

# 3-D FEM ANALYSIS OF PLASTIC BEHAVIOR INSIDE MMC COMPOSITES CONSIDERING DAMAGE THEORY

A. Abedian\*, H. Farahpour\*\*

Department of Aerospace Engineering,  
Sharif University of Technology, Tehran, Iran

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## Abstract

The delamination phenomenon has been studied in macroscopic scale by different approaches. Here, using micro-mechanical models, it is tried to find the possible relationship between the problem and the stress fields around a fiber. Using the ideas and the results on the free surface cracking of unidirectional composite, the FEM model is developed. The model verification is done by comparing the results by the results obtained in presence of damaged incorporated by the damage theory. Here, on the free surfaces fiber/matrix debonding is applied as a composite damage and the results show a good agreement with the available results.

## 1 Introduction

Although the delamination phenomenon in laminated composite materials has been studied by so many researchers, the basic reasons for this structural defect, which makes the design with these materials very complicated, are not fully known, yet. Using different approaches (i.e. experimental, analytical, and numerical), several damaging parameters responsible for composites delamination have been found and characterized. The results obtained by these studies, which are mainly common in their macroscopic scale, refer the delamination on the free edge of laminates to different stress components. In brief, the results

are grouped as follows. The shear stress ( $t_{xz}$ ) is responsible for delamination in angle ply laminates, the cross-ply laminates are affected by  $s_x$  and  $t_{yz}$ , and finally  $s_x$ ,  $t_{yz}$ , and  $t_{xz}$  cause delamination in  $[\pm\theta/90/0]$  laminates[1].

The classical laminate theory, despite its weakness and limiting assumptions, has been widely used for analysis of delamination. With this theory, it is assumed that the layers of the laminate are in plane stress condition, i.e. all of the out of plane stress components are vanished. Also, no slipping between the layers occurs. In this way, the equilibrium equations on the free edge are not completely satisfied. In fact, the stress state in this zone is three dimensional, which is totally different with the initial assumptions, explained earlier.

A quick search in the literature also shows that the numerical methods (like Finite Element Analysis) have also played a considerable role in studies involving delamination phenomenon in laminate composites. For example, a 2-D FEM model has been used for calculation of the inter-laminar stresses and also the amount of strain energy release due to the delamination damage occurred at the free edge of a laminate under tensile, bending, and torsion loads. Also, using similar FEM models, a vast number of studies on the analysis of delamination have been performed [8]. It also worth to note that size-wise, most of the models considered in these studies are of the scale of laminate thickness. Note that in this scale a large number of fibers exist. However, the delamination study using

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\*E-mail address: [abedian@sharif.edu](mailto:abedian@sharif.edu)

\*\* Email address: [h\\_farahpour@alum.sharif.edu](mailto:h_farahpour@alum.sharif.edu)

micro-mechanical modeling has received no or a very little attention. Here, in the present study, it is tried to look at the delamination phenomenon using micro-mechanical FEM models in the size of fiber diameters. The history of this idea goes back to the studies performed on the end effects of unidirectional fiber reinforced composite materials, where it has been observed that during fabrication process these materials experience fiber/matrix cracking on the free surface of the composite [9-14]. In these studies it has been shown that the available shear-lag theories are incapable of explaining the reasons for such a defect which brings along serious setbacks in designing with these materials. In contrary to the shear-lag theory, which ignores the radial and hoop stress components at the fiber end on the free surface, in these studies singular radial and hoop stresses were found and proven and for the reasoning a new theory so called "Overlapping Hypothesis" was introduced [9-10]. In brief, this hypothesis was developed paying attention to the geometrical deformation of the composite constituents in the vicinity of the free surface during cooling from the processing temperature. The main focus was put on the differences in coefficient of thermal expansions of the composite constituents. According to these findings, it could be easily seen that a singular tensile radial stress component occur at the fiber/matrix interface on the free surface which could most probably cause the old problem of free surface cracking in unidirectional composites. More details of the hypothesis could be found in [9-14].

