NONLINEAR ANALYSIS BY STRENGTH EVALUATION AND FABRICATION METHOD OF THINWALLED AIRFRAMES

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Abstract

The simplest stability analysis of the airframe panels of up-to-date passenger and transport aircrafts with the cells of the skin with the size of 400÷500 mm into the length and 200÷300 mm into the width, with the thickness from 0.8 to 1.5 mm, depending on aggregate, shows that the breaking stresses of their local stability are located in the region of 10÷20 mPa. But it is assumed at designing, and practice confirms it, that the panels often experience membrane stress above 100 mPa. But it is assumed at designing, and practice confirms it, that the panels often experience membrane stresses of more than 100 mPa. As a result, significant parts of the airframe at some loadings may be in a state of nonlinear deformation far beyond the stability of the skin. In some cases, geometric nonlinearity can be also accompanied by nonlinearity of material deformation. Of course the aforesaid is not a discovery, but in the airframe design practice still prevails the linear analysis, not taking into account the deformation beyond stability limits. Nonlinear analysis remains still a challenge, but since the linear analysis does not appear sufficiently correct procedure of strength evaluation, solution of this problem is extremely actual for practice.

The decision of the problem is restrained by inadequate for practice theoretical support and insufficient automation of the systems of nonlinear equations solution algorithms as well as large sizes of these systems. The latter is caused by both the sizes and structural complexity and by the quality of the available finite-element base, since the finite element method (FEM) is usually used for the solution of practical problems. And the quality of finite elements (FE) is determined by convergence. Despite the large number of studies undertaken so far to create efficient FE, in the manuals for the use of practically all known modern programs of the finite-element strength analysis quite strict requirements for the ratio of the FE plates and shells sizes are still contained. Non-compliance leads to the sharp loss in the accuracy of results. However, the observance of requirements is accompanied by the increase in the systems of equations and not less sharp increase of calculation effort, which becomes an obstacle to the solution of many practically important problems. On the way to improve the convergence it was possible to create FE for the simulation of plates and shells, and also beam-type elements to a considerable degree free from the typical limitations to the ratio of dimensions and characterizing by the improved convergence when solving both the linear and nonlinear problems. Elements in the process of solution of systems have six degrees of freedom at the nodes (three displacements and three rotation angles) and are able to take moments about the surface normal at the nodes.

1 Linear Problem

The limited volume of the publication does not allow to cover all aspects of creation of the mentioned FE, therefore we shall consider only the basic characteristic features of the model of a quadrilateral element of a plate in a hope, that the reader is familiar with the FEM basis and is capable to generalize the received results on more complicated models. The results of works [1,2,3,4] show that a noticeable improvement of the convergence can be obtained by reduction of requirements for the FE deformation consistency. In this case the mathematical correctness of result suffers, however practical benefit proves to be considerably more essential. Following this example it is possible to act as follows. Let us conceive that quadrilateral FE is composed of four triangular ones in two layers, with the common nodes along the contour, as shown in figure 1.

To simulate displacements within each triangle it is possible to use an incomplete cubic polynomial. If for the displacement components along the axes *x, y, z* we accept notations *u, v, w,* then for *u* and *v*, which are responsible for the deformation in the FE plane, polynomials can be represented in the form

$$
\begin{aligned}
& u = u_l + u_n, \quad u_l = \sum_{i=1}^{3} u_i \cdot L_i, \\
& u_n = u_{xy} \cdot \left(xy - a_{xy} - b_{xy} \cdot x - c_{xy} \cdot y\right) \\
& + u_{yy} \cdot \left(y^2 - a_{yy} - b_{yy} \cdot x - c_{yy} \cdot y\right) \\
& + u_{yyy} \cdot \left(y^3 - a_{yyy} - b_{yyy} \cdot x - c_{yyy} \cdot y\right) \\
& + u_{xx} \cdot \left(x^2 - a_{xx} - b_{xx} \cdot x - c_{xx} \cdot y\right) \\
& + u_{xxy} \cdot \left(x^2 y - a_{xxy} - b_{xxy} \cdot x - c_{xxy} \cdot y\right),
\end{aligned}
$$

$$
\begin{aligned}
& v = v_l + v_n, \quad v_l = \sum_{i=1}^{3} v_i \cdot L_i, \\
& v_n = v_{xy} \cdot \left(xy - a_{xy} - b_{xy} \cdot x - c_{xy} \cdot y\right) \\
& + v_{xx} \cdot \left(x^2 - a_{xx} - b_{xx} \cdot x - c_{xx} \cdot y\right) \\
& + v_{xxx} \cdot \left(x^3 - a_{xxx} - b_{xxx} \cdot x - c_{xxx} \cdot y\right) \\
& + v_{yy} \cdot \left(y^2 - a_{yy} - b_{yy} \cdot x - c_{yy} \cdot y\right) \\
& + v_{xyy} \cdot \left(y^2 x - a_{xyy} - b_{xyy} \cdot x - c_{xyy} \cdot y\right).
\end{aligned}
$$

