TOOLS FOR DYNAMIC ADJUSTMENT OF AIRCRAFT SEPARATIONS IN WAKE VORTICES

Laurent Planckaert*, Patricia Coton*, Mathieu Balesdent**

**ONERA/DCSD 5, Bd Painlevé 59045 Lille FRANCE,*
***ONERA/DPRS 29, avenue de la division Leclercq 92322 Chatillon FRANCE*

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Abstract

To avoid the risk induced by wake vortices, minimum separation standards between aircraft in the arrival and departure flow have been defined which, nowadays seem to be over conservative.

The aim in the medium / long term of the work presented in this paper is to develop tools for aircraft guidance in the wake turbulence using predictive modelling of the vortices and measurements performed on board or transmitted from outside. Thus, this will enable to perform simulation tests on new operation procedures authorizing more throughput and to give some specifications for new sensors.

The proposed avoidance tools are based on the frame of potential field methods, using flatness properties of the flight mechanic model of the aircraft in order to build feasible trajectories.

1 Introduction

In today's efficient operations, aircraft are either separated by wake turbulence rules or by runway and radar separations that apply when wake turbulence separations are not required. To avoid the risk induced by wake vortices, minimum separation standards between aircraft in the arrival and departure flow were defined by IACO in the seventies. These standards are currently based on the maximum takeoff weight of aircraft (MTO weight). The aircraft are distributed in 3 categories according to their MTOW and minimum separation distances are defined in a pair wise manner.

These rules are, in general, over conservative in that they are fixed regardless of the prevailing meteorological conditions impact on the transport and decay of the wake turbulence. Moreover, the existing wake vortex separations seem to be rough because quite different aircraft pairs are considered to require the same distance separation. Consequently, they have a direct negative impact on the landing and takeoff capacity of the runways.

In the last years, numerous works have been done to study less conservative operation concepts to sequence arriving or departing aircraft: time-based separations, weather-dependent separation and pair wise separation (with a more refined categorization).

However, these new concepts mainly depend on static characteristics of the aircraft and gains in performance could be waited from taking into account some dynamic features (actual circulation of the wake, gross weight of the following aircraft…) and from the possible ability of the follower to manage with the wake vortex encounter.

Improvement of the modelling of the involved physical phenomena (wake formation and development under action of the atmospheric environment, impact on the follower aircraft), advances in anemometric measurement instruments based on LIDAR and prospect that each aircraft could receive information on the type, configuration and environment of the preceding aircraft make it possible to consider a dynamic management of spacing between aircraft. Thus, knowing the disturbance field to come, one will be able to define new trajectories, to avoid areas where the disturbance is the strongest or get through these areas with enough reserve of maneuverability. Moreover, ground based or airborne instruments like LIDARS need specifications (range, accuracy…) according to their use in weather dependant or dynamic separation concepts.

So there is a need to get a quick evaluation of new operation concepts or new instruments. This evaluation includes a long term planning of the trajectory which enables to give an alert on the severity of the wake encounter and a mid-term planning of the trajectory to alleviate the effect of the wake.

The proposed guidance/avoidance tools are based on potential field methods, and use flatness properties of the flight mechanics model of the aircraft in order to build feasible trajectories. Optimization techniques using flatness are less time consuming than other methods like dynamic inversion because it enables to reduce the problem dimension. Potential field methods for guidance applications have the advantage to take into account easily constraints on trajectory.

2 Trajectory generation using flatness property of aircraft system [1,2]

2.1 Definition of flatness

A system defined by the equation:

$$f(x,\dot{x},u)=0$$

where x is the state vector and u the input, is flat if there exists an output vector z_d with independent components so that :

$$z=h(x,u,u^{(1)},...u^{(\varepsilon)})$$

and two functions A and B so that :

$$x=A(z,z^{(1)},...,z^{(\eta)})$$

$$u=B(z,z^{(1)},...,z^{(\mu)})$$

where ε, η and μ are finite integers.

