THE METHOD FOR NONLINEAR ANALYSIS OPTIMIZATION IN AERONAUTICAL RESEARCH

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Abstract

This paper introduces a method that utilizes the response surface methodology into genetic algorithm to solve the aeronautical nonlinear optimization problem. This method is used in a transonic flutter optimization of a horizontal tail and the analysis result shows that this method can be used in aeronautical nonlinear multidisciplinary optimization.

1 General Introduction

There are huge nonlinear analytical problems in the aeronautical discipline, such as transonic aero-elasticity, unsteady aerodynamic research, structural post bucking, material nonlinearity and far-field aero-acoustic calculation. The solutions of those problems always need so much iterations and repeated linear calculations resulting in enormous calculation efforts and time consumed. The nonlinear optimization becomes more difficult.
Because of the complexity and unpredictability of nonlinear problems and more and more nonlinear problems emerging in the advanced aircraft development, solving the nonlinear problems becomes the main challenge of aircraft design. In the past, we used to adopting relative larger safe coefficient to cover the nonlinearity issue, which could solve the problem in a rough way. However, that kind of method could not realize the optimal design. The research on the nonlinear system optimization has been put on the agenda based on above reasons.

1.1 Nonlinear System Optimization

Aircraft design is a complicated system design which is composed of many disciplines. Each real system is nonlinear system, so analysis and optimization of nonlinear system mean so much to aircraft design.
Nowadays, when it comes to nonlinear system analysis and calculation, approximate linear iteration, semi-experimental, semiempirical formula are the main methods. These methods could realize analysis and calculation to some extent. However, when utilize with existing optimal methods, it will greatly increase the analysis amount. Therefore, optimization on nonlinear problems becomes more difficult or even cannot realize.
Transforming the problem to the one in parametric space is also difficult in nonlinear optimization. Nonlinear system itself involves in many design variables and constraints. How to find the critical, sensitive design variable and then change them to the optimization variables share the same difficulty.

1.2 Optimization Algorithm

Classical optimization algorithms, such as Newton ’ s method, Sequential Linear Programming (SLP)、Steepest Descent Method have difficulty in realizing nonlinear optimization. In recent years, Stochastic algorithms, such as Simulated annealing, Genetic Algorithms (GA), are applied in nonlinear system optimization. On the one hand, the optimization is not influenced by the system’s exact mathematic form. The Stochastic algorithms usually use the value of objective function as the search information and evaluate the individual with fitness value rather than with the differential coefficient or other assistant information [1]. On the other hand, such algorithms take the code (binary, gray-code) of the design variables as calculation object rather than the variable itself. Therefore, it adapts to large scale variables optimization.

This paper adopts genetic algorithm (GA) as the optimization algorithm.GA is formed based on Darwin's theory of natural selection and Mendelism. It is a self-adaptive global probability search method and originated from 1960s based on the research on the nature and man-made self-adaptive system [2]. It is first proposed by professor Holland of Michigan University, U.S.A.

1.3 Response Surface Method

In the optimization, the response of system state is needed continuously. However, nonlinear system calculation is so complicated and time consumed that it is necessary to use an approximate method to decrease the computation required. On such condition, response surface method could offer a solution.

The response surface (RS) methodology approximately builds a function model between the variables interested in and the response value. The response surface is the combination of math and statistical method [3]。The RS method fit the complicated and unknown function with one degree or quadratic polynomial in limited area. It is easy to calculate and it includes test design, model building, model reasonability testifies, looking for the best combination condition and other tests and statistical technology.

The RS method has some disadvantages, such as, when the design variable become large, it is harder to construct an approximate polynomial explicitly to analyze. Furthermore, when the model has high nonlinearity or the distribution of error is not normal distribution, it is difficult to find the reasonable optimal design. Although the commercial software make the establishment, visualization, error analysis of RS more convenient, for a certain system model , how to select design variables, sample points and other control parameters still need to be researched.

In this paper, Fraunhofer SCAI DesParO is used to build the RS.