Here axes *x, y* lie in the FE plane, the axis *z* - along normal to surface. u_i, L_i are nodal displacements and triangular coordinates [3];

$$a_{xy} = \sum_{i=1}^{3} \frac{a_i \cdot x_i \cdot y_i}{\Delta}, \quad b_{xx} = \sum_{i=1}^{3} \frac{b_i \cdot x_i^2}{\Delta}, \quad \text{etc;}$$

a_i, b_i, c_i, Δ are coefficients and the determinant in expressions for triangular coordinates [3,4]; *i* is an ordinal number of the node. The direction of numbering is not important.

Evidently, the polynomials describing the behavior of *u* and *v* are presented in the form of two components — linear and nonlinear. Linear components are determined by the nodal displacement values. Nonlinear components go to zero at nodes. To determine the first 3 coefficients in non-linear components of each polynomial through nodal factors you can use the values of the coordinate derivatives of the orthogonal direction at nodes. So for u_n the defining nodal values will be $\frac{\partial u_n}{\partial y}$. Correspondingly, for v_n — $\frac{\partial v_n}{\partial x}$. It is possible subsequently to define the remained 4 coefficients from stationary condition of energy, by considering them as internal unknowns [3], general for all four triangles.

Displacement *w* in this case describes deflection and change in the surface curvature of the plate. Depending on what curvature it is necessary to simulate, polynomials for *w* are characterized by summands with the highest degrees. So for generation $\frac{\partial^2 w}{\partial x^2}$ polynomial takes the form

$$
\begin{aligned}
& w = w_l + w_n, \quad w_l = \sum_{i=1}^{3} w_i \cdot L_i, \\
& w_n = w_{xy} \cdot \left(xy - a_{xy} - b_{xy} \cdot x - c_{xy} \cdot y\right) \\
& + w_{xx} \cdot \left(x^2 - a_{xx} - b_{xx} \cdot x - c_{xx} \cdot y\right) \\
& + w_{yy} \cdot \left(y^2 - a_{yy} - b_{yy} \cdot x - c_{yy} \cdot y\right) \\
& + w_{xxy} \cdot \left(x^2 y - a_{xxy} - b_{xxy} \cdot x - c_{xxy} \cdot y\right) \\
& + w_{xxx} \cdot \left(x^3 - a_{xxx} - b_{xxx} \cdot x - c_{xxx} \cdot y\right).
\end{aligned}
$$

It is possible to notice, that such form of the polynomial provides the linear law of variation $\frac{\partial^2 w}{\partial x^2}$. For definition of 5 coefficients of the polynomial w_n by means of nodal factors it is possible to use 3 nodal derivatives $\frac{\partial w_n}{\partial x}$. Another two missing conditions can be obtained as follows. Let us write the derivative of a polynomial

$$\frac{\partial w_n}{\partial y} = w_{xy} \cdot (x - c_{xxy}) - w_{xx} \cdot c_{xx} + w_{yy}(2 \cdot y - c_{yy}) + w_{xxy} \cdot (x^2 - c_{xxy}) - w_{xxx} \cdot c_{xxx}.$$

Then for $\frac{\partial w_n}{\partial y}$ write down the linear approximation

$$\frac{\partial w_n}{\partial y} = \left(\frac{\partial w_n}{\partial y}\right)_1 \cdot L_1 + \left(\frac{\partial w_n}{\partial y}\right)_2 \cdot L_2 + \left(\frac{\partial w_n}{\partial y}\right)_3 \cdot L_3.$$

Here in brackets are nodal values of the derivative. To obtain the first additional condition let us differentiate the x linear approximation and substitute into nodes the values of the derivative of a polynomial $\frac{\partial w_n}{\partial y}$

$$w_{xy} + b_{xx} \cdot w_{xxy} = \sum_{i=1}^{3} \left(\frac{\partial w_n}{\partial y}\right)_i \cdot \frac{b_i}{\Delta}.$$

