WHY COMMERCIAL JET PLANES CRUISE AT $30\,000\,FT$?

CONCEPTUAL OPTIMAL CRUISE ALTITUDE

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Abstract

All commercial jet planes fly at a cruising altitude of about $30\,000\,ft$, whereas regional turboprop airplanes fly lower and business jet planes fly higher. Why ?

The aim of this paper is to demonstrate that the conceptual cruise altitude corresponds to an optimum. The "conceptual" cruise altitude is the altitude the aircraft designer chooses, in his office, when the airplane does not exist. And this choice is fundamental for the final performance of the airplane. It must not be confused with the "operational" cruise altitude, that is to say the altitude the pilot chooses for its cruise. This operational cruise altitude is generally the operational ceiling where a given airplane gets its best performances.

It will be shown that when the conceptual cruise altitude increases, both the finesse of the airplane $L/D = f$ and the operational empty weight OWE, increases. The increase of finesse leads to an improved performance, while, the increase of OWE reduces the performance. Dealing with these two conflicting effects leads to an optimal cruising altitude.

This optimal altitude can come to light by computation through a conceptual design software. This kind of approach will be illustrated. But in this paper, special emphasis will be given to the physical meaning of this optimum. So that the fundamental aspects of this process could be highlighted.

1 Introduction

The basis of conceptual aircraft design are firstly recalled in order to enlighten the method and describe the process. The process starts with the evolution of wing loading with altitude. Then the consequence of the variation of wing surface on finesse, $L/D = f$, and operational empty weight, OWE, is evaluated. Due to these two effects, the existence of the conceptual optimal cruise altitude can be demonstrated.

2 Conceptual aircraft design method

During the conceptual phase of airplane design the fundamental issue is to be able to carry

- ❶ from Paris to New York — a range R
- ❷ 200 passengers — a payload weight W_{pl}
- ❸ being the best airplane on market . — minimum take-off weight $MTOW_{mini}$

2.1 Requirement parameters

Then the airplane design is a three parameters game [1], two — R and W_{pl} to define the mission of the airplane — and one — $MTOW_{mini}$ — is the optimization objective. This last one is close to the price of the airplane. Another objective could be chosen, like the fuel weight, close to the operating flight cost. And a mix of the two is the Direct Operating Cost. All these parameters are an

1. These three parameters can be seen as the requirement objectives.

expression of the flight cost, so they are linked to the market demand, (Section 2.3, p. 2).

This is the design approach adopted by the MANUFACTURERS like BOEING and AIRBUS.

2.2 Design variables

Now the objectives are known, but how to reach it ? For that, the aircraft designer can choose design variables.

It can be shown that the airplane design problem fundamentally comes down to choose two design variables : the wing area or wing loading, $\frac{W}{S}$, and the thrust of the engines or thrust ratio, $\frac{T}{W\,g}$. And actualy, the choice of these two variables is equivalent to the choice of the cruise altitude and the choice of the cruise Mach number, (Table 1, p. 3).

An optimal altitude and an optimal Mach number arise from the quest of optimum performances. These two parameters are the fundamentals ones in the design process, although they are usually taken as prescribed values and not as variable parameters.

The search of the optimal Mach number is not the purpose of this study, although it is a fascinating subject and the result is of great operational interest. Our subject in this paper is the optimization of the sole conceptual altitude at a constant Mach number. As a consequence, the choice of the wing loading related to altitude will determine the thrust ratio.

REMARK 2.1 In the design process, two parameters more fundamentals than the altitude and Mach number, are the range R, and the size M_{TOW}, of the aircraft. It can be demonstrated that the best performances are obtained for an optimal range and an optimal size. But this is not the aim of this paper and usually the airline companies give these parameters as requirements.

2.3 Optimisation criteria

Specific empty weight $\widehat{M_{VOE}}$ - The first criterion, that can be called a "specific empty weight"

$$\widehat{M_{VOE}} = \frac{M_{VOE}}{N_{pax}\,R} \qquad (1)$$

that is to say the "*kg*" of empty weight per pax carried and kilometre — or 100*km* — travelled, corresponds to a kind of economic efficiency of the investment made at the aircraft purchase. The empty weight M_{VOE} is linked to the purchase price of the aircraft, whereas both N_{pax} and R represent the revenue per flight for the company. Finally, $\widehat{M_{VOE}}$ is more or less a return on investment.