2.2 Trajectory planning

To generate a trajectory $z_d(t)$ on a time segment [t_0, t_f], you just have to use the input law

$$u_d=B(z_d,z_d^{(1)},...,z_d^{(\mu)})$$

It is necessary that z_d is μ-derivable with respect to t.

2.3 Criterion optimization

To generate a trajectory minimizing a criterium J

$$J=\int_{t_0}^{t_f} q(x,u)dt$$

is equivalent to solve the following problem

$$J=\int_{t_0}^{t_f} L(z,z^{(1)},...,z^{(\alpha)})dt$$

2.4 Trajectory tracking

By using :

$$v=z_d^{(\mu)}+\sum_{k=0}^{\mu-1}K_k\left(z_d^{(k)}-z^{(k)}\right)$$

with

$$K(p)=p^{\mu}+\sum_{k=0}^{\mu-1}K_k p^k$$

being a polynomial whose roots have negative real part. The control law

$$u=B(z,z^{(1)},...z^{(\mu-1)},v)$$

enables to ensure an asymptotic tracking of the desired trajectory z_d .

2.5 Flatness of aircraft model

Using the classical flight mechanics equations with wake vortex perturbations:

$$m\frac{d\vec{V}}{dt}=m\vec{g}+\vec{F}_{aero}+\Delta\vec{F}_{WV}+\vec{F}_{Thrust}$$

$$\frac{d\vec{\sigma}}{dt}=\vec{M}_{aero}+\Delta\vec{M}_{WV}$$

It is possible to demonstrate under non restrictive assumptions that the aircraft flight dynamics model without wake perturbations is flat. The assumptions used are that the inputs effects on forces are negligible on Y and Z axes.

V, γ, φ, β (respectively the speed, path angle, bank angle and sideslip angle of the aircraft) are the flat outputs. Then the four inputs T, δ_m, δ_l, δ_n (respectively thrust, pitch, roll, and yaw control surface settings) can then be written as follows :

$$T = h(V, \dot{V}, \gamma, \dot{\gamma})$$
$$\delta_m = k(V, \dot{V}, \ddot{V}, \gamma, \dot{\gamma}, \ddot{\gamma}, \gamma^{(3)})$$
$$\delta_l = l(\phi, \dot{\phi}, \ddot{\phi}, \beta, \dot{\beta}, \ddot{\beta})$$
$$\delta_n = n(\phi, \dot{\phi}, \ddot{\phi}, \beta, \dot{\beta}, \ddot{\beta})$$

2.6 Generation of trajectory

Considering the longitudinal flight mechanics model, as

$$\dot{x} = V\cos\gamma$$
$$\dot{z} = -V\sin\gamma$$

If we impose the trajectory then (V, γ) is unique.

If we now look at the lateral model, a given trajectory may results from different (β, ϕ).

Considering that the pitch angle and angle of attack remain in the vicinity of a mean value during the motion and that the sideslip angle β_d remains equal to 0 all along the trajectories, it is possible to express V, γ, ϕ in a unique manner as functions of $\dot{x}_d, \dot{y}_d, \dot{z}_d$. The trajectory needs to be 4-derivable. The inputs $P, \delta_m, \delta_l, \delta_n$ are directly calculated from x_d, y_d, z_d.