2 Optimization Example

This paper introduces a method that utilizes the RS methodology and the GA method to solve nonlinear optimization problem. In this method, the response value gotten from the RS is adopted instead of calculating the individual's fitness by the nonlinear analysis, which obviously reduces the optimization time and overcomes the difficulty in the nonlinear problem. The method has been used in a transonic flutter optimization of a horizontal tail wing.

The non-decayed vibration of aircraft parts caused by aero-elasticity is defined as flutter. The amplitude of flutter is rather large and will destroy the structure of aircraft. For the flutter margin design, under certain flight height, the quicker the aircraft flies, the greater the flutter speed would be. For that reason， the enhancement on flutter speed is an important goal in flutter optimization [4-5]. There are several factors affecting flutter [6]: the bending stiffness and torsional stiffness of lifting surface, center of gravity, center of stiffness, focus point's position, concentrated mass on lifting surface, the flight height, compressibility of air. In flutter optimization of composite structure, it is usually to change the direction of stiffness of wing and couple of deformation to control the aero-elasticity deformation of the wing, that is aero-elasticity tailoring. Aero-elasticity tailoring is such a method that through altering the ply direction, thickness and ply percentages to realize optimization.

Whatever theoretical calculation or wing tunnel test, just single state' computation needs so much resource and time, not to say optimization. For example, the time spent on the analysis of the transonic flutter speed is nearly 10 thousand times of the analysis of the subsonic. Considering those difficulties stated above, a new method is applied in the transonic flutter speed optimization. It started considering the stiffness property of laminate, then select the critical design point, and then build the response surface, finally resolve the transonic flutter speed optimization.

2.1 Example Introduction

Firgure1 is the wing model used in the optimization. The white area is the original design element of upper skin.

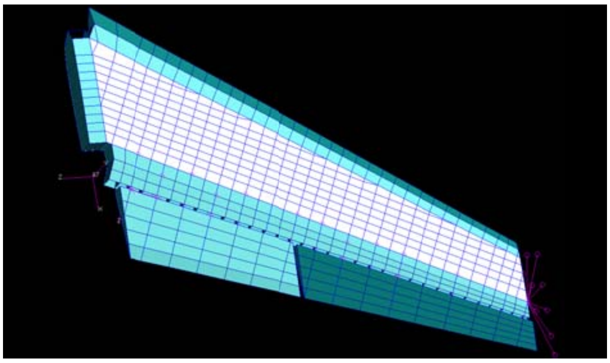


Fig.1.The Fem Model of the Horizontal Tail Wing

This method improves the transonic flutter speed through changing the stiffness direction. In the beginning of optimization, it is hard to find the critical area and the limited on the calculation make the whole skin elements as design elements impossible. Therefore, the subsonic flutter optimization is carried out before the transonic flutter speed.

In order to obtain the most sensitive skin area to flutter speed and begin the transonic flutter optimization with a initial value, the thickness of the original design element decreases to 0.5mm($0/90/\pm45 = 1/1/2$) and then go on with subsonic flutter speed optimization. According to the traditional design, the transonic flutter speed equals the corresponding subsonic flutter speed plus a certain coefficient. Therefore, when the subsonic flutter speed increases to some value, that state will selected as the beginning point of transonic flutter speed optimization.

The original design elements are the finite elements in the upper and lower skin of the wing box counted 558. The first step is to rank the elements according to the subsonic flutter sensitivity and then chose the elements with high sensitivity value as the design elements in the transonic flutter optimization. Each design element is defined with different composite laminate that is the thickness and ply percentages. The ranking line of the design elements is shown in Fig.2. The selected elements with high sensitivity value on upper wing are showed in Fig.3 and thickness of these elements and surrounding areas are uniformed .The upper and lower wing each has four sensitive areas, just like the Fig.4(a) for lower skin, and Fig.4 (b)show for upper skin.