Specific fuel consumption $\widehat{W_{fuel}}$ - The second criterion is the specific fuel consumption

$$\widehat{W_{fuel}} = \frac{W_{fuel}}{N_{pax}\,R} \qquad (2)$$

that is to say the "*kg*" of fuel weight per pax carried and kilometre — or 100*km* — travelled. The "*kg*" unit of fuel weight can be transformed by a "litre" in order to go back to a classical specific fuel consumption. This $\widehat{W_{fuel}}$ is close to one of the main costs of the aircraft per flight, the fuel, which impact the flight revenue of the company.

The minimization of these two criteria is profitable to the economic yield of the aircraft. The first one, $\widehat{M_{VOE}}$, is relative to the purchase investment linked to the flight revenue, and the second one, $\widehat{W_{fuel}}$, is relative to the fuel operating costs linked to the flight revenue.

Economic profitability criteria $N_{pax}\,R$ - These two criteria, weight and consumption, are sensitive to the same term $N_{pax}\,R$, which can be called "economic profitability criteria". Consequently when this product is maximized the profitability of the aircraft, as seen by the airline company, increases. Why ? Because the flight revenue is proportional to the number of passengers but also to the travelled distance.

Requirements		*Design variables*		
Range		**wing area**		**cruise altitude**
Payload	$\Longleftarrow$	and	$\Longleftrightarrow$	and
$MTOW_{mini}$		**thrust of the engines**		**cruise Mach number**

Table 1 Design requirements and design variables

REMARK 2.2 It can be shown that an optimal payload ratio u_W exists which maximizes the economic profitability criteria $N_{pax}R$. This optimal ratio is linked to an optimal number of passengers, but also as a consequence, to an optimal fuel weight ratio, which is equivalent to an optimal range. The empty weight aircraft gives around 50 % of its maximum weight for passenger and fuel. This is the optimization of this balance between passenger and fuel which drives toward the maximum of $N_{pax}R$. This optimum exists since at the two extremum the aircraft revenue is zero. This is the case for an aircraft with the maximum of passenger but with no fuel, so going nowhere. And the same for an aircraft with the maximum fuel, but with no passengers. Evaluations show that the optimum is not so far from a balance between the weight of passenger and fuel. And these value are those used in this study.

Direct Operating Cost DOC **-** The quest for an optimized aircraft is mainly built on two criteria : the specific empty weight $\widehat{MVOE}$, and the specific consumption $\widehat{W_{fuel}}$. But these two criteria don't give the same airplane, so how to choose the right one ? One way is to find an equivalence between each of them, and then find a kind of mean as a final criteria. This is the role that the Direct Operating Cost DOC can play which can integrate the two costs, purchase price, $\widehat{MVOE}$, and operating cost, $\widehat{W_{fuel}}$, into the global cost of the flight. In that way the DOC, or better the specific Direct Operating Cost, $\widehat{DOC}$, can be considered as a kind of center of gravity of the two previous costs.

2.4 Two design approaches

From a mathematical point of view the design process could be changed, with almost the same result, for example with a given $MTOW$ and given W_{pl}, but with a maximum range R_{maxi} as the optimization objective. This approach can be called "academic", with the advantage of a simpler process since the weights are constant. Then the physical meaning of the optimum on cruise altitude will be easier to explain. We will use the **"ACADEMIC APPROACH"** for the demonstration, and at the end the connection with the **"MANUFACTURER APPROACH"** will be done.

Approaches	$MTOW$	Payload	Range
MANUFACTURER	$\underleftarrow{MTOW_{mini}}$	W_{pl}	R
ACADEMIC	$MTOW$	W_{pl}	$\underrightarrow{R_{maxi}}$

ACADEMIC APPROACH : more simple — computer and analytic

What is the connection between the two approaches ?

3 Conceptual design models

Three fundamentals forces act on an aircraft : its aerodynamics, weight and thrust. For each of them, a model has been developed and compared to current airplane data.

With these models, the flight dynamic equations give the performance of the aircraft, as presented in this chapter.

All these models are classical conceptual design models, [Kro06], [Tor86] with an accuracy around 10 %, and sometimes better. However the wing and fuselage weight models result from new developments and include structural analysis in the evaluation of the box beam of wing and fuselage. That way the accuracy reaches 6 %, whereas the accuracy of classical statistic models is around 16 %.

For the performances of engines, a new model was developed with an identification process on 50 engines data. The accuracy on specific fuel consumption is better than 4 %.