To generate a trajectory in the (x_d, y_d) plane at $\beta_d = 0$, the following relationship is used :

$$\tan\psi_d \approx \frac{\dot{y}_d}{\dot{x}_d}$$

which leads to

$$\dot{\phi}_d = (\dot{\psi}_d \cos\theta_0 - q_0 \sin\phi_d) f(\phi_d) + g(\phi_d)$$

with

$$\dot{\psi}_d = \frac{\ddot{y}_d \dot{x}_d - \ddot{x}_d \dot{y}_d}{\dot{x}_d^{\,2} + \dot{y}_d^{\,2}}$$

where

$$f(\phi_d) = \frac{mV(\tan\theta_0 \cos\phi_d \sin\alpha_0 + \cos\alpha_0) + \frac{1}{2}\rho S V^2 \left(\frac{C_{yp} l}{V} \tan\theta_0 \cos\phi_d - \frac{C_{yr} l}{V}\right)}{\cos\phi_d \left(\frac{1}{2}\rho S l V C_{yp} + mV \sin\alpha_0\right)}$$

and

$$g(\phi_d) = \frac{\frac{1}{2}\rho S V^2 C_{yp} \tan\theta_0 q_0 \sin\phi_d - mg \sin\phi_d \cos\theta_0 + mV \tan\theta_0 q_0 \sin\phi_d \sin\alpha_0}{\left(\frac{1}{2}\rho S l V C_{yp} + mV \sin\alpha_0\right)}$$

2.7 Trajectory tracking in wake turbulence

The effects of the wake vortices are difficult to formulate according to the flat outputs. Thus, in order to follow the desired trajectory in disturbed environment, these effects are compensated by means of appropriate increments of thrust and deflection of the control surfaces. Moreover, it is possible to compensate in the same process the neglected terms of the flat approximate model.

For instance, using the following longitudinal flight model, where $\Delta Z, \Delta M$ are the force and torque induced by the wake

$$m\dot{V} = -\frac{1}{2}\rho S V^2 f\left(C_z - \frac{2\Delta Z}{\rho V^2}\right) - mg\sin\gamma + T\cos(\alpha + \overline{\omega})$$

$$-mV\dot{\gamma} = \Delta Z - \frac{1}{2}\rho S V^2 Cz + mg\cos\gamma - T\sin(\alpha + \overline{\omega})$$

$$B\dot{q} = \frac{1}{2}\rho S l V^2 \left(C_{m0} + C_{m\alpha}\alpha + C_{mq}\frac{ql}{V} + C_{m\dot{\alpha}}\frac{\dot{\alpha} l}{V} + C_{m\delta m}\delta m\right) + \Delta M$$

it is possible to alleviate the wake perturbation effects as well as the $-T\sin\alpha$ term. The necessary condition not to modify the desired trajectory is to keep the same history of (V, γ). Therefore, from the two forces equations it is possible to deduce an increment of (α, T). The increment of δ_m is then obtained so as to cancel the effect of ΔM while generating the desired increment $\Delta\alpha$ with the following equation.

$$B\Delta\ddot{\alpha} = \frac{1}{2}\rho S l V^2 \left(C_{m\alpha}\Delta\alpha + (C_{mq} + C_{m\dot{\alpha}})\frac{\Delta\dot{\alpha} l}{V} + C_{m\delta m}\Delta\delta m\right) + \Delta M$$

3 Wake vortex avoidance strategy

Wake vortices may have strong effects on following aircraft while their influence volume around the trajectory of the generator aircraft is rather small (one wing span in height * two wing spans in width). Consequently to avoid the encounter of hazard area often needs only small changes of the trajectory.

Hazard area determination results from wake vortex features, following aircraft abilities to cope with the perturbation and pilots' perception of risk. A global vision can be obtained through hazard criterions.

Two typical applications were envisaged in this work:

- Long term planning of the trajectory: having an a priori knowledge of the wake vortex perturbation in the vicinity of the landing trajectory of the aircraft, determine a new "safe" admissible trajectory
- Mid term planning: having knowledge on a shorter range ahead from the aircraft, adapt the trajectory dynamically as new information is available to the aircraft. A good initial trajectory for this application could be the one found using all the a priori knowledge of the wake vortex features.