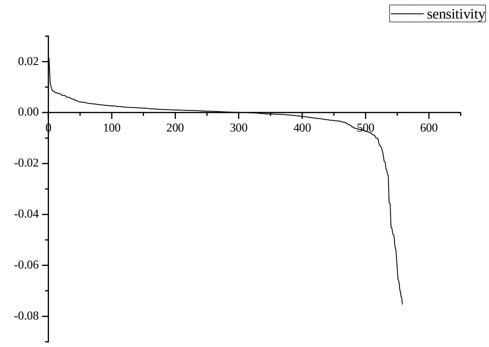


Fig.2.The Sensitivity Ranking of the Design Elements

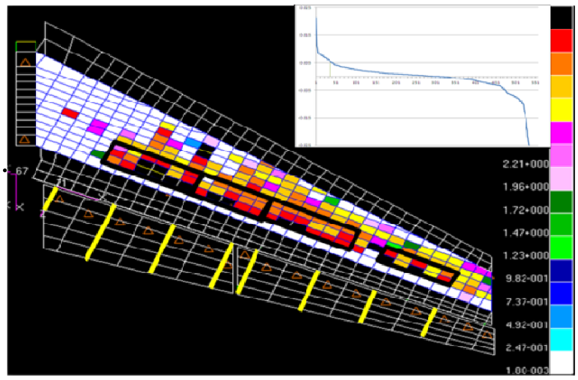
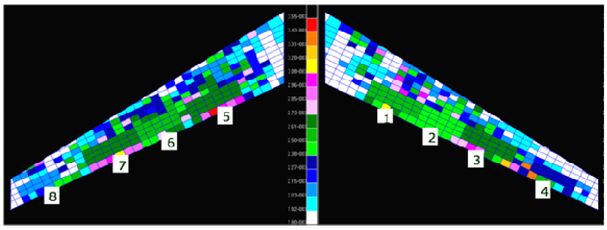


Fig.3.The Selected Elements with High Sensitivity Value on Upper Wing



(a) (b)

Fig.4. The Design Elements for Transonic Flutter Optimization

2.2 The Optimization Process

So as to realize the whole wing's transonic flutter speed optimization, combining with the above chapters, the whole optimization process is formed just showed in Fig.5.

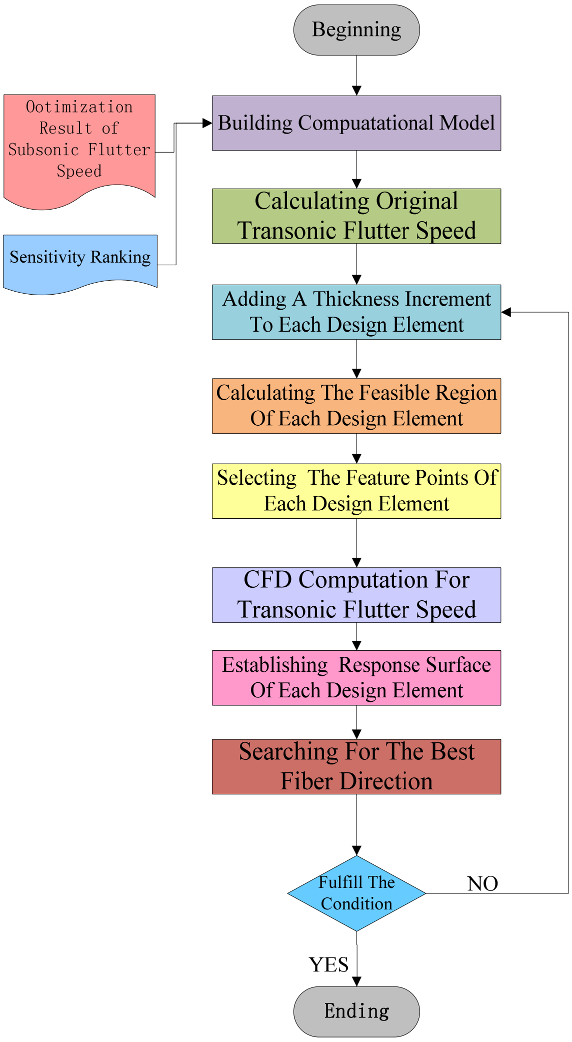


Fig.5.Processes for Transonic Flutter Speed Optimization

2.3 Response Surface Establishment

The first step to build a response surface is to select the sample points. The most usual method to select sample points mainly start from the interpolation points and then get 2n+1 points. Analyzing each sample point, a response value will achieved. With these 2n+1 response values, the 2n+1 coefficients in response formula can be determined so as to certify the response formula[7].