The present study is designed to see what happens during the manufacturing process at the free edge of a laminate where fibers with different angles in separate layers meet the free surface. The question is that whether an overlapping type hypothesis could be found for explaining the possible delamination activities in laminated composites during manufacturing process? This may then be used to see what happens to the laminate free edge under

subsequent service loads like mechanical, thermal, or combined thermo- mechanical loads either with constant or cyclic nature. In this study the following steps are taken. To find the stress nature in a laminate under a thermal load it is necessary to look at the elastic behavior of the laminate at first. Therefore, to eliminate the effects of inelastic deformation or any property change with temperature, only one degree temperature change was applied to the laminate. A two-layer unidirectional and also a four-layer symmetric cross-ply laminate are considered here. The details of the results could be found in [15], however, a brief review of the findings will be presented in the discussion section. Then the effects of inelastic behavior of the composite constituents on the stress-strain fields inside a two-layer cross-ply and a four-layer symmetric cross-ply laminates are studied, which the results are discussed in detail in the following sections. Note that the stress-strain fields were obtained in different locations of the laminates by considering appropriate boundary conditions, which are given in the modeling section. Finally, it will be shown that how the strain energy release due to the damage inflicted by the excessive plastic deformation could affect the composite internal stress and strain fields.

## 2 Material and geometry modeling

Depending on the size of the fibers the micro-mechanical modeling of laminate composites would be different. For the glass/epoxy composite used in [15], where only one degree of temperature change was applied, due to the small diameter of the fibers, it was necessary to assume two thin layers (with only one row of fibers) in the middle of the four-layer symmetric cross-ply laminate. In this way, it will become possible to model the laminate with only two adjacent cross fibers. However, there would be not such a limitation for two-layer unidirectional or two-layer cross-ply laminates. Note, as it was explained earlier, this part of the study would make it clear that what would be the stress fields

around the fibers in different kinds of laminates and also in different locations of a laminate without taking any nonlinear effects into account. To include the material nonlinearity of the constituents of a laminate, the SCS-6/Ti-24Al-11Nb inter-metallic matrix composite (MMC) is considered. There are two points in studying this composite. One, the stress fields around the fibers in different locations under the processing temperature and in presence of plastic deformation and also the effects of material property change with temperature would be known. Two, due to the large diameter of the fiber used, this MMC four-layer symmetric cross-ply laminate would be micro-mechanically modeled with an acceptable approximation, if one assumes just one row of fibers in every layer of the laminate (as was suggested in [16], see Fig. 1). This would brush aside any doubts around the result obtained with modeling of small size fiber laminates explained earlier.

Here, for studying the stress fields at free edge or inside the cross-ply and unidirectional laminates under consideration four regions as schematically shown in Fig. 2, should be considered. In region four, which is far enough from the free edges of the laminate, it is assumed that the edge effects would diminish or in other words the state of internal stress would dominate, which here is referred to as the inner zone. For simulating different regions of cross-ply two or four-layer symmetric laminates, either with small or large diameter fibers, only one model but with different boundary conditions would be used. Fig. 3 shows the schematic of the model incorporating two fibers of each layer.

In each layer, the fiber parallel to the free edge is the most critical fiber, though the next adjacent fiber to it is also important to watch. However, in this study, due to the limitations of the computer resources for the 3-D modeling only one fiber of each layer, i.e. the one adjacent to the free surface, is modeled, see Figs. 3(b) and 3(c). The boundary conditions on areas ABCD (free), BB'C'C (free), AA'D'D (coupled displacement in

the y-dir), A'B'C'D' (coupled displacement in the z-dir), and DCC'D' (coupled displacement in the x-dir) should be applied to the model of Fig. 3(c) to simulate the mentioned laminates. However, the boundary condition on area ABB'A' would only be distinct between the laminates being simulated by the model. The symmetric boundary condition identifies the four-layer symmetric cross-ply, while the condition of coupled displacement in the x-dir. defines the two-layer cross-ply laminate. Note that the model of Fig. 3(c) could also be used for simulating the inner zone of the composite, i.e. the location identified as region 4 in Fig. 2. The only difference here is the boundary conditions on the free edges of ABCD and BB'C'C which should be changed to the symmetric conditions. Additionally, due to the existing symmetric conditions, the model could be further reduced as shown in Fig. 3(d).