4 Conceptual optimal cruise altitude with ACADEMIC APPROACH

For the ACADEMIC APPROACH, assumptions are a constant take-off weight M_{TOW} with a constant payload weight W_{pl} and the quest for a maximum range R.

For each method, numerical results will be presented to demonstrate the existence of this optimum altitude, but they will also be completed by physical arguments so as to explain as simply as possible the origin of this optimum.

This method can be divided in four steps. First of all, we will demonstrate that the wing area increases with the cruise altitude. Next, we will examine the impact of this evolution on the finesse and the different weights constituting the aircraft. Finally, the evolution with the altitude of the aircraft aerodynamic, the finesse, and the weights will lead to find an optimum altitude. In the last paragraph, as a conclusion, all these results will be summarized and some principles linked to the optimal cruise altitude will be expressed.

4.1 Wing loading and altitude

Due to the lift equation — Lift equal to Weight —

$$W = \frac{1}{2}\rho\, S V^2 C_L = 0.7\, p_s\, S M^2\, C_L$$

or

$$\frac{W}{S} = \frac{1}{2}\rho\, V^2 C_L(\alpha) = 0.7\, p_s\, M^2\, C_L(\alpha)$$

the wing loading, $\frac{W}{S}$, follows the pressure, p_s, or the air density, ρ. Thus, the wing area, S, should increase when the altitude increases. The lift coefficient C_L is assumed not to vary too much. We assume that the cruise is performed at a constant cruise Mach number and is imposed, which is realistic, and therefore the optimum lift coefficient C_L corresponds to the maximum finesse or lift over drag ratio

$$C_{Lf} = \sqrt{e_O \pi A \sqrt{1 - M^2}\, C_{Do}} \qquad (3)$$

Where A denotes the aspect ratio of the wing, and e_O the Oswald coefficient.

As a matter of fact, the lift coefficient of the aircraft C_{Lf} varies by 20 % for an altitude variation of 30 000 ft. This is quite small relative to the variation of the inverse of air density, $\frac{1}{\rho}$, which is multiplied by almost five for the same range of altitude. And as announced, it can be seen in figure (1), p. 4, the wing surface is actually following $\frac{1}{\rho}$.

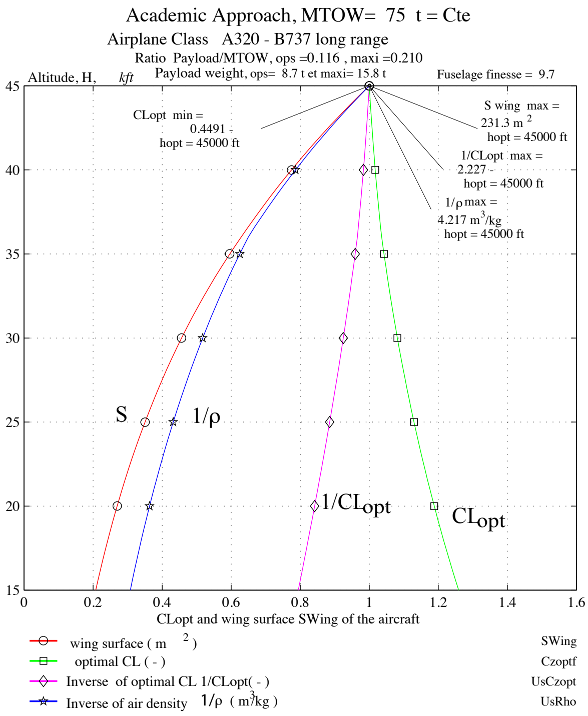

Fig. 1 Evolution of the wing area and CL_{opt} with the altitude. It must be noted that on x abscissa axis of this figure each parameter has been divided by its minimum or maximum value.

REMARK 4.1 The variation of C_{Lf} with the altitude is due to the variation of the minimum drag coefficient C_{Do} with the altitude. Which itself is due to the variation of wing surface (see next paragraph).

