

Fig. 3 P2MP problem and MP2MP problem.

Table 2 Relationship between the number of initial aircraft groups and the number of possible topologies.

No. of Initial Aircraft Groups	P2MP Problem	MP2MP Problem
1	1	1
2	1	2
3	3	19
4	15	691
5	105	54746
6	945	6617011

2 Method Outline and Features

The main objective of formation organisation methods is to quickly select efficient "topologies" – the combination of 1) whether to group aircraft together, and 2) in which order to group/separate formations. The problem here is that the number of possible topologies rapidly increases with the initial number of aircraft groups (Table 2), making any brute force method unfeasible. The number of topologies are, in fact, of $O(n^n)$ for P2MP problems and of $O(n^{2n})$ for MP2MP problems, where n is the initial number of aircraft groups.

The method proposed in this paper makes best use of the fact that the region where aircraft will save fuel if they rendezvous/separate with other aircraft can be calculated analytically. For example, the region where aircraft with airspeed limitation should rendezvous with other aircraft can be calculated by solving the following equations.

$$E_0/L \geq D_s\rho_1 + D_f\rho_2 \tag{1}$$

$$\rho_2 = \sqrt{1+\rho_1^2 - 2\rho_1\cos\theta} \tag{2}$$

where E_0 is the amount of energy required, D_s and D_f are the minimum drag (taking minimum airspeed into account) before and after rendezvous, and $\rho_1 \equiv L_1/L, \rho_2 \equiv L_2/L$ (Fig. 4). The location of separation point is not taken into account in this calculation because, the aircraft under consideration can save the largest amount of energy when it flies in formation to its the destination.

2.1 Topology Selection

In the proposed method, we use the above knowledge to create two envelopes, R-envelope and S-envelope, to quickly select efficient topologies. R-envelope is the convex polygon containing the region where, if the aircraft rendezvouses with other aircraft, it has the possibility to save energy (Fig. 5). S-envelope is computed from the intersection of R-envelopes, and is the convex polygon containing the region to where the aircraft has to fly with the other aircraft if it is to save energy. The reason convex polygons are used instead of the true regions is because the true regions are represented by nonlinear functions, and therefore are difficult to compute their intersections (Fig. 6). Notice that computing S-envelopes is a region-to-region mapping, compared to point-to-region mapping in the case of R-evnelopes. However, this region-to-region mapping is computationally expensive. Instead, an approximate of the S-envelope is used. The approximate S-envelope is defined as the convex hull of the S-envelopes for the vertices of the intersection of R-envelopes and several internal points of the intersection (Fig. 7). The number of internal points used is carefully selected, so that the approximate S-envelope is sufficiently close to the true S-envelope, while keeping the computation cost as low as possible. Currently, the author uses the geometric centre of gravity of the intersection, and the dividing points between the vertices and the centre of gravity.

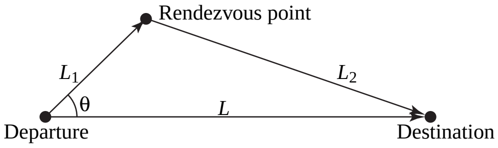


Fig. 4 Symbol definition.

Algorithm 1 Topology Selection Process (full)

```
while Replacements can be made do
    for all Pairs of paths do
        Compute R-envelopes
        Compute intersections of R-envelopes
        if Intersection of R-envelope exists then
            Compute S-envelopes
            Compute intersections of S-envelopes
        end if
    end for
    Select pair of path to replace
    Replace selected paths
end while
```

Using the two envelopes, the proposed method selects the topology by the algorithm stated in Algorithm 1 (see also Fig. 8). In case of P2MP problems, the selection process can be shortened to Algorithm 2, as the aircraft can always start flying in a formation. Algorithm 2 has already been used in Hino [5], and as already stated, showed satisfactory results.

After the topology has been selected, inserted waypoints (i.e. rendezvous/separation points) are optimised using sequencial quadratic programming (SQP). SQP optimises waypoints so that the total energy consumption is minimal, whilst ensuring no aircraft requires excessive energy for the benefit of others. During the optimisation, no change in topology is made.