One more note to make is that due to the large number of 3-D elements required for meshing the model, it is not possible to perform sufficient mesh refinement. Therefore, in the most critical area of the model, i.e. the fiber/matrix interface, the sub-modeling capability of Ansys (the commercial FEM software) is used for refining the mesh in this area. In the interface region, nonlinear 8-node quadrilateral elements are used, while in the rest of the model where the changes in the stresses and displacements are nearly uniform 4-node linear elements are incorporated. As for the material modeling, for studying the effects of inelastic behavior of the composite constituents, the SCS6/Ti-24Al-11Nb (IMC) composite is simulated from its processing temperature of 900°C to the room temperature. Here, the onset and progression of plastic deformation is analyzed by applying the Prandtl-Reuss associated flow rule with the von-Mises equivalent stress as the yield criterion. As in [14], the isotropic hardening rule is adopted. The elastic modulus of the fiber is considered to be  $E=413.7$  GPa, the Poisson's ratio  $\nu=0.33$ , and coefficient of thermal expansion  $CTE=4.86 \times 10^{-6}$

( $K^{-1}$ ). The properties of the matrix with temperature are given in [14].

To examine the influence of the damage (or strain energy release) inflicted by excessive inelastic deformation of the matrix material, the properties of SiC/AlMgSi composite which was considered in [16] is applied to the model for region 4 (see Figs. 2 and 3(d)). Since the results including the damage parameters could be found in [16], here the stress and deformation fields for this composite ignoring the energy release due to plastic deformation will be obtained. A comparison of these results will demonstrate the necessity for considering the damage effect at the laminate free edge. The properties of AlMgSi matrix are shown in [16]. Note the properties of SiC fiber, i.e.  $E=450$  Gpa,  $\nu=0.17$ , and  $CTE=4.5 \cdot 10^{-6}$  ( $K^{-1}$ ). Here, the composite is cooled down from  $200^{\circ}C$  to  $20^{\circ}C$ . One more note to make is that the results of this section not only help in better understanding of the decision on applying the damage effects when studying the free edge stresses, but also could dictate consideration of fiber/matrix debonding at the fiber end on the laminate free edge. A detailed explanation and the material parameters for calculating the damage effects are presented in [16].

### 3 Results and Discussion

For better understanding of the stress fields in different regions of the previously mentioned laminates, it is necessary to review the principles of thermal stress generation and also the overlapping hypothesis in advance. This hypothesis explains the physical reasons behind the generation of radial and hoop stresses at the fiber end on the free surface which are normally ignored by the shear-lag theories.

Consider two coaxial cylinders (representing the fiber and the matrix around it) made up of two dissimilar materials with specifically different CTE's where the CTE for the core cylinder to be smaller than that of the outer cylinder and assume the assembly goes through a  $1^{\circ}C$

temperature change, see Fig. 4. If the axisymmetric model of the coaxial cylinder is heated and if the interface of the two plates is detached, due to higher CTE of the outer cylinder the plate that represents the matrix will experience higher longitudinal and circumferential expansions compared to those of the fiber. Now, for a perfect bound between the fiber and matrix, the matrix will come under axial compression while the fiber will experience a tensile axial load when eliminating the axial difference in elongations of the fiber and matrix. The outer cylinder under the axial compression, which is applied to its inner rim, will deform as shown in the figure. In other words, the inner radius at the cylinder end will reduce, while it shows some increase compared to the initial radius when moving toward the mid-length of the cylinder. Now, for a perfect bound case, it is necessary to eliminate the matrix overlapping at the fiber end by pushing back the matrix cylinder and fill the gap between the fiber and matrix along the rest of the length of the matrix cylinder by pulling it toward the fiber. This will cause a large compressive radial stress at the interface right at the end of the cylinder assembly and a tensile radial stress through out the rest of the cylinder length at the interface. Similarly, for the circumferential direction, the matrix will see a compressive hoop stress while it will be tensile for the fiber. Now, for a  $1^{\circ}C$  reduction in temperature (or cooling) the sign of the mentioned stresses will reverse and the large tensile radial stress at the fiber end will cause cracking at the interface on the free surface of unidirectional composites. For more details one can refer to [9].

In the following subsections, the effects of the inelastic behavior of the constituents of the laminate composites on the stress fields around the fiber at the free edge of the laminates in the different regions mentioned before (as shown in Fig. 2) will be explained. Finally, the importance of the damage (or the strain energy release due to excessive plastic deformation of the matrix) in

studying the stress fields will be shown and proven.