4.2 Finesse and altitude

The aircraft finesse is directly a function of the minimum drag coefficient C_{Do}, (Equation 4, p. 5). Then the variation of the finesse with altitude, will be examined through the variation of

C_{Do} with altitude.

$$
\begin{aligned}
f_{max} &= \frac{C_{L_{opt}}}{C_{D_{opt}}} = \frac{\sqrt{e_O\,\pi A\,\sqrt{1-M^2}\,C_{Do}}}{2\,C_{Do}} \\
f_{max} &= \frac{\sqrt{e_O\,\frac{\pi}{4} A\,\sqrt{1-M^2}}}{\sqrt{C_{Do}}}
\end{aligned}
\tag{4}
$$

When the wing area increases with a constant fuselage, the minimum drag coefficient C_{Do} decreases and progressively tends to the profile drag coefficient of the wing, which is much lower than the complete aircraft. To give an simple illustration of this result, the aircraft tends progressively to a flying wing with the fuselage completely submerged into the wing. This phenomenon is modelised by the expression of the minimum drag coefficient C_{Do} which is the sum of the profile drag coefficient of the wing, C_{Do_W}, and the drag coefficient of the fuselage C_{Do_F}.

$$
\begin{aligned}
\underleftarrow{C_{Do}} &= \underrightarrow{C_{Do_W}} + \underleftarrow{\underleftarrow{C_{Do_F}}} \\
\underrightarrow{C_{Do_W}} &= \underrightarrow{\frac{S_{w,W}}{S}}\,\underrightarrow{C_{Do_{W,\ell}}}\underleftarrow{(\Re_W)} \\
\underleftarrow{C_{Do_F}} &= \underleftarrow{\underleftarrow{\frac{S_{w,F}}{S}}}\,\underrightarrow{C_{Do_{F,\ell}}}\underleftarrow{(\Re_F)}
\end{aligned}
\tag{5}
$$

The local profile drag coefficient of the wing, $C_{Do_{W,\ell}}$, and the drag coefficient of the fuselage, $C_{Do_{F,\ell}}$, are proportional to the friction drag coefficient C_f, and are related to the local surfaces, that is to say both the wet areas of the wing $S_{w,W}$ and fuselage $S_{w,F}$. The profile drag of the wing, C_{Do_W}, and fuselage, C_{Do_F}, are related to the wing reference area so that they can be directly added.

A first effect due to the Reynolds number $\Re$ – When the altitude increases while keeping a constant speed, the Reynolds number, $\Re$, decreases proportionally to the air density [2], ρ. Therefore, both the local profile drag coefficient of the wing, $C_{Do_{W,\ell}}$, and fuselage, $C_{Do_{F,\ell}}$, increase. Nevertheless, $C_{Do_{W,\ell}}$ increase slower than $C_{Do_{F,\ell}}$ as the wing chord increases due to the increase of wing area. Both these coefficients follow the evolution of the friction drag coefficient C_f, which represent 5 % of variation for the wing and 10 % for the fuselage.

A second effect due to the ratio of the wing wet area over the reference area $\frac{S_{w,W}}{S}$. For the wing, the previous effect concerning the Reynolds number represents only one third of the increase of C_{Do_W}. The other two thirds are due to the increase of the wing wet area $S_{w,W}$ compared to the reference area S. As the wing surface increases the part of the fuselage relative to the aircraft geometry, decreases. In that way, the wing surface which "experiences the wind", that is to say the wet surface, increases and tends to toward the total wing surface, including the part inside the fuselage. This surface is the reference surface S. In others words when the wing surface increases, the wet wing surface $S_{w,W}$ increases faster than the reference surface S.

A third effect due to the ratio of the fuselage wet area over the reference area $\frac{S_{w,F}}{S}$ – Contrary to the wing where both the wing wet area and the reference area increased with the altitude, here only the reference area increases as the fuselage shape is fixed and thus its wet area, $S_{w,F}$, remains constant. In this case, the evolution of this ratio is far more important than for the wing. As a result, the drag coefficient of the fuselage, C_{Do_F} decreases readily thanks to the decrease of the ratio $\frac{S_{w,F}}{S}$, itself due to the increase of the reference area S. This is the dominant effect on the drag coefficient of the airplane C_{Do}, even if there is a small increase of 10 % of the local drag coefficient $C_{Do_{F,\ell}}$ due to the effect of the Reynolds number on the friction coefficient C_f.

To summarize - The drag coefficient of the wing C_{Do_W} slightly increases by 20 % for a variation of altitude h of $30\,kft$ — from $15\,kft$ to $45\,kft$ —. One third of this evolution is due to the decrease of the Reynolds number — increase of the fric-

2. For an operational flight with a constant C_L the Reynolds number is proportional to square root of the air density.

tion drag coefficient Cf — while the two other thirds are due to the stronger increase of the wet wing area $S_{w,W}$ compared to the total area of the wing, that is to say the reference area S, in the ratio $\frac{S_{w,W}}{S}$. This phenomenon is directly linked to the fixed shape of the fuselage.