Algorithm 2 Topology Selection Process (shortened)

```
while Replacements can be made do
    for all Pairs of paths do
        Compute S-envelopes
        Compute intersections of S-envelopes
    end for
    Select pair of path to replace
    Replace selected paths
end while
```

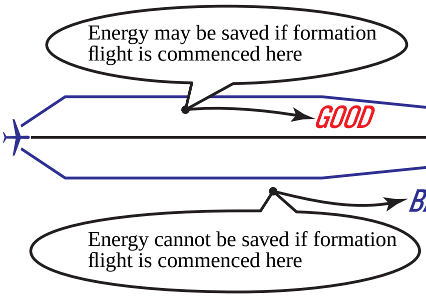


Fig. 5 Definition of R-envelope.

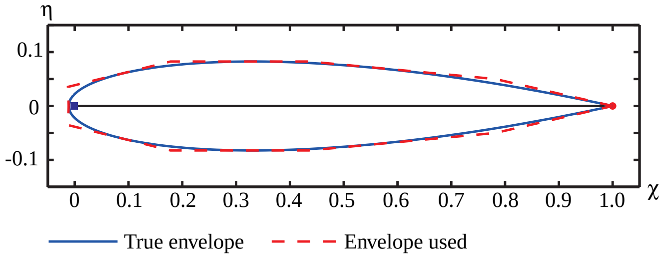
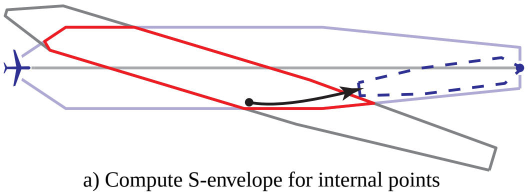


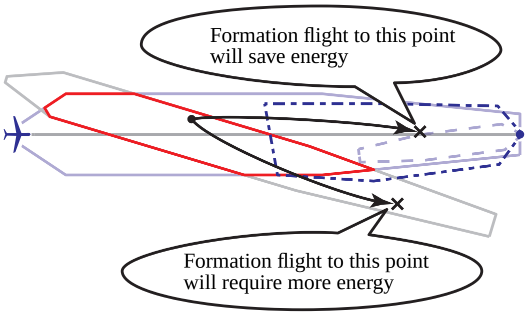
Fig. 6 R-envelope for 2 MARS07AFs originally flying at 15 m/s with no time of arrival or energy constraints. Normalised by original path length. Original path is along $\eta = 0, \chi \in [0, 1]$.

2.2 Reflecting Constraints

Various constraints on aircraft can be taken into account when computing the envelopes. The most common constraints on aircraft are 1) airspeed limitations, 2) time of arrival and 3) available energy. Airspeed limitations are the easiest constraint to take into account, and only the original airspeed affects the shape of the enve-

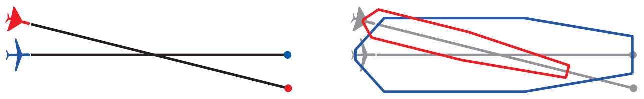


a) Compute S-envelope for internal points



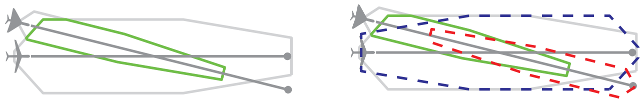
b) Compute convex hull of all the S-envelopes for internal points

Fig. 7 Definition of S-envelope. S-envelope computed from the intersection of R-envelope in red bold line. Approximate S-envelope for the intersection (blue dashed line) computed as the convex hull of S-envelope computed from the internal points of the intersection (blue broken line).



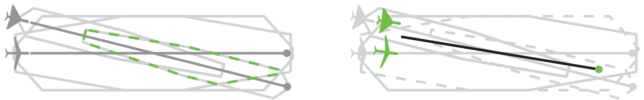
0) Original paths

1) Compute R-envelope



2) Compute intersection of R-envelope

3) Compute S-envelope



4) Compute intersection of S-envelope

5) Select and replace paths

Fig. 8 Topology selection process.