#### 4 Inelastic Stress Analysis

Before presenting any discussion about the inelastic results obtained in the current study, it is so important to see how reliable is the chosen model. The model verification here is done by comparing the stress and plastic deformation in the inner zone of the composite, (i.e. in region 4 shown in Fig. 2) with the results presented in [16]. Fig. 5(a) shows the equivalent stress in region 4 obtained in this study using the same material properties as in [16]. However, the damage parameters offered in that reference were not included. As was discussed before, it happens because the current research work is aimed at showing the importance of considering damage when studying the free edge of a laminate. With a quick comparison of the results in Fig. 5(a) with the results obtained in [16], see Fig. 5(b), one can conclude that the stress levels are pretty much the same. However, the plastic deformation obtained in [16] (see Fig. 6(a)) is about 17% higher than what the current study shows, see Fig. 6(b). As expected, the damage effect or the excessive energy release shows up by inflicting some extra inelastic deformation, the thing that through out this study will be shown to repeat if the damage is even considered as fiber/ matrix debonding.

Now with the confidence obtained regarding the modeling, in the following, the stress situation in two-layer and 4-layer symmetric cross-ply laminates in regions 1, 2, 3 and 4 as shown in Fig. 2 will be discussed in turn.

#### 5 Two-Layer Cross-Ply Laminate-Regions 1&2

Since in two and four-layer laminates the models of regions 1 & 2 are similar in configuration, boundary conditions, and loading, here only one of these models will be discussed. In these regions, based on Figs. 2 and 3, one fiber has a free end while the modeled part of the fiber in crossed layer has the conditions of the inner

zone. Here, the stresses and plastic strain around and along the length of the fiber with free end are of interest. Fig. 7(a) shows the plastic strain on the free surface (paths 2 & 3) and the location that the fibers cross each other and have the minimum distance to one another (i.e., path 1). As it is seen, along paths 2 & 3 right at the interface of the fiber and matrix on the free surface large plastic strains occur, and these values decay with moving away from the interface. However, along path 1, as expected, large plastic strains occur at the interface of both fibers, but the values near the fiber with free surface is higher than the values for cross fiber. That is because large thermal stresses occur at the fiber/matrix interface. What is important here is that paths 1 and 3 intersect the interface of the layers of the laminate. The plastic strain and the stress values along this interface are very significant and vital for making any decision on what could be the main reason for causing the delamination defect. However, it is too early for making such conclusions.

The distribution of plastic strain along both fibers is also shown in Fig 7(b). Due to the free surface condition (as was explained before by the overlapping hypothesis) large plastic strains occur at the fiber end but at a small distance away from this location the values sharply decrease, see path 1 in Fig. 7(b). However, this is not the case for path 2 along the cross fiber. The distribution of plastic strain along this path specially where the fibers have the least distance to each other shows some increase in values, but generally these values are small compared to the values along path 1. It seems the change in values of the local fiber volume fraction could be a sufficient reason for explaining the above discussed subject.

Due to the presence of free surface, it is expected that singular stress components happen at the fiber end. In cooling, based on the overlapping hypothesis, the radial stress at the fiber end will be tensile and singular, while away from the fiber end it will become compressive. The singularity

of the stress is shown here with reducing the size of FEM elements at the fiber end which is done using sub-modeling facility of ANSYS the FEM software. Fig. 8(a) shows the radial stress for three different levels of mesh size in the neighborhood of the fiber end. With refining the mesh, the stress values sharply increase. The radial stress distribution around the fiber circumference on the free surface for the three sizes of mesh is also shown in Fig. 8(b). Again, the increase in the stress values with the decrease in mesh size is evident. Also, the sinusoidal distribution of the radial stress around the fiber, which is explainable with the change in local fiber volume fraction at different locations around the fiber, is as expected. Note the maximum value of the stress at location  $\theta=0^\circ$ , where the distance of the fiber to the free surface is the least. This location is the only place around the fiber that does not obey the rule of local fiber volume fraction. In fact, the high value of the stress is against what one would expect considering the value of local fiber volume fraction at this location. This would be due to the neighboring free surface and will be better understood when the results of four-layer cross-ply laminate is discussed.