3.1 Hazard metrics

A lot of risk metrics have been proposed in the past. From piloted simulations in wake turbulence experiments, the pilots' danger perception was correlated with flight parameters, and best candidate risk metrics were based on roll behaviour. NASA Ames study [3] has shown that Φ_{max} metrics was very efficient and enable to define a risk criterion. An improved Φ_{max} criterion taking into account the effect of altitude was also established:

$$CRIT_{\Phi} = \frac{\Phi}{a_1 H + b_1}$$

More recent works in the European S-WAKE project [4] lead to the same conclusions. Others hazard criterions were developed such as Roll Control Ratio (RCR) which is relative to the ratio of the vortex-induced roll acceleration to the maximum acceleration that the pilot can command at an altitude H .

$$CRIT_{RCR} = \frac{\ddot{\Phi}_{WV}}{(a_2 H + b_2)\ddot{\Phi}a / c_{max}}$$

In order to improve these roll criterions, another sub criterion was developed by Airbus: the glide slope deviation criterion (ΔGS is the deviation of the glide slope induced by the wake vortex encounter).

$$CRIT_{\Delta GS} = (\Delta GS + 4\Delta\dot{G}S)K_{\Delta GS}$$

with $K_{\Delta GS} = \min\left(\left(\frac{H}{b}\right)^2, 1\right)$ and b the wingspan.

3.2 Long term planning

In order to obtain a pre-planned trajectory, an 'a priori' knowledge of the perturbation field all along the trajectory is needed. In this case, the problem to solve is to find the trajectory defined by the history of the coordinates ($x(t), y(t), z(t)$) of the plane such that the hazard metrics criterions are not violated during the flight and that the trajectory remains in an admissible region such as $g(M) < 0$. Therefore, what is to be done is quite equivalent to solve the following optimisation problem:

$$\int_0^{t_f} \max(Crit(M) - Threshold, 0)dt + f\left(y_{t_f}, z_{t_f}\right)$$

$$\text{with } g(M) < 0 \text{ and } M = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

In order to reduce the size of the problem the functions $x(t), y(t), z(t)$ are projected on a basis of B-Spline functions. The trajectory defined by the vector function $M(t)$ is expressed as:

$$M(t) = \sum_i B_{i,k}(t) P_i$$

where $B_{i,k}(t)$ is a B-spline function of order k (if we want $M(t)$ to be of class C^k , the number of control points must be greater than k+1). The variables that we have to optimize are the control points P_i . The trajectory is inside the control polygon defined by the P_i .

3.3 Mid term planning

From the knowledge of the field perturbation measured a few seconds ahead of the aircraft, it is possible to modify the pre-planned trajectory to take into account the actual position and intensity of the vortices.

The method used for obstacle avoidance is based on the potential field method [5,6]. The interest of this method lies in that it doesn't need a pre planned initial trajectory, is easy to deploy and is less calculus demanding than the spectral optimization methods. Its main drawbacks are the difficulty to manage multiple obstacles (problem of local minima) and the possible oscillatory behaviour in narrow passage.

The aircraft is subjected to an artificial force potential field U.

$$U(M) = U_{att}(M) + \sum_k U_{rep_k}(M)$$

The attractive potential U_{att} provides the rallying to a final position, and the repulsive potentials U_{rep_k} move away the aircraft from the obstacles.

The total "artificial" energy of the vehicle is calculated,

$$L = T - U - \sum_k U_{rep_k}$$

where T is the kinetic energy and U the "real" potential energy.

Then by application of lagrangian formalism, external forces F can be calculated:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\delta L}{\delta x} = F$$

In our case, a repulsive potential is calculated from the risk criterion at a point M. The attractive potential is introduced to keep the trajectory inside a desired area.

4 Application in simulation cases

A flight mechanics model of a generic transportation aircraft which is similar to an A300 airplane was used to carry out the simulation study.

4.1 Wake vortex effect model

Effects of the wake vortices on aerodynamic forces and torques applied on the following aircraft are estimated using the strip theory method [7]. The main lifting surfaces (wings, horizontal tail, rudder) of the aircraft are divided in strips (Fig.1). Each strip is characterized by its surface, its lifting force gradient and its aerodynamic center.