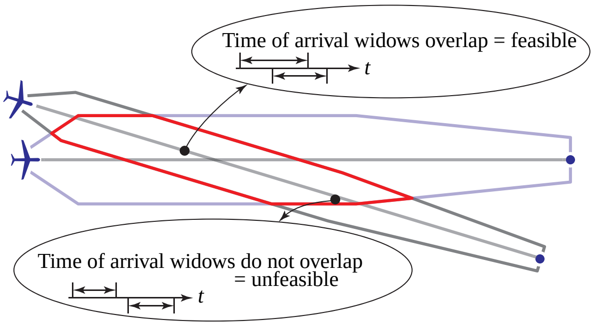


Fig. 9 Time of arrival windows within R-envelope intersection.

lope. However, airspeed limitations, especially maximum airspeeds, must be considered carefully when calculating intersections of envelopes. This is because for some places within the intersection, the time of arrival windows that allow aircraft to save energy may not overlap, and therefore be unfeasible (Fig. 9). On the other hand, constraints on time of arrival and energy greatly affect the shape of the envelope. Constraints on time of arrival affects the envelope's "width," whilst constraints on available energy affects the envelope's "length" (Fig. 10).

Envelopes can also be designed for problems with twos degrees of freedom (Fig. 11). In this case, information on feasible time of arrival windows are explicitly reflected on the envelope, and therefore less care needs to be taken when computing intersections.

Derivation of the various envelopes noted above is be omitted due to lack of space.

3 Features of the proposed method

The proposed method has several notable features. First, the topology selected by the proposed method is always feasible, and never inefficient as constraints on aircraft are reflected in the envelope. Topologies obtained during anytime in the selection process are also guaranteed to be feasible and efficient. Second, the computation cost is very low, and the proposed method can select efficient topologies within a few seconds. Monte Carlo test is used to measure the time required to select topologies. Problems with 2-10

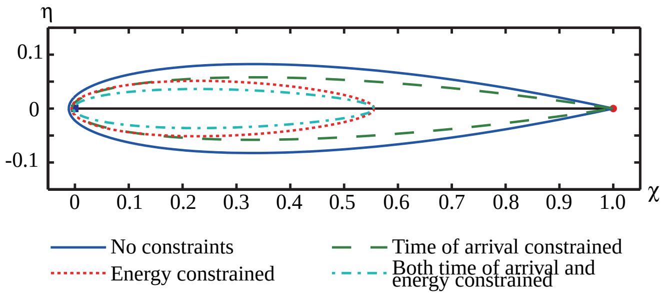
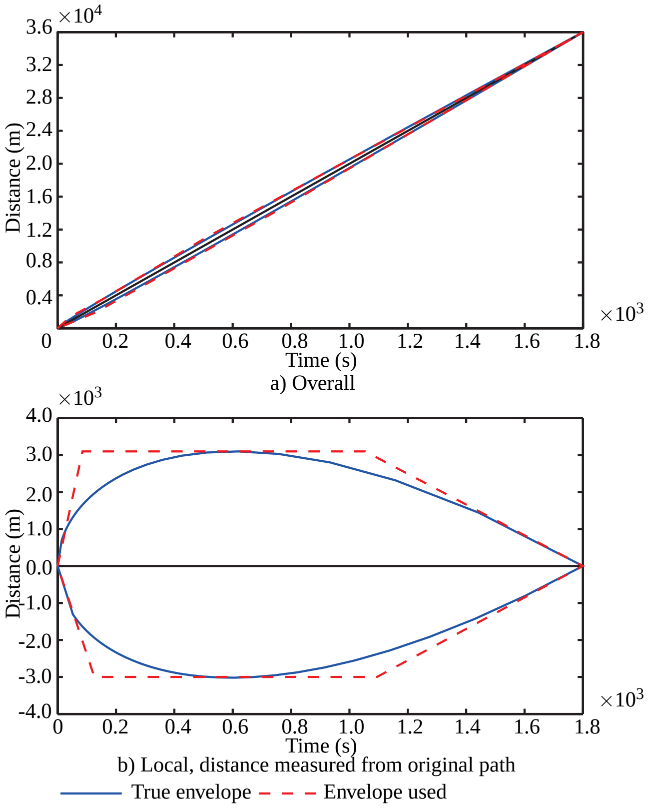