Similarly, the other stress components like the hoop and axial stresses will be singular which are shown in Figs. 8(c) & 8(d) here. As it is seen, the maximum of the stresses also occur at location  $\theta=0^\circ$  around the fiber. Note also the tensile nature of the hoop stress which may inflict matrix cracking on the free surface. Consequently, considering these stress values, the plastic strain at the fiber end should go to infinity, provided no limiting values like the matrix breakage is set. Fig. 9 shows the changes in plastic strain with the mesh size at the fiber end. As a result, for the time being, it could be justifiable if one considers the large deformation as a damage which happens only as fiber/matrix debonding at the fiber end on the free surface. Other types of damage, like matrix cracking, could also be considered, which for the moment

it is ignored here. The plastic deformation in presence of fiber/matrix debonding at the fiber end along two paths of 1 and 2 are shown in Fig. 10(a). As it is seen, with applying the debonding, the maximum value of the plastic strain moves to the root of the fiber/matrix separation. The dashed lines in the figure show the plastic strain along path 2 before and after debonding. The corresponding radial stress values for the same path are also shown in Fig. 10(b). As was explained before, in presence of damage, the values of plastic deformation show some increase, while the stress levels may decrease or remain the same. Here, due to the presence of the free surface in the neighborhood of path 1 the maximum value of the plastic strain occur at the root of the debonding along path 2, see Fig 10(a). This would be explained more when the same results for the 4-layer laminate is discussed.

## 6 Conclusions

From the results obtained in this study, one may draw some preliminary conclusions as listed below, though it is too early for making any recommendations regarding the delamination phenomenon;

- 1- The FEM model in the inner zone region provides satisfactory results in comparison to the numerical and experimental results published in [16].
- 2- The radial and hoop stress components around the fiber end are tensile and singular in nature that may cause fiber/matrix debonding or matrix cracking on the free surface.
- 3- On the free edge of laminate composites large stress and strain fields appear at the fiber end that depending on the boundary conditions may extend to the interface region of layers causing delamination.
- 4- Considering the fiber/matrix debonding damage at the fiber end on the free edge of the composite alters the stress and strain fields in a way that layers values of

the mentioned quantities appear at the interface of the layers.

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## 8 Figures

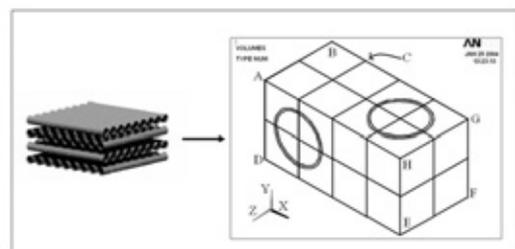


Fig. 1. Micro-mechanical model of Four-layer symmetric cross-ply laminate

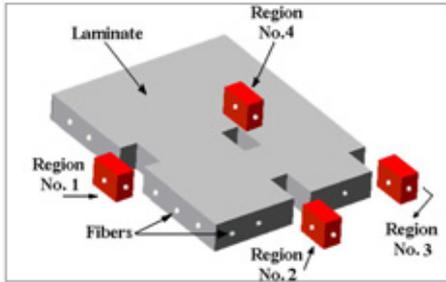


Fig. 2. The schematic of the regions considered in the cross-ply and unidirectional laminates.

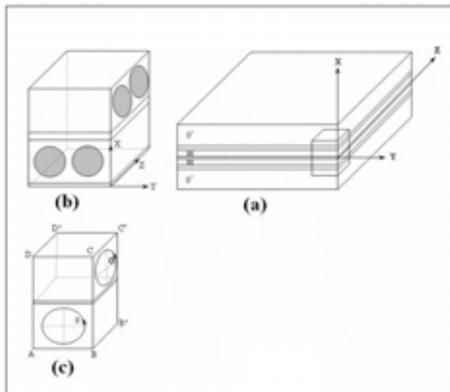


Fig. 3(a, b, c). 3-D modeling of adjacent fiber to the free surface for each layer