For complicate structure, it is time consumed to carry out nonlinear response computation, so when deal with such systems, it is better to minus the sample points without influencing the computation accuracy. On the other hand, the sample points selected with some method are meaningless or are not realistic to real system, so it is necessary to design the boundary and constraints of sample points according to real system.

The design variables of transonic flutter speed are the related parameters of design elements, that are ply direction, thickness and ply properties. The ply direction and the ply properties are not easy to describe with simple mathematical variables (actually they are discrete variables). It is very difficult to construct the response surface between them and the transonic flutter speed because the combinations of the variables are too complex. Therefore, the three-dimensional stiffness of the skin laminate is selected to describe the mechanical characters of the composite laminate. Through designing the three-dimensional stiffness, stiffness sample points are selected to build the response surface.

For the symmetric and balanced laminate, three-dimensional stiffness is stated with formula (1).This method converts the discrete variables to continuous variables and then the three-dimensional stiffness is used to set up the response surface as independent variables. After the optimization, the variables are treated as discrete variables according to the structure design principle.

$$\begin{cases} (ET)_x = E_x t = A_{11} - \dfrac{A_{12}^2}{A_{22}} \\ (ET)_y = E_y t = A_{22} - \dfrac{A_{12}^2}{A_{11}} \\ (GT) = G_{xy} t = A_{66} \end{cases} \qquad (1)$$

(ET)x---the equivalent in-plane tensional/compressive stiffness of the laminate in the direction X; *(ET)y*--- the equivalent in-plane tensional/compressive stiffness of the laminate in the direction Y; *(GT)* ---the equivalent in-plane shear stiffness of the laminate; t---thickness of the laminate; *A11, A12, A22, A66*---the in-plane, tensile stiffness coefficient.

The in-plane, tensile stiffness coefficient are calculated following these formula [8].

$$A_{ij} = \sum_{k=1}^{N} t_k \left(\overline{Q}_{ij}\right)_k \; (i = 1,2,6; j = 1,2,6) \qquad (2)$$

N---layers of the laminate; t_k---thickness of single layer;

$\overline{Q}_{ij}$ *(i=1,2,6;j=1,2,6)*are given in formula 3：

$$\begin{cases}\overline{Q}_{11} = m^4Q_{11} + 2m^2n^2(Q_{12} + 2Q_{66}) + n^4Q_{22} \\ \overline{Q}_{12} = m^2n^2(Q_{11} + Q_{22} - 4Q_{66}) + (m^4 + n^4)Q_{12} \\ \overline{Q}_{22} = n^4Q_{11} + 2m^2n^2(Q_{12} + 2Q_{66}) + m^4Q_{22} \\ \overline{Q}_{16} = m^3n(Q_{11} - Q_{12}) + mn^3(Q_{12} - Q_{22}) - 2mn(m^2 - n^2)Q_{66} \\ \overline{Q}_{26} = mn^3(Q_{11} - Q_{12}) + m^3n(Q_{12} - Q_{22}) + 2mn(m^2 - n^2)Q_{66} \\ \overline{Q}_{26} = mn^3(Q_{11} - Q_{12}) + m^3n(Q_{12} - Q_{22}) + 2mn(m^2 - n^2)Q_{66}\end{cases} \quad (3)$$

Q_{ij} (i=1,2,6;j=1,2,6)-- reduced stiffness. the relationship with engineering constants are:

$$\begin{cases}Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}} \\ Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}} \\ Q_{66} = G_{12} \\ Q_{12} = \mu_{12}Q_{22} = \mu_{21}Q_{11}\end{cases} \quad (4)$$

$E_1, E_2, \mu_{12}, G_{12}$--- engineering constants of single layer.