On the other hand, the drag coefficient of the fuselage, C_{DoF}, steeply decreases for a variation of altitude h of $30\,kft$ — from $15\,kft$ to $45\,kft$ —, as its value is divided by four. This phenomenon is due to the increase of the wing reference area S in the ratio of the wet area of the fuselage $S_{w,F}$ over S. And the decrease of this ratio when the altitude increases is dominant on all other effects.

> **Finally, the minimum drag coefficient C_{Do} of the airplane decreases steeply with the altitude h,**
> **it follows the C_{Do} of the fuselage.**
> This phenomenon is directly linked to the increase of the wing area which progressively dominates the fuselage within its own geometry, and leads the C_{Do} of the aircraft to tend towards the C_{Do} of the wing alone, that is to say far smaller.

4.3 The evolution of weights with altitude

With a constant total weight $MTOW$ and constant payload weight W_{pl}, when the altitude changes, the evolution of weights are focused on the operational empty weight OWE and the fuel weight W_{fuel}.

The operational empty weight OWE is the sum of the wing weight W_w, the engine weight W_e, the fuselage weight W_{fu} and equipment weight W_{eq}. With the altitude variation, only the wing and engine weights are likely to vary.

$$MTOW = \underrightarrow{OWE} + \underleftarrow{W_{fuel}} + W_{pl} \qquad \underrightarrow{h}$$

with

$$\underrightarrow{OWE} = \underrightarrow{\underrightarrow{W_w}} + \underrightarrow{W_e} + W_{fu} + W_{eq} \qquad \underrightarrow{h}$$

Then, when altitude will change, an exchange will carry out between the wing weight, W_w, plus engine weight, W_e, and the fuel weight W_{fuel}. So the less is the empty weight, the better is the fuel weight and the performance.

Due to the wing area increase with the altitude, the wing weight increases as well as the engine weight. In fact, even if the increase of finesse induces a reduction of drag force, the dominant effect which determine the size of engine is the proportionality of thrust with the air density. So, in the end, the size of engine should increase.

In conclusion, the two weights — wing W_w, engines W_e — increase when altitude increases, so the fuel weight, W_{fuel}, decrease with altitude.

4.4 The optimal cruise altitude exists

Now have a look at the performance knowing the effect of altitude on finesse and fuel available. The ratio of fuel weight to total weight is denoted

$$\widetilde{W_{fuel}} = \frac{W_{fuel}}{MTOW}$$

The performance is modelized by the Bréguet Range equation

$$\begin{aligned} R &= \frac{f\,V}{Cs_J\,g}\ \mathrm{Log}\,\frac{1}{1-\widetilde{W_{fuel}}} \\ R &\approx Cte\ f\,\widetilde{W_{fuel}}\left(1+\frac{\widetilde{W_{fuel}}}{2}\right) \approx Cte\ f\,\widetilde{W_{fuel}} \end{aligned}$$

In a first simplified approach, we can say that the range R is proportional to the finesse f and the fuel weight W_{fuel}. We have shown that the finesse increases with altitude and the fuel weight decreases with altitude. Thus, there are two conflicting effects leading to an optimum altitude and resulting from the trade-off between the finesse, f, on one hand and the fuel weight, W_{fuel}, on the other hand (Figure 2).

The maximum range is obtained for an altitude of $28\,000\,ft$, whereas the minimum specific fuel consumption is reached around $38\,000\,ft$. Therefore, the minimum DOC will be placed between these two altitudes. Then the operational cruise altitude, for this kind of aircraft, is found.