To obtain the second additional condition let us differentiate the y linear approximation and also substitute the values of the derivative of a polynomial $\frac{\partial w_n}{\partial y}$ into nodes

$$w_{yy} + c_{xx} \cdot w_{xxy} = \sum_{i=1}^{3} \left(\frac{\partial w_n}{\partial y}\right)_i \cdot \frac{c_i}{\Delta}.$$

As a result, for the expression of 5 coefficients of the polynomial w_n via 6 nodal values of the derivatives $\left(\frac{\partial w_n}{\partial x}\right)_i, \left(\frac{\partial w_n}{\partial y}\right)_i$ we obtain a system with a nonsingular square matrix.

For obtaining $\frac{\partial^2 w}{\partial y^2}$ polynomial looks like

$$w = w_l + w_n, \quad w_l = \sum_{i=1}^{3} w_i \cdot L_i,$$

$$w_n = w_{xy} \cdot (xy - a_{xy} - b_{xy} \cdot x - c_{xy} \cdot y) + w_{xx} \cdot (x^2 - a_{xx} - b_{xx} \cdot x - c_{xx} \cdot y) + w_{yy} \cdot (y^2 - a_{yy} - b_{yy} \cdot x - c_{yy} \cdot y) + w_{xyy} \cdot (y^2 x - a_{xyy} - b_{xyy} \cdot x - c_{xyy} \cdot y) + w_{yyy} \cdot (y^3 - a_{yyy} - b_{yyy} \cdot x - c_{yyy} \cdot y)$$

The algorithm of expression of the polynomial w_n coefficients through the nodal values of its derivatives is similar to that described above, taking into account that the coordinates *x, y* and the corresponding coefficients should be interchanged.

The results of numerical experiments show that sufficient approximation for the derivative $\frac{\partial^2 w_n}{\partial x \partial y}$, necessary for accounting of the plate torsion, is the expression, built on the basis of differentiation respectively of the *y, x* linear approximations of derivatives $\frac{\partial w_n}{\partial x}, \frac{\partial w_n}{\partial y}$

$$\frac{\partial^2 w_n}{\partial x \partial y} = \frac{1}{2} \cdot \sum_{i=1}^{3} \left(\left(\frac{\partial w_n}{\partial x}\right)_i \cdot \frac{c_i}{\Delta} + \left(\frac{\partial w_n}{\partial y}\right)_i \cdot \frac{b_i}{\Delta} \right).$$

The obtained expressions can be easily used to generate the strain energy functionals of triangular FE by the known way [3, 4]. In this case into the number of nodal kinematic factors will enter $u, v, w, \frac{\partial u_n}{\partial y}, \frac{\partial v_n}{\partial x}, \frac{\partial w_n}{\partial x}, \frac{\partial w_n}{\partial y}$. Here, taking into account the method of introduction of polynomials into examination, should be accomplished a passage to the derivatives of general polynomials from the derivatives of nonlinear components according to the formulas $\frac{\partial u_n}{\partial y} = \frac{\partial u}{\partial y} - \sum_{i=1}^{3} \frac{c_i}{\Delta} \cdot u_i$, $\frac{\partial v_n}{\partial x} = \frac{\partial v}{\partial x} - \sum_{i=1}^{3} \frac{b_i}{\Delta} \cdot v_i$ etc. As a result, into the number of nodal kinematic factors will enter $u, v, w, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$. Evidently, there arose rather unusual situation where the angular position under deformation in the plane is determined by two factors: $\frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}$.

It is possible to get rid of it, having entered into consideration the rotation angle θ about the *z*-axis by the known mean: $\frac{\partial u}{\partial y} = -\theta + \frac{\gamma}{2}$, $\frac{\partial v}{\partial x} = \theta + \frac{\gamma}{2}$. Here γ is the angle of displacement in the node. The latter should either be attributed to internal unknowns or without significant loss of the calculation accuracy of displacements set equal to 0, and that was carried out. As a result, in the node we have: $u, v, w, \theta, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$.

After the described transformations the strain energy functional of quadrilateral FE is obtained by simple summation of the functionals of triangles. In order to avoid doubling of the FE stiffness, since the triangles form two layers, the obtained energy functional should be divided in 2. Then evaluation of internal unknowns via nodal ones [3,4] and final formation of the FE stiffness matrix is carried out.