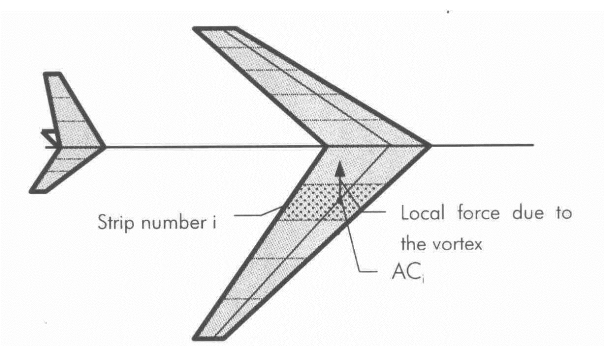

Fig 1. Principles of the strip theory method

The components $\begin{bmatrix} 0 \\ v_{0_i} \\ w_{0_i} \end{bmatrix}$ of the wake induced wind on each strip i are calculated in the plane orthogonal to the vortex axes and containing the aerodynamic center of the strip, then projected in the body frame $\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = R^{-1} \begin{bmatrix} 0 \\ v_{0_i} \\ w_{0_i} \end{bmatrix}$

which enables to get the variation of the angle of attack and sideslip angle on each strip

$$\Delta\alpha_i = -\frac{w_i}{V} \quad \text{and} \quad \Delta\beta_i = -\frac{v_i}{V}$$

The effect of the wake on the aerodynamic forces and torques is finally obtained through integration of the lift induced on every strip by the aforementioned variations.

4.2 Profile of the wake induced wind

The profile of the tangential speed in a plane orthogonal to the vortex axis is often written as a function of the distance from a given point M of the plane to the centre of the vortex and its circulation. An example of profile is given by the Burham and Hallock formula [8]:

$$V_\theta(M) = \frac{\Gamma}{2\pi} \frac{r}{r^2 + r_c^2}$$

with

- r : distance from M to the center of the vortex
- r_c : viscous radius of the vortex

The wake induced wind speed at M is the sum of the contributions of right and left vortices.

Let m be the mass of generator aircraft and b its wingspan

$$\frac{1}{2}\rho SV^2 Cz = mg = \rho V \int_{-b/2}^{b/2} \Gamma(y)dy = \rho Vkb\Gamma_0$$

$$\text{So } \Gamma_0 = \frac{mg}{\rho Vkb}$$

with $k = \frac{\pi}{4}$ for an elliptic lift distribution on the wing, $k = 1$ for a uniform distribution.

Moreover, the circulation $\Gamma(t)$ is a solution of an Ordinary Differential Equation (ODE): decay equation, and the location of the centre of the vortex is also obtained through an ODE resolution: the transportation equation.

4.3 Example of long term planning

A case of final approach is presented. The trajectory optimization was carried out from an altitude of 150m above the runway. This altitude is a classical GO/NOGO decision point for landing. One of the wake vortices (circulation 400m^2/s) is at 75m of altitude in the runway axis which is sufficient to saturate the ailerons. It is assumed that there is no transportation of the vortices.

The trajectory is researched in a family of class C^4 B-Splines curves. At least 5 control points are needed. A point is added in the vicinity of the touch point to improve tangent conditions of the trajectory to the runway axis. A supplementary point is added to increase the richness of the possible trajectories. The initial control points are regularly spaced along the x-axis. The first point is fixed to the initial condition value.

In this example, the criterion to minimize is:

$$\int_0^{t_f} \frac{\max\left(|\delta l| - \delta l_{\max}, 0\right)^2}{\delta l_{\max}{}^2} dt + \left(\frac{y(t_f)}{b}\right)^2 + \left(\frac{z(t_f)}{b/2}\right)^2$$

Using B-Spline decomposition, the variables to optimize are the y and z components of the control points (12 DOF). In this application, Nelder-Mead's algorithm was chosen to optimize the trajectory because it doesn't require estimation of criterion gradient.