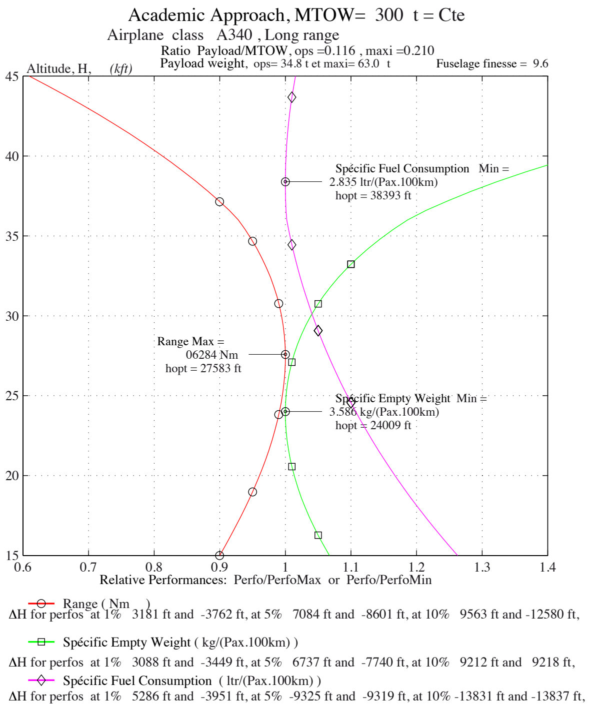

Fig. 2 Specific Performances of an A330 or B777 Airplane Class as a function of altitude. A conceptual optimal cruise altitude exists.

> The performance, that is to say the maximum range, is therefore the product of the finesse by the fuel weight.
>
> Each parameters is varying on the opposite side when the altitude increases,
>
> **Thus an optimum altitude exists.**

4.5 Principles linked to the optimal cruise altitude

Considering our previous observations, it is possible to state two principles leading to the existence of an optimal conceptual cruise altitude. During the preliminary design of an aircraft, the engineer will choose the cruise altitude :

– as high as possible in order to get a good aerodynamic performance, that is to say a high finesse.
– but not too much because both the wing and engines weights will increase, which will lead to a higher empty weight and finally less fuel to perform the mission

These two principles are illustrated on the figure (3), p. 8 showing the evolution with the altitude of the weight and aerodynamic functions in the Bréguet range formula and reported to the optimum values so as these functions cross at the optimal altitude.

If the finesse of the aircraft increases due to, for example, an improvement in the aerodynamic, h_{opt} will decrease because in this case there is no more reason to fly high to reach a good aerodynamic performance.

On the other hand, if the empty weight becomes worse following, for example, a concern during the structural design, h_{opt} will also decrease because in this case we are already limited by the weight and the situation does not become worse when increasing the altitude. And in this case, it is also difficult to increase the aerodynamic by increasing the altitude.

4.6 Cruise speed effect

In the previous section, the existence of an optimal altitude has been demonstrated physically as the result from the trade-off between two conflicting effects with the altitude. But the same powerful approach can be applied by changing an input parameter in the design process — cruise speed, size of the aircraft, range, etc.

For example, how the optimal altitude is affected by a reduction of the cruise Mach number ?

At a given altitude, if the cruise speed is lower, the wing area will be higher — lift equation. A higher wing area leads to a higher finesse on one hand and higher empty weight, so less fuel, on the other hand. As the optimal altitude is roughly at the intersection of the finesse curve and the fuel curve, the optimal altitude decreases when the cruise Mach number decreases (Figure 3).

Practically, this results proves that the cruise altitude of a turboprop commuter aircraft is lower than a jet aircraft.

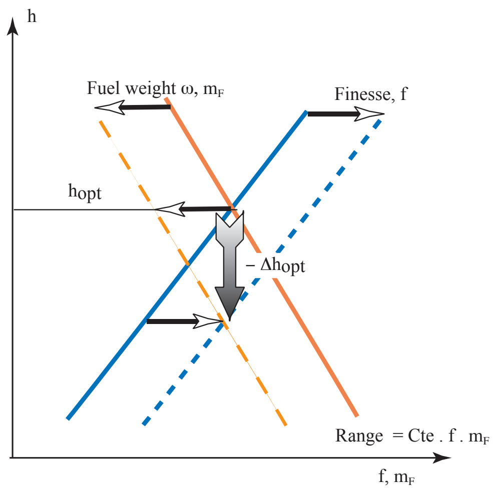

Fig. 3 Effect of cruise speed on the conceptual optimal cruise altitude

5 Conceptual optimal cruise altitude with MANUFACTURER APPROACH

An optimal cruise altitude has been demonstrated with the ACADEMIC APPROACH, that is to say while considering a constant total weight $MTOW = Ct$, as well as a constant payload weight $W_{pl} = Ct$ leading to find the maximum range. As said before, this approach favors physical explanations but is not directly linked to the operational requirements from the customer.

On the other hand, the MANUFACTURER APPROACH gives an answer to the operational requirements from the customer, in particular by considering a constant given range $R = Ct$, as well as a constant payload weight $W_{pl} = Ct$ leading to find the minimum total weight $MTOW_{mini}$. Now, the link between the MANUFACTURER APPROACH and ACADEMIC APPROACH will be given.