$\overline{Q}_{ij}$ change with ply direction, A_{ij} have nothing to do with stacking sequence and are determined only by ply percentages. Therefore, considering a laminate compose with ply α, β, γ, A_{ij} can be stated in those forms：

$$A_{ij} = (N_\alpha \overline{Q}_{ij,\alpha} + N_\beta \overline{Q}_{ij,\beta} + N_\gamma \overline{Q}_{ij,\gamma}) \, t_k \text{ (i=1,2,6;j=1,2,6)} \quad (5)$$

$$\begin{cases}(ET)_x = \left(N_\alpha \overline{Q}_{11,\alpha} + N_\beta \overline{Q}_{11,\beta} + N_\gamma \overline{Q}_{11,\gamma}\right) t_k - \frac{\left(N_\alpha \overline{Q}_{12,\alpha} + N_\beta \overline{Q}_{12,\beta} + N_\gamma \overline{Q}_{12,\gamma}\right)^2 t_k}{\left(N_\alpha \overline{Q}_{22,\alpha} + N_\beta \overline{Q}_{22,\beta} + N_\gamma \overline{Q}_{22,\gamma}\right)} \\ (ET)_y = \left(N_\alpha \overline{Q}_{22,\alpha} + N_\beta \overline{Q}_{22,\beta} + N_\gamma \overline{Q}_{22,\gamma}\right) t_k - \frac{\left(N_\alpha \overline{Q}_{12,\alpha} + N_\beta \overline{Q}_{12,\beta} + N_\gamma \overline{Q}_{12,\gamma}\right)^2 t_k}{\left(N_\alpha \overline{Q}_{11,\alpha} + N_\beta \overline{Q}_{11,\beta} + N_\gamma \overline{Q}_{11,\gamma}\right)} \\ (GT) = \left(N_\alpha \overline{Q}_{66,\alpha} + N_\beta \overline{Q}_{66,\beta} + N_\gamma \overline{Q}_{66,\gamma}\right) t_k\end{cases} \quad (6)$$

As soon as the longitudinal, transverse, tortional stiffness and laminate thickness are already known, formula 5 can be taken into formula 1 and then with formula 6 then the equivalent stiffness can be calculated [9].

The feasible stiffness region of each design element could be built as soon as the engineering constants of single layer, thickness and ply percentages of laminate are already known. The stiffness feasible region of design element may be the same or different depending on the thickness region of each design element.

After analyzing the stiffness feasible region, points that could present the features of the feasible region are selected. The stiffness feasible region and part of definition points of one design element are shown in Fig.6. Each design element has m definition points then the transonic flutter speed is calculated by CFD relatively. Finally, the response surface of each element is built by Fraunhofer SCAI DesParO with m definition points and m transonic flutter speed relatively. If the wing has n design elements, n number of response surface will be built. One of the response surfaces is shown in Fig.7. To build the whole RS of the wing, m*n nonlinear transonic flutter computation will be needed, that is a huge scale computation for the RS preparation before optimization. In this example, the definition points of each design element are 9. There are 8 design elements and it is need to build 8 RS. The calculation for transonic flutter speed is 72times.