2 Nonlinear Problem

The solution of geometrically nonlinear problem of deformation to a considerable degree is based on the ideas of the previous section. Assuming displacements unlimited, it is more convenient to pass to the vector notation. The radius-vector of the initial surface of triangular FE can be presented in the form $\{r\}_0 = \sum_{i=1}^{3} \{r_i\} \cdot L_i$. For the deformed surface, as in the previous section, we can write down the radius-vector as a sum of linear and nonlinear components $\{\tilde{r}\} = \{\tilde{r}\}_l + \{\tilde{r}\}_n$. Here $\{\tilde{r}\}_l = \sum_{i=1}^{3} \{\tilde{r}_i\} \cdot L_i$; $\{\tilde{r}\}_n = u_n \cdot \{\tilde{e}_1\} + v_n \cdot \{\tilde{e}_2\} + w_n \cdot \{\tilde{e}_3\}$; $\{\tilde{r}_i\}$ are the values of the radius-vector of the deformed state in the nodes; $\{\tilde{r}_i\} = \{r_i\} + \{u_i\}$; $\{\tilde{e}_1\}, \{\tilde{e}_2\}$ are the mutually orthogonal unit vectors, which lie in the plane, passing through FE nodes in the deformed state. The directions of these unit vectors are defined with the help of vectors $\left\{\frac{\partial \tilde{r}}{\partial x}\right\}_l = \sum_{i=1}^{3} \{\tilde{r}_i\} \cdot \frac{b_i}{\Delta}$, $\left\{\frac{\partial \tilde{r}}{\partial y}\right\}_l = \sum_{i=1}^{3} \{\tilde{r}_i\} \cdot \frac{c_i}{\Delta}$ so that bisectors of the angles between the vectors $\{\tilde{e}_1\}, \{\tilde{e}_2\}$ and $\left\{\frac{\partial \tilde{r}}{\partial x}\right\}_l, \left\{\frac{\partial \tilde{r}}{\partial y}\right\}_l$ coincide. The unit vector $\{\tilde{e}_3\}$ is defined as the right vector product of the unit vectors $\{\tilde{e}_1\}, \{\tilde{e}_2\}$.

Strain components can be written down as:

$$\varepsilon_x = \sqrt{\left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor \left\{ \frac{\partial \tilde{r}}{\partial x} \right\}} - 1, \quad \varepsilon_y = \sqrt{\left\lfloor \frac{\partial \tilde{r}}{\partial y} \right\rfloor \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}} - 1,$$

$$\gamma = \frac{1}{A \cdot B} \left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor \left\{ \frac{\partial \tilde{r}}{\partial y} \right\},$$

$$A = \sqrt{\left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor \left\{ \frac{\partial \tilde{r}}{\partial x} \right\}}, \quad B = \sqrt{\left\lfloor \frac{\partial \tilde{r}}{\partial y} \right\rfloor \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}}.$$

Having carried out substitutions, we shall receive

$$\varepsilon_x = \sqrt{\left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor_l \left\{ \frac{\partial \tilde{r}}{\partial x} \right\}_l + 2 \cdot \left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor_l \left\{ \frac{\partial \tilde{r}}{\partial x} \right\}_n + \left(\underline{\left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor_n \left\{ \frac{\partial \tilde{r}}{\partial x} \right\}_n} \right)} - 1,$$

$$\varepsilon_y = \sqrt{\left\lfloor \frac{\partial \tilde{r}}{\partial y} \right\rfloor_l \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}_l + 2 \cdot \left\lfloor \frac{\partial \tilde{r}}{\partial y} \right\rfloor_l \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}_n + \left(\underline{\left\lfloor \frac{\partial \tilde{r}}{\partial y} \right\rfloor_n \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}_n} \right)} - 1,$$

$$\gamma = \frac{1}{A \cdot B} \left(\left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor_l \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}_l + \left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor_l \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}_n + \left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor_n \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}_l + \left(\underline{\left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor_n \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}_n} \right) \right).$$