The optimized trajectory on figure 2 was found in 11 iterations. This trajectory moves away laterally from the vortex where the aircraft intersects the plane containing the vortex centres. Bank angle remains at a low value (<2°) while aileron setting stays slightly below its saturation value (30°).

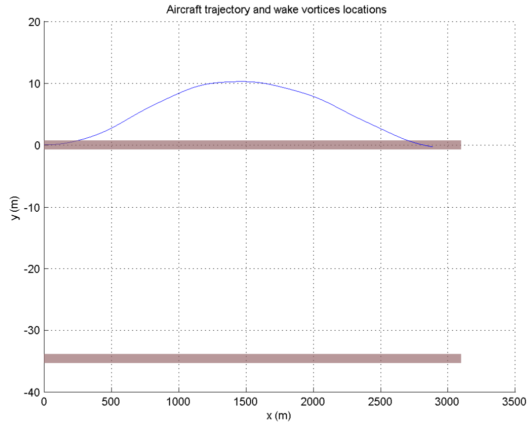

Fig. 2.1: Aircraft optimized trajectory (in red : footprint of the vortices)

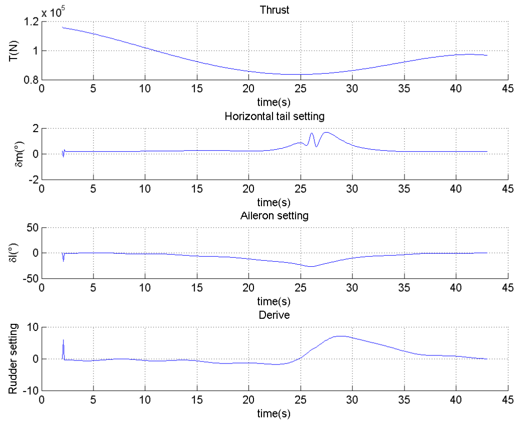

Fig. 2.2 Input history during optimized trajectory

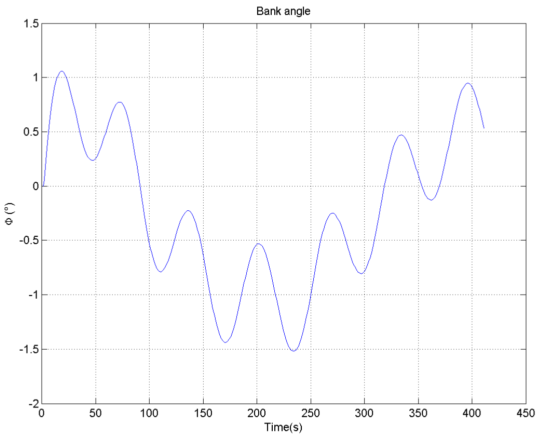

Fig. 2.3: Bank angle history

4.4 Example of mid term planning

The case studied here is an approach-landing phase, in the configuration:

- trajectory of generator aircraft
  - A300 type
  - Heading 0°
  - Slope −2°
  - ground speed 70,7m/s
  - location at t=0s : x=0 , y=−15.7m, h=870m (left vortex on the runway axis)
- Initial trajectory of the following aircraft
  - Generic aircraft
  - heading 0°
  - slope −3°
  - ground speed 70,7m/s
  - location at t=100s: x=0 , y=0, h=930m

It is supposed that the aircraft has the ability to measure the wind velocity field in a plane orthogonal to its trajectory 300m ahead.

The vortex has a Winkelmans profile [9], and decay and transportation equations are based on the Sarpkaya decay model [10]. Each vortex has an action on the other so that the two vortices have a descent motion.