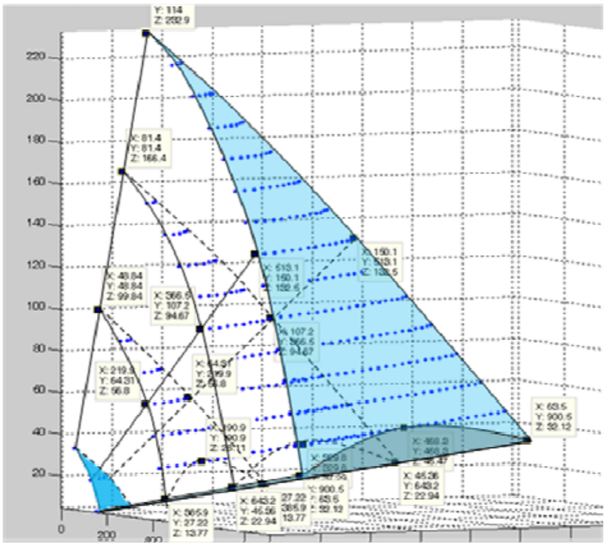


Fig.6. Stiffness Feasible Region and Feature Points

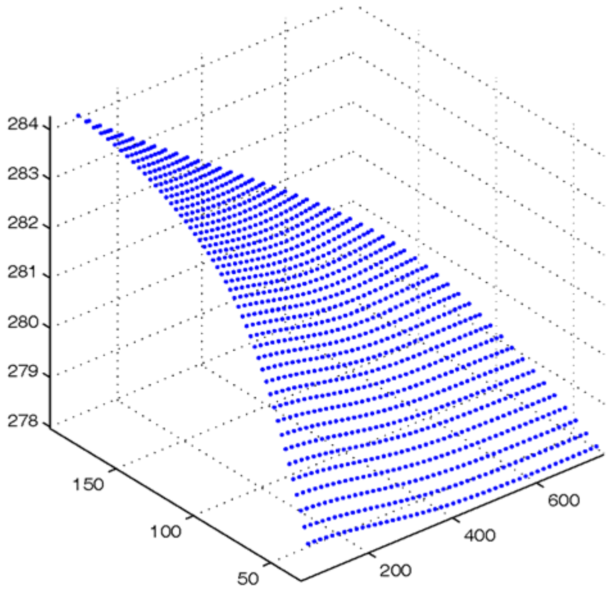


Fig.7. One of Response Surfaces with Certain Thickness

2.4 Precision Validation of Response Surface

The black line (represents the result searched in RS) and red line (represents the result calculated by ZaerO) in Fig.8 show the comparison between response value and accurate calculation of 90 design points. This step is carried out before the whole optimization so as to testify the precision of RS. The figure indicates that the transonic flutter speed achieved from RS nearly the same as the accuracy calculation value.

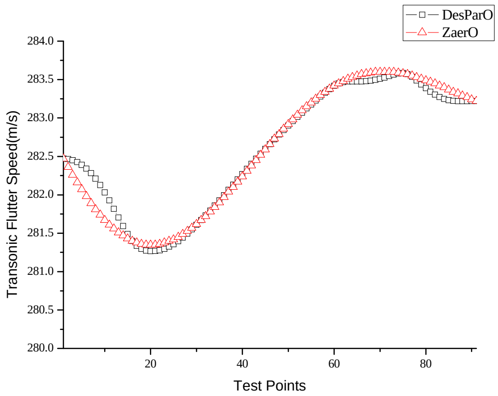


Fig.8. Comparison Between Response Value and Accurate Calculation

Furthermore, the values in table 1 are very close to reference value. Both the fitting line and the precision prove that the RS built with this method is reliable.

Table1.PrecisionValidation of Response Surface

Index	Real Value	Reference Value
RMSE	4.69E-05	0
R2	0.977215	1

2.5 Optimization

Based on the RS built by above method, GA is adopted to optimize the variables of each design element including adding the limited thickness of laminate, adjusting the ply percentages and the fiber direction to get the best transonic flutter speed of the wing. During the optimization, the fitness of each individual is obtained by searching the RS, which greatly accelerates the optimization processes and makes GA used in solving nonlinear optimization problem possible. The GA optimization needs several steps to reach the final optimization results.

In this method, increment on thickness of each design element every step is so small that the influence on transonic flutter speed could be taken as independent.

After each optimization, the best adding position and angle will be determined. The wing model will change according to the result and then go to the next step of optimization.

3 Optimization result

After 7 steps optimization, the transonic flutter speed of wing model increases from 283m/s to 284.68m/s (showing in Fig.9).