Curvature components can be written down in the form:

$$\kappa_x = -\frac{\lfloor \tilde{e}_3 \rfloor}{A_l^2} \left\{ \frac{\partial^2 \tilde{r}}{\partial x^2} \right\}, \quad \kappa_y = -\frac{\lfloor \tilde{e}_3 \rfloor}{B_l^2} \left\{ \frac{\partial^2 \tilde{r}}{\partial y^2} \right\},$$

$$\kappa_{xy} = -\frac{2 \cdot \lfloor \tilde{e}_3 \rfloor}{A_l B_l} \left\{ \frac{\partial^2 \tilde{r}}{\partial x \partial y} \right\},$$

$$A_l = \sqrt{\left\lfloor \frac{\partial \tilde{r}}{\partial x} \right\rfloor_l \left\{ \frac{\partial \tilde{r}}{\partial x} \right\}_l}, \quad B_l = \sqrt{\left\lfloor \frac{\partial \tilde{r}}{\partial y} \right\rfloor_l \left\{ \frac{\partial \tilde{r}}{\partial y} \right\}_l}.$$

It is possible to note that the last expressions are given with the deviation from the classical record [5]: the vector $\tilde{e}_3$ and coefficients A_l, B_l are considered constants in the limits of FE. However, this form does not contradict limit relations and ensures significant simplification of calculations and better convergence.

Having entered into consideration vectors

$$\{\varepsilon\} = \lfloor \varepsilon_x \, \varepsilon_y \, \gamma \rfloor^T, \quad \{\kappa\} = \lfloor \kappa_x \, \kappa_y \, \kappa_{xy} \rfloor^T,$$

$$\{t\} = [E]\{\varepsilon\}, \quad \{M\} = [D]\{\kappa\},$$

let us write down the expression of the plate strain energy

$$U = \frac{1}{2} \int_F \left(\lfloor \varepsilon \rfloor \{t\} + \lfloor \kappa \rfloor \{M\} \right) dF.$$

Here $[E], [D]$ are the matrixes of membrane and bending stiffness [4]. In the general case of nonlinear deformation the recorded functional has a high degree of nonlinearity relative to the required unknowns. For the iterative minimizing of the functional by Newton's method when solving problems let us construct its Taylor expansion in increments of unknowns entering into $\{\varepsilon\}, \{\kappa\}$, in the neighborhood of some initial value U_0. It is assumed that the increments are small, at least in the case of convergence

$$U \approx U_0 + \int_F \left(\lfloor \delta\varepsilon \rfloor \{t\} + \lfloor \delta\kappa \rfloor \{M\}\right) dF$$

$$+\frac{1}{2}\int_F \left(\lfloor \delta\varepsilon \rfloor [E] \{\delta\varepsilon\} + \lfloor \delta\kappa \rfloor [D] \{\delta\kappa\} + \left[\delta^2 \varepsilon\right]\{t\}\right) dF.$$

Here δ is a symbol of increment.

Having fulfilled substitutions and having integrated over FE area we shall receive

$$U \approx U_0 + \lfloor \delta X \rfloor \{g_\Delta(X_0)\} + \frac{1}{2} \cdot \lfloor \delta X \rfloor [H_\Delta(X_0)] \{\delta X\},$$

where $\{g_\Delta(X_0)\}, \{H_\Delta(X_0)\}$ are the gradient and the Hessian matrix of functional with X_0; $\{\delta X\}$ is the increment of nodal unknowns.

Immediately after substitutions into the number of unknowns in the node will enter $\delta u, \delta v, \delta w, \delta \frac{\partial u_n}{\partial y}, \delta \frac{\partial v_n}{\partial x}, \delta \frac{\partial w_n}{\partial x}, \delta \frac{\partial w_n}{\partial y}$. Entered here derivatives should be expressed in terms of global factors [3, 4]. Such as:

$$\delta \frac{\partial u_n}{\partial y} = \left\lfloor \delta \frac{\partial \tilde{r}}{\partial y} \right\rfloor \{\tilde{e}_1\} - \left\lfloor \delta \frac{\partial \tilde{r}}{\partial y} \right\rfloor_l \{\tilde{e}_1\}.$$ But

$$\left\lfloor \delta \frac{\partial \tilde{r}}{\partial y} \right\rfloor_l \{\tilde{e}_1\} = \sum_{i=1}^{3} \lfloor \delta \tilde{r}_i \rfloor \{\tilde{e}_1\} \cdot \frac{c_i}{\Delta} = \sum_{i=1}^{3} \lfloor \delta u_i \rfloor \{\tilde{e}_1\} \cdot \frac{c_i}{\Delta},$$

$$\left\lfloor \delta \frac{\partial \tilde{r}}{\partial y} \right\rfloor \{\tilde{e}_1\} = -\lfloor \delta\varphi \rfloor \{k_2 \cdot \tilde{e}_2 + k_3 \cdot \tilde{e}_3\},$$

$$k_2 = -\frac{\partial \tilde{r}}{\partial y}\tilde{e}_3, \quad k_3 = \frac{\partial \tilde{r}}{\partial y}\tilde{e}_2.$$