Fig. 10 R-envelopes with various constraints for 2 MARS07AFs originally flying at 15 m/s. Aircraft only have 99% of the required energy for energy constrained envelopes. All envelopes normalised by original path length. Original path along $\eta = 0, \chi \in [0, 1]$.



a) Overall

b) Local, distance measured from original path

Fig. 11 Time of arrival constrained R-envelope for 1 MARS07AF flying along an airway. Original airspeed 20 m/s.

Table 3 Test condition and coefficient of determination of topology selection speed measurement.

Item	Laptop PC	Desktop PC
Processor	Intel® Core i7™ L640	Intel® Core i7™ 870
Frequency	2.13 GHz	2.93 GHz
RAM	4.00 GB	16.00 GB
OS	Windows 7 (64bit)	
Coefficient of determination	0.9971	0.9998

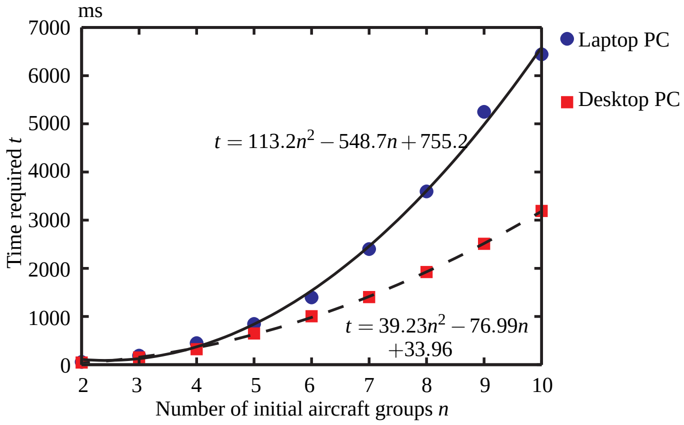


Fig. 12 Time required to select topologies. Average of 1000 cases.

initial aircraft groups are evaluated. 1000 cases are generated for each number of initial aircraft group. The test is run on two separate PCs (Table 3). Test results (Fig. 12) show that topologies can be, on average, selected within 7 seconds for problems with 10 or less initial aircraft groups, and that the proposed method has a computation cost of $O(n^2)$ in selecting topologies. Also, in the proposed scheme, optimisation of waypoints, which is computationally expensive, takes place only once, reducing computation cost even more. Furthermore, the waypoints added during the selection process acts as a good initial guess for the optimisation, speeding up the optimisation. Finally, as the proposed method visually indicates which aircraft have the potential to save energy, people (e.g. UAV operators or air traffic controllers) can take over the decision making process at any stage of the topology selection.

Table 4 Mote Carlo test conditions.

Aircraft groups			
Group		Homo.	Hetero.
1	MARS07AF	1	0
	FCTR UAV	0	1
2	MARS07AF	3	3
	FCTR UAV	0	0
1	MARS07AF	2	1
	FCTR UAV	0	1
Departure points			
Randomly generated within	$x \in [-500, 500]$ (m) $y \in [-250, 250]$ (m)		
Destinations			
Randomly generated within	$x \in [-500, 500]$ (m) $y \in [39750, 40250]$ (m)		

4 Test cases

Monte Carlo test is conducted to evaluate the proposed method. 100 cases are randomly generated for both homogeneous and heterogeneous aircraft groups, with no time of arrival constraints. The details of test conditions such as aircraft groups are listed in Table 4. Only the aerodynamic performance and speed limitations of the FCTR UAV (Fig. 5 and Table 5) is used, and constraints on flight time are disregarded. The reason paths are put so close together is because, by doing so, selecting optimal topology is more difficult.

The performance of the method is evaluated using energy parameter ε:

$$\varepsilon = \frac{E_{\text{optimal}} - E_{\text{selected}}}{E_{\text{original}} - E_{\text{optimal}}} \tag{3}$$

which is the ratio of how much more energy could have been saved and how much energy could have been saved.