The repulsive potential used is based on the RCR criterion:

$$U_{rep} = \frac{|\Delta Cl_{WV}|}{|Cl_{\delta l}\delta l_{\max}|}$$

As we want that the trajectory stay near the non disturbed one (glide slope 3°), the attractive potential acts so that the trajectory stay in a cone around the landing trajectory.

$$U_{att} = \frac{\frac{h}{d} - \tan\sigma_0}{\tan\sigma_0}$$

The value of σ_0 was 0.3° (cone semi-angle), h is the altitude and d the distance to the runway.

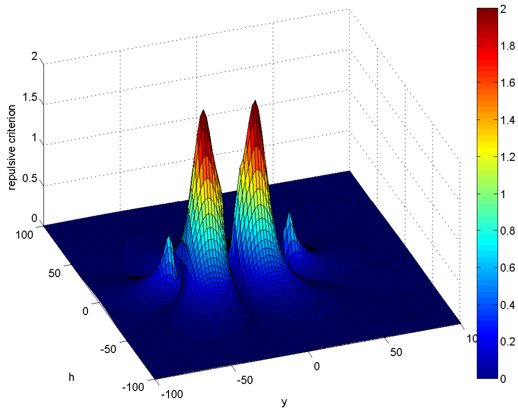

Fig. 3: Repulsive potential in a plane orthogonal to aircraft trajectory

4.4.1 *Description of the avoidance algorithm*

The different steps of the methods are :

1. Initialization phases : on the interval $[t_0, t_0 + (n-1)\Delta t]$ the trajectory is identical to the nominal one
2. Standard phases
   a) At time t, a prediction of speed and location of the aircraft at $t + (n-1)\Delta t$ is available. An initial guess point M at $t + n\Delta t$ is calculated on the tangent to the circle at this point
   b) If $U_{rep}(M) > Threshold$ or $U_{att}(M) > 0$ a new M is calculated in the direction of $-gradU_{rep} - gradU_{att}$ so as to reduce the artificial potential
   c) If $M_{new} = M$
      If $U_{rep}(M) > Threshold$ then
      GO Around decision
      else
      goto e)
   d) If this new M location is not acceptable return to step b)

e) While $U_{rep}(M) < Threshold$ and $U_{att}(M) > -1$ a new point is calculated to reduce the attractive potential

f) A new circle trajectory is defined from location at $t + \Delta t$ to M

g) t is increased of Δt. The control settings on $[t, t + \Delta t]$ are calculated using flatness properties. Goto a)

4.4.2 Simulation results

A value for the RCR criterion of 0.6 was chosen in order to leave some margin of aileron control. Using the initial trajectory, the aircraft should intersect the vortex core plane at x=13000m and the RCR criterion would exceed 1. The planned trajectory avoiding the right vortex is moving upwards and to the left, towards the middle of the two vortices (Fig 4.1 and 4.2). The comparison between planned and actual (using flatness control) trajectory is also shown (Fig. 4.1). Just after the vortex avoidance phase, there is an overshooting in lateral displacement due to the algorithm of avoidance (use of circle to extrapolate the trajectory and imposed minimum radius turn).

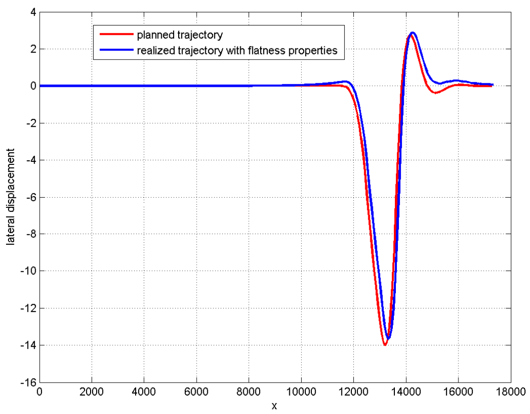

Fig 4.1 Planned and achieved trajectory (lateral displacement)