On computation it is possible to set $k_2 = 0, k_3 = 1$, since the inclination of the deformed surface to the reference plane is usually small and decreases rapidly with the decrease of FE sizes. Here $\delta\varphi$ is the incremental vector of the angle of rotation. As a result

$$\delta \frac{\partial u_n}{\partial y} = -\lfloor \delta\varphi \rfloor \{\tilde{e}_3\} - \sum_{i=1}^{3} \lfloor \delta u_i \rfloor \{\tilde{e}_1\} \cdot \frac{c_i}{\Delta}.$$

Similarly:

$$\delta \frac{\partial v_n}{\partial x} = \lfloor \delta\varphi \rfloor \{\tilde{e}_3\} - \sum_{i=1}^{3} \lfloor \delta u_i \rfloor \{\tilde{e}_2\} \cdot \frac{b_i}{\Delta},$$

$$\delta \frac{\partial w_n}{\partial x} = -\lfloor \delta\varphi \rfloor \{\tilde{e}_2\} - \sum_{i=1}^{3} \lfloor \delta u_i \rfloor \{\tilde{e}_3\} \cdot \frac{b_i}{\Delta},$$

$$\delta \frac{\partial w_n}{\partial y} = \lfloor \delta\varphi \rfloor \{\tilde{e}_1\} - \sum_{i=1}^{3} \lfloor \delta u_i \rfloor \{\tilde{e}_3\} \cdot \frac{c_i}{\Delta}.$$

After the transformations the number of nodal unknowns is reduced to 6: three increments of displacement and three projections of the rotation angle increment.

The gradient and the Hessian matrix of quadrilateral FE result from simple summation of corresponding factors of triangles. Internal unknowns here are considered as the general ones. To avoid doubling of FE stiffness as triangles form two layers, the received gradient and matrix should be divided by 2. Then the definition of internal unknowns through nodal ones [3,4] and final formation of the gradient and Hessian matrix of FE is carried out.

In connection with the numerical realization of the described algorithm it is necessary to note two circumstances essentially influencing on the process. Direct use of the above expressions for deformations leads to not quite satisfactory results. Difficulties proceed from underlined components which induce false stiffness. The situation definitely improves if preliminary to integrate these components over the area and having divided into the area to use at calculations as constants independent of coordinates. At the same time for the entering into these components derivatives $\frac{\partial w_n}{\partial x}, \frac{\partial w_n}{\partial y}$ it is possible to use linear approximations similar to presented in the previous section 1.

3 General conclusion

As it follows from the previous consideration, quadrilateral FE is not required to be flat. This condition applies only to triangles that form it. Thus quadrilateral FE itself appears to some extent the shell at least depressed. When more correct calculation of the initial form is necessary, the previous consideration remains in force. It is required only to introduce clarity into the relations connecting internal static factors and the deformation

$$\{t\} = [E]\{\varepsilon - \varepsilon_0\}, \quad \{M\} = [D]\{\kappa - \kappa_0\}.$$

Here it is better to enter vectors $\{\varepsilon_0\}, \{\kappa_0\}$ as a result of a preliminary deformation of the initially flat piece of plate. So it is easier to be saved from the unacceptable mistakes connected with approximation of calculations.

4 Results

As a test for efficiency of the developed FE in solving of linear problems of deformation the influence on results of the FE form in the plan at the action of elementary loadings has been investigated. There is very illustrative test in which one FE simulated overhung beam at a bend from a plane and in a plane under action of transverse force and the moment on the free end.

At a bend from a plane the element at a large length-to-width ratio practically exactly simulates the beam.

In figure 2 some results of solution of the problem of a bend by force in a plane are shown. For comparison, the results obtained with the help of programs ANSYS and NASTRAN, as well as from the polynomial solution of the problem [6] are given. Here, the plate thickness h = 1 mm, modulus of elasticity E = 68030 mPa, Poisson's ratio μ = 0.3. In this example with the growth of length-to-width ratio nodal displacements of the element tend to beam ones.