The test results are shown in Figs. 14 and 15. In both cases 60% of the topologies selected by the proposed method had an energy parameter under 5%, which means that the proposed method was successful in selecting optimal or near-optimal topologies.

Fig. 13 FTCR UAV

Table 5 Specification of FTCR UAV

Item	Value	Unit
Weight	2.5	kg
Wingspan	1.38	m
Wing area	0.227	m^2
Aspect ratio	5.32	
Cruise speed	18.0 - 35.0	m/s
Flight time	8	min

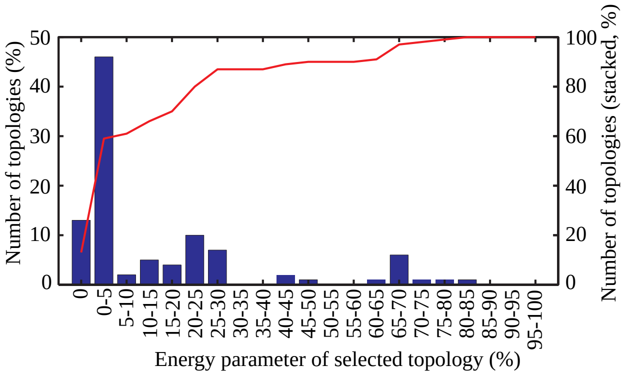


Fig. 14 MCT result for homogeneous formations.

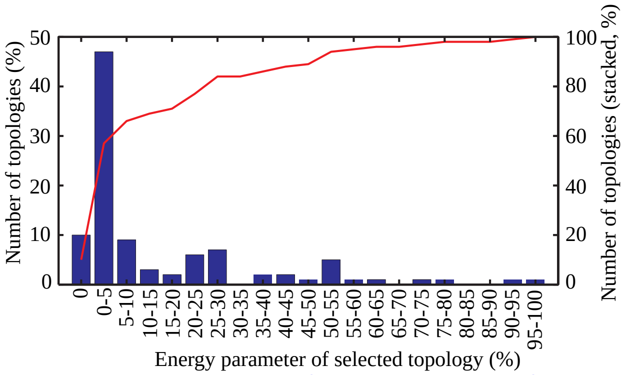
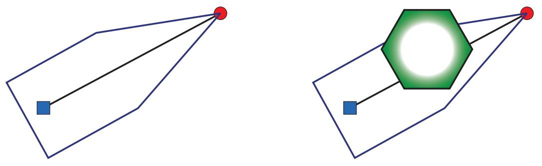
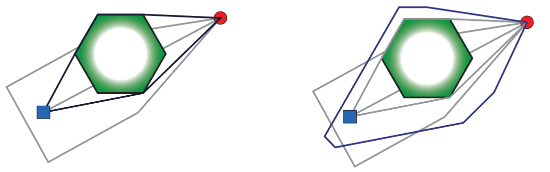


Fig. 15 MCT result for heterogeneous formations.



a) Original path

b) If obstacles exist



c) Compute two shortest paths that avoid obstacles

d) Expand envelope around it

Fig. 16 Envelope expanded around obstacles.

5 Conclusion

In this paper, formation organisation method that can be used in real time applications is proposed. The objective of the proposed method is to quickly select topologies that can save energy by formation flying. The method is characterised by the use of two envelopes that incorporate various constraints on aircraft. These envelopes visually indicate where aircraft should rendezvous/separate. By using these envelopes, feasible and efficient topoologies can easily be selected by both computers and humans. Test cases showed that the method can select efficient topologies, with 60% of the selected topologies having an energy parameter parameter under 5%. Also the time required to select topologies is less than 7 seconds for problems with 10 or less initial aircraft groups.

The next steps of this research will be to

1. run test cases with time of arrival constraints
2. incorporate obstacle avoidance
3. application to real life problems such as trans-ocean flights

Test cases with time of arrival constraints are currently being run. For obstacle avoidance, the author is currently considering "expanding" the envelope around obstacles (Fig. 16). Results on these problems will be shown at the conference.

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