With the ACADEMIC APPROACH , the range was varying with the altitude with a constant total weight $MTOW$. From this result, the range must be kept constant in the MANUFACTURER APPROACH. From the maximum range at the optimal cruise altitude given by the ACADEMIC APPROACH, $h_{opt_{Aca}}$, some fuel must be added to keep the same range when flying above and below the optimal altitude, $h_{opt_{Aca}}$. Thus, the $MTOW$ reaches its minimum value at $h_{opt_{Aca}}$.

When comparing the figure (4), p. 9 on the left computed with the ACADEMIC APPROACH with the figure (4), p. 9 on the right computed with the MANUFACTURER APPROACH, it appears that

- The altitude leading to the minimum total weight $MTOW_{mini}$ from the MANUFACTURER APPROACH is equal to the altitude leading to the maximum range from the academic method,
- The altitude leading to the minimum specific empty weight $\widehat{MVOE}$ from the MANUFACTURER APPROACH is lower than the altitude computed with the ACADEMIC APPROACH, by approx. $-7200\,ft$. In addition, the minimum specific empty weight obtained at the optimal altitude is also lower by $2\,\%$ for the MANUFACTURER APPROACH.
- The altitude leading to the minimum specific consumption $\widehat{W_{fuel}}$ from the MANUFACTURER APPROACH is also lower than the same altitude computed with the ACADEMIC APPROACH, by approx. $-4600\,ft$. In addition, the minimum specific fuel consumption $\widehat{W_{fuel}}$ obtained at the optimal altitude is higher by $1.7\,\%$ for the MANUFACTURER APPROACH.

To summarize, when moving from the
ACADEMIC APPROACH to the
MANUFACTURER APPROACH
the optimal cruise altitudes decrease by approximately $5000\,ft$
Except for the $MTOW_{mini}$ which corresponds to the maximum range
from the ACADEMIC APPROACH
The minimum specific empty weight $\widehat{MVOE}$ decreases by $2\,\%$
and the minimum specific fuel consumption $\widehat{W_{fuel}}$ increases by $2\,\%$

5.1 Empty weight with the MANUFACTURER APPROACH

The empty weight is affected by the variation of $MTOW$, it increases above and below the optimal altitude corresponding to the minimum