At a bend by the moment nodal displacements of the element are closer to the corresponding displacements obtained from the polynomial solutions [6] at any length-to-width ratios.

According to the accepted FE generation algorithm its form can vary over a wide range. At the same time its properties change rather monotonically with the change of the form. So at a deviation from the shape of a canonical rectangular there is no abrupt change of its quality. As numerical experiments have shown in similar conditions for the estimation of displacements it is usually required by an order of magnitude less of developed FE, in comparison with similar elements of ANSYS or NASTRAN.

In the nonlinear problem quality of FE is practically kept at a bend from a plane. At a bend in a plane the convergence is a little bit worse. In the figure 3 it is shown the initial and nonlinear deformed state of the final element of the plate circumscribing a bend of the overhung beam in its plane under action of moments M = 300 Nm, applied at the nodes on the free end. The length of the element L = 200 mm, width b = 20 mm, thickness h = 2 mm, modulus of elasticity E = 6803 mPa, Poisson's ratio μ = 0.33. Displacements of the end point of the FE middle line: -47.5 mm - horizontal; 113 mm - vertical. The dashed line shows the elastic line of the exact solution for the bar that is equivalent with respect to stiffness. Here, the corresponding displacements are: -53.45 mm, 114.1 mm. In the problems with so significant deformation the limiting length-to-width ratio for calculations with adequate accuracy should not exceed 8. In the case of smaller deformation the ratio may be higher.

The presence of final elements that allow with satisfactory accuracy take into account so significant deformations, can substantially reduce the number of FE, not going beyond the constructive discretization in computing the real products. There appears an opportunity to pass to the practical decision of problems of deformation beyond the bounds of stability of the real products.

Figure 4 shows the finite-element schemes of the airframe units with the separation of panels being deformed beyond the limit of stability. In this case special FE are used, approximately considering the deformation beyond the limit of stability by the use of the additional shape function for the deflection

$$w = f \cdot \sin\left(\frac{\pi \cdot x}{L}\right) \cdot \cos\left[m \cdot \pi \left(\frac{x}{L} - \frac{k \cdot y}{b}\right)\right] \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right)$$
$$+ \sum_{i=1}^{5} \sum_{j=1}^{5} f_{ij} \cdot \sin\left(\frac{i \cdot \pi \cdot x}{L}\right) \cdot \sin\left(\frac{j \cdot \pi \cdot y}{b}\right).$$

The latter simulates the deformation of a simply supported plate under the combined loading of tension or compression with shear.

Now more attention of the aircraft technologists is attracted by engineering processes concerning slow molding of integrally milled aviation panels in the modes of creep and relaxation of material. The attractiveness of these processes is determined by the fact that the material during processing in the minimal degree loses resource and other strength characteristics of the state of delivery. However, the design of technological equipment for these processes encounters certain difficulties. To design the form of equipment it is required to solve the labour-consuming problems of elasto-plastic deformation with great displacements of large-sized panels of complex forms. Since there usually presents a significant part of elastic deformation in panels at molding, it is necessary to involve optimal control algorithms for taking into account the elastic 'rebound'. On solving these problems the number of FE has a defining value. Application of the developed FE has allowed transference of all problems practically unsolvable before in the category of success.

Figure 5 shows the final and predicted (with attachments) forms of the stabilizer panel surface within the limits of elasto-plastic molding technology in the relaxation mode. Figure 6 shows a typical finite-element scheme for the large-sized panel.

Conclusion

The model of quadrilateral FE with the improved convergence is offered for solving the problems of plate and shell deformation. It requires further investigations for theoretically correct improving of the model convergence in the nonlinear version.

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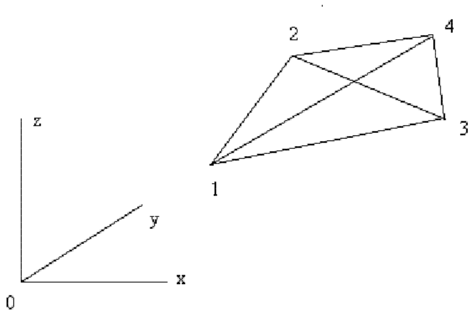
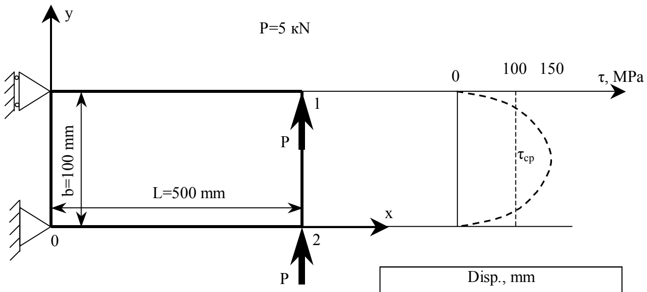