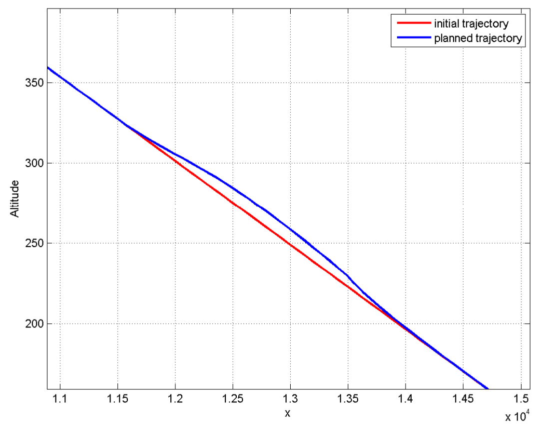

Fig 4.2 Vertical displacement

The hazard criterion is well respected but there is a second bump on the curve (Fig. 4.3) caused by the too early rallying of the aircraft to the glide slope. The control inputs are presented Fig. 4.4 and 4.5.

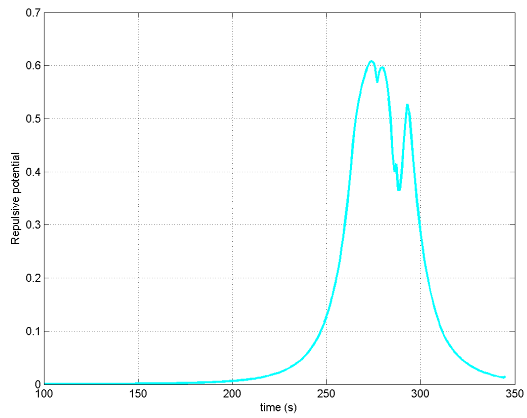

Fig 4.3 History of hazard criterion

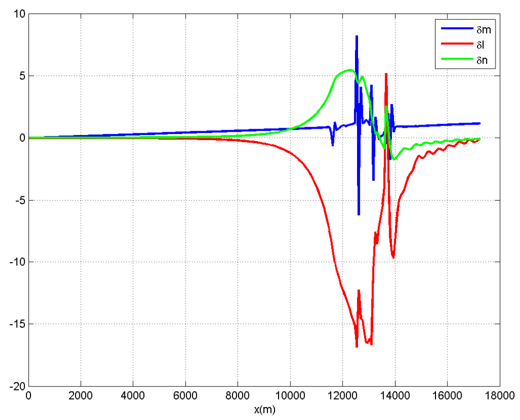

Fig 4.4 History of control settings

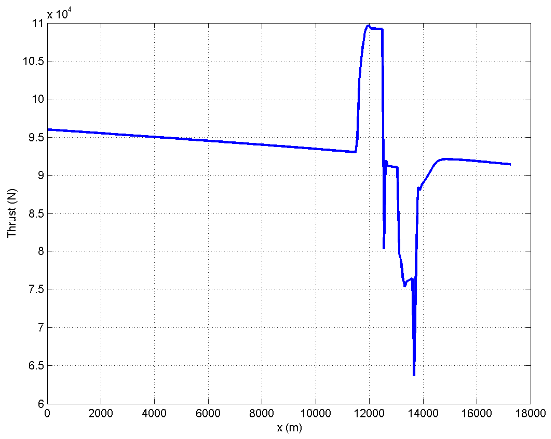

Fig 4.5: History of thrust

5 Conclusion

The tool presented in this paper aims at defining and adjusting trajectories to avoid wake vortices. It could be used to contribute to the definition of in-board or ground-based disturbance sensors. It may indeed help to determine the minimum detection distance of the wake vortices as well as the dimensions of the plane to cover. It also may be useful to test new operation procedures.

Moreover, the method used has a lot of flexibility (potential to deal with multiple criterions and trajectory constraints), is easy to implement and not time consuming but requires some adjustment.

Use of more accurate methods based on receding horizon control are under study.

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