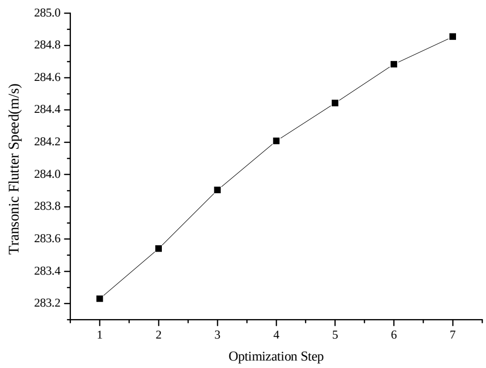


Fig.9. Variation of The Transonic Flutter Speed

During each step, the contribution to the transonic flutter speed of each design element and the corresponding ply direction. In order to gain the utmost increment on flutter speed with per weight input, it is need to select the best according to the efficiency of per unit area. Take step one as example. Adding a certain thickness ply of 74° can get the highest speed. The efficiency of per area of each design element is showed in Fig.11.

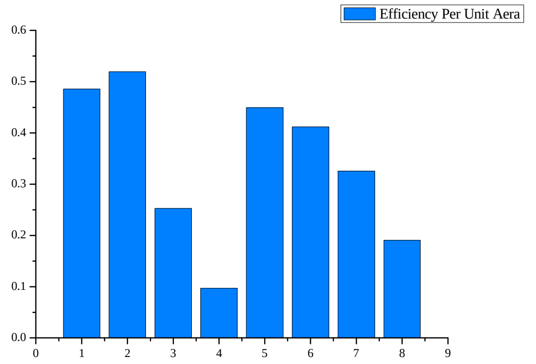


Fig.10. Efficiency per Unit Area of Each Design Element

The material adding to design element 2 is 2.87E-4m3. Supposing the density of composite material is 1.5g/cm3. It can be calculated that the improvement on flutter speed is 0.72 m/skg at per unit material input.

The ultimate state will determined after each optimization. The accurate speed of this state is calculated by ZearO. From figure**，it can be visibly induced that the value from RS are nearly the same as the accurate value. This proves the reliability of the RS furthermore.

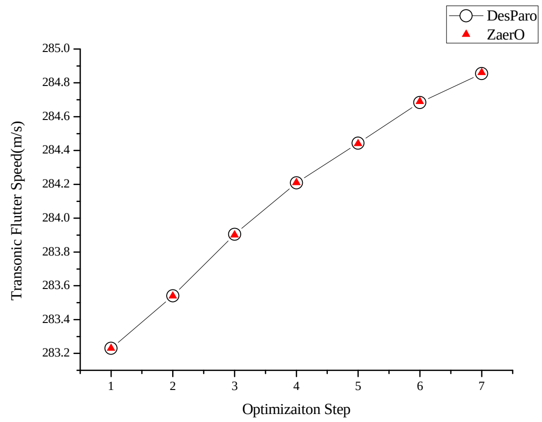


Fig.11. Comparation Between Two Methods At Optimal Design

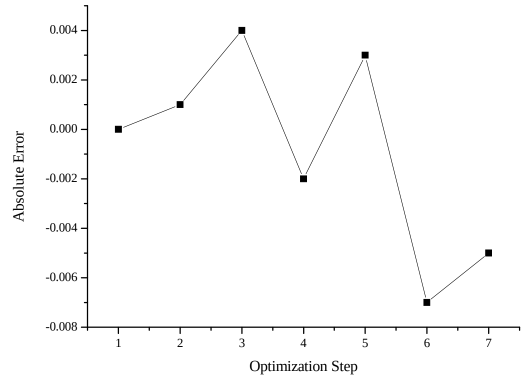


Fig.12. Absolute Error of Optimal Result

From above figure, it shows that the absolute error is limited between -0.5%-0.5%. The optimization result is reliable.

4. Conclusion

Through the optimization example, it is proved that this method could be used in aeronautical nonlinear multidisciplinary optimization. The response surface built by this method is reliable and error between response value and accuracy value are no more than 0.5%. The optimization of transonic flutter speed also is proved to be efficient

The accuracy of the whole optimization totally depends on the construction and the revision of the RS. This method acquires relevant support of RS software and GA optimization system.

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