Fig.1



Disp., mm		
	[6]	FEM
u_1	-11.02	-11.05
υ_1	75.48	74.86
u_2	11.02	11.00
υ_2	75.48	74.87

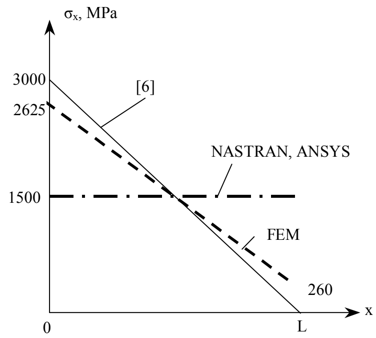


Fig.2

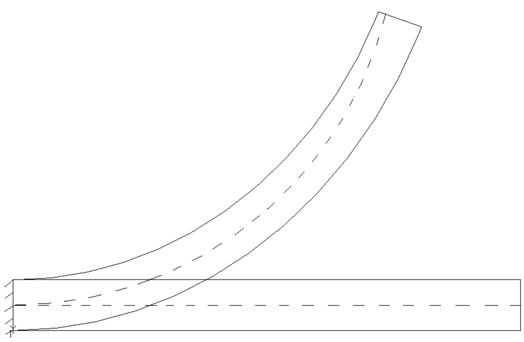


Fig.3

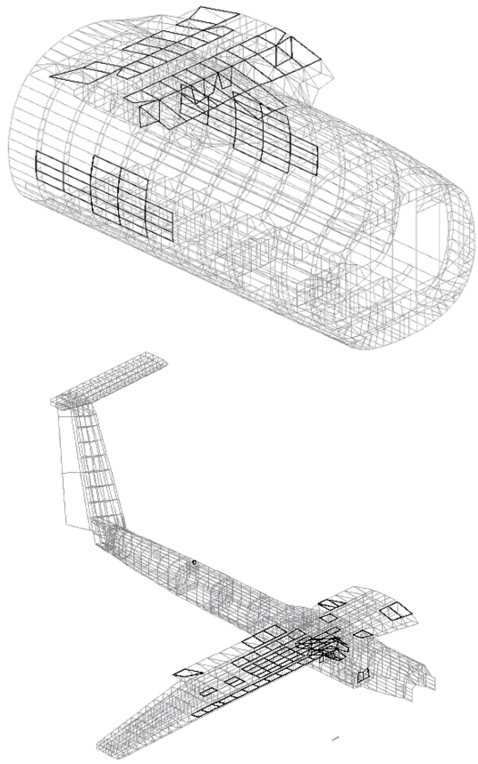


Fig.4

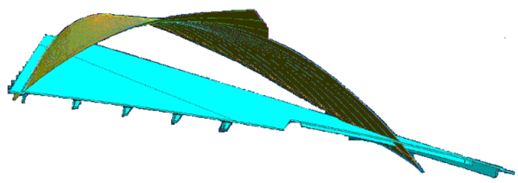


Fig.5

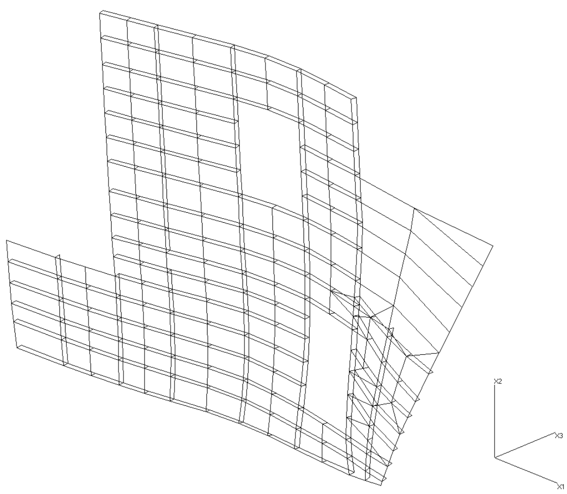


Fig.6