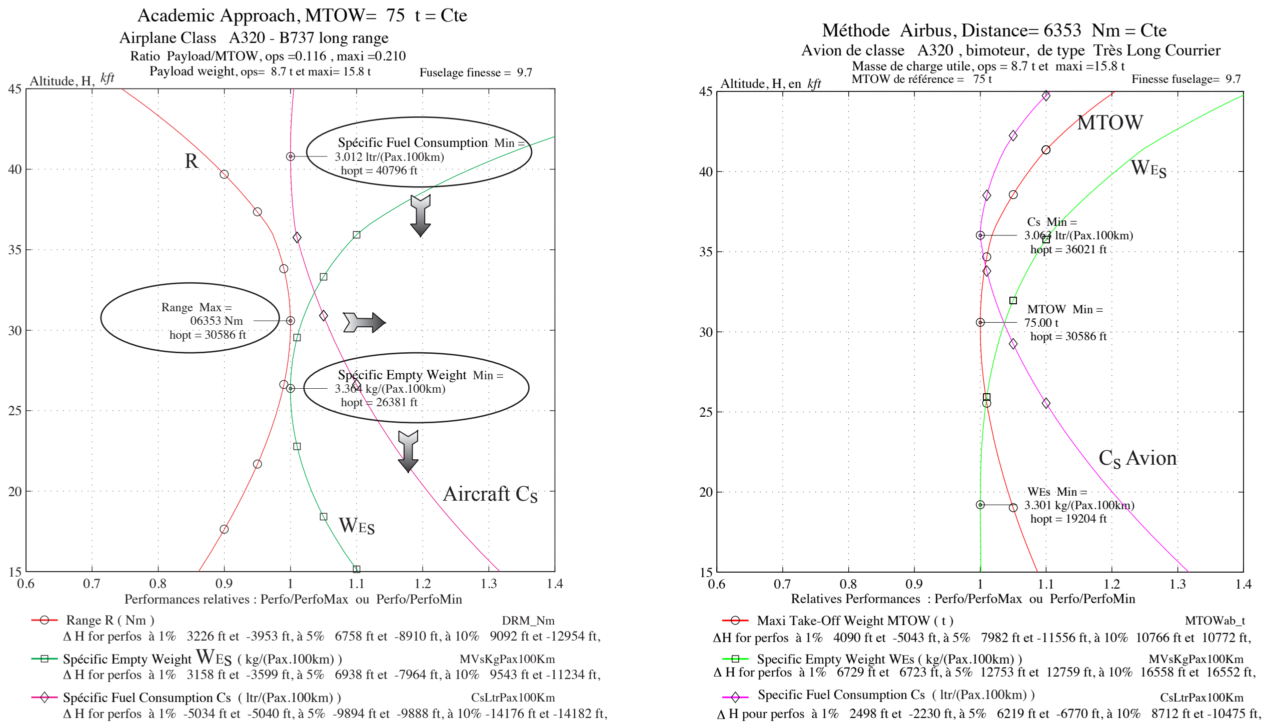

Fig. 4 On the left, the optimum of the range R, the specific fuel consumption $\widehat{W_{fuel}}$ and specific empty weight $\widehat{MVOE}$ for a single-aisle type aircraft with the ACADEMIC APPROACH. And on the right, the optimum of the weight $MTOW$, the specific fuel consumption $\widehat{W_{fuel}}$ and specific empty weight $\widehat{MVOE}$ for the same type of aircraft with the MANUFACTURER APPROACH.

$MTOW$, while following the increase of the wing weight and engines weight due to the increase of the $MTOW$. With the academic method, it was also increasing to follow the effects of altitude variation, but without the influence of this increase of $MTOW$. This is therefore an additional effect.

Concerning the specific empty weight (Equation 1, p. 2), it isn't anymore affected by the range R, contrary to the academic method where the range was lower below and above the optimal altitude, leading naturally to get an increased specific empty weight on both sides of this altitude. There was a kind of "attraction" of this optimal altitude of the specific empty weight by optimal altitude of the range. On the other hand, with the MANUFACTURER APPROACH, both the range and the number of passengers, $RNpax$, are constant and the optimal altitude of the specific empty weight is no more attracted up by the optimal altitude corresponding to the minimum range or minimum weight. It's therefore lower.

5.2 Fuel weight MANUFACTURER APPROACH

As for the empty weight, the specific fuel weight is no longer affected by the variation of range, contrary to the ACADEMIC APPROACH, because with the MANUFACTURER APPROACH both the range and number of passenger are constant. Therefore its optimal altitude isn't attracted up anymore by the optimal altitude corresponding to the minimum range.

Nevertheless another effect appears. The optimal altitude remains attracted down by the optimal altitude corresponding to the $MTOW_{mini}$, because the fuel weight increases on both sides of this altitude to reach the constant range whatever the altitude.

This second effect dominates the first one since the optimal altitude decreases.

6 Conclusion

When the aircraft designer decides what will be the value of the cruise altitude of his future aircraft, he can choose an optimal cruise altitude, which optimizes the performances. This result is shown in this paper and summarized below.

The performances are evaluated through the range R, which is roughly the product of the finesse of the airplane $L/D = f$ with the fuel weight W_{fuel}. Both are dependent on the size of the wing, that is to say the wing surface.

Due to the lift equation, the wing surface is strongly linked to the altitude, so that the wing surface increases when the conceptual cruise altitude increase.

When the wing surface increases, the finesse also increases because the minimum drag coefficient of the airplane, C_{Do}, decreases towards the C_{Do} of a wing, which is considerably lower. On the other hand, when the wing surface increases the wing weight increases, thus the empty weight also, O_{WE}, leaving less room for the fuel weight, W_{fuel}.

The increase of finesse leads to better performance, however, the increase of O_{WE} reduces the performance. Thus, two conflicting effects appear, which reveal an optimal altitude.

❶ The aircraft designer is in quest for altitude for his airplane in order to “gain” a good finesse more performances, more range R

❷ But he has to “pay” for this altitude with wing weight and engines weight less fuel, less performance, less range R

As a side result, thanks to this approach, it can be shown, that this optimal altitude h_{opt} increases with the cruise speed of the airplane. And for example, this approach is suitable for finding and understanding an optimal cruise Mach number.

References

[Kro06] Ilan Kroo. Aircraft design : Synthesis and analysis. Course AA 241 http ://adg.stanford.edu/aa241/, Stanford University, 2006.

[Tor86] Egbert Torenbeek. *Synthesis of subsonic airplane design*. Delft University Press, 1986.

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