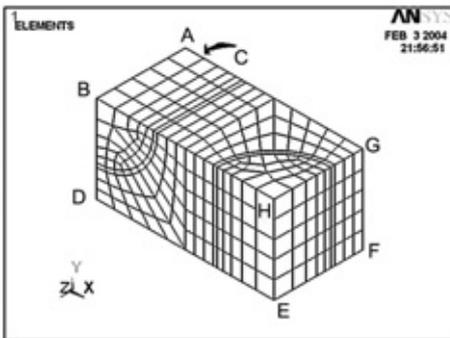


Fig. 3(d). Reduced model due to the existing symmetric conditions

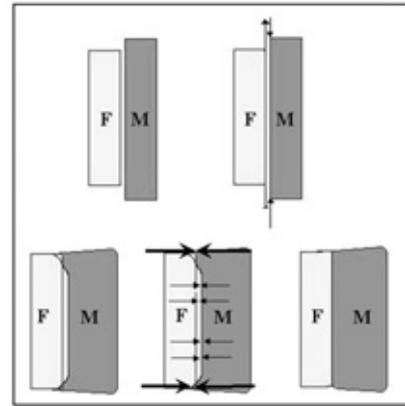


Fig. 4. Overlapping model and radial stress distribution

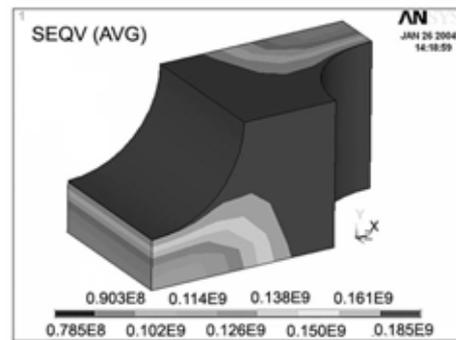


Fig. 5(a). The equivalent stress in region 4 obtained in this study.

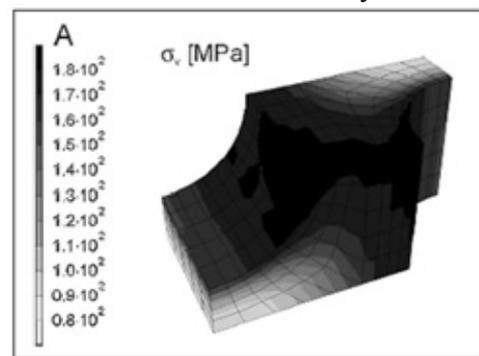


Fig. 5(b). Results in region 4 obtained in [16]

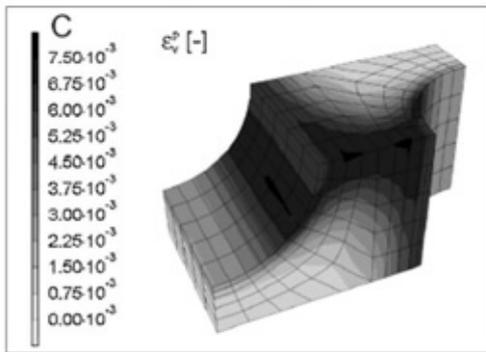


Fig. 6(a). The plastic deformation in region 4 obtained in [16]

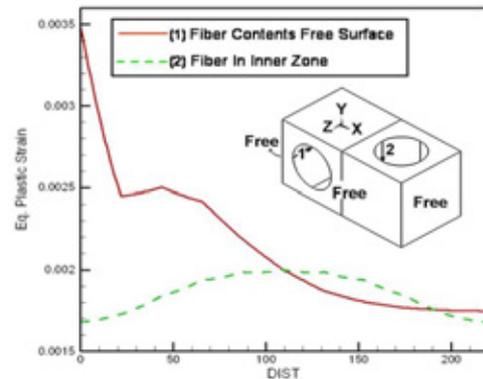


Fig. 7(b). The distribution of plastic strain along both fibers

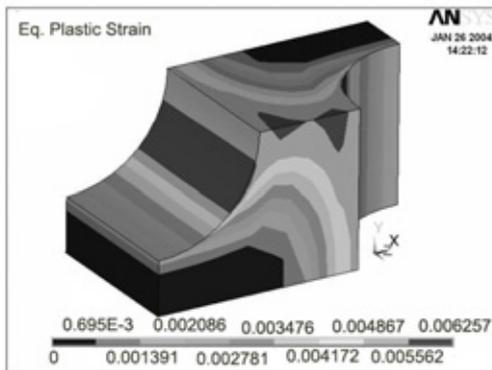


Fig. 6(b). The plastic deformation in region 4 obtained in the current study

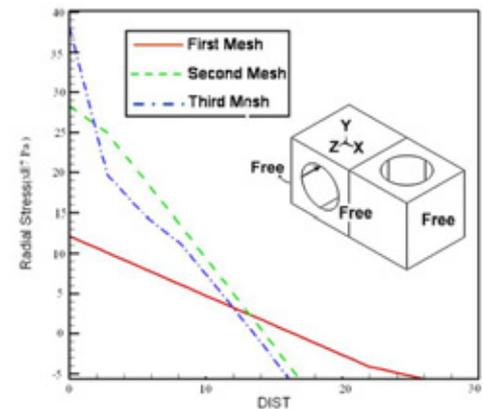


Fig. 8(a). The radial stress for three different levels of mesh size in the neighborhood of the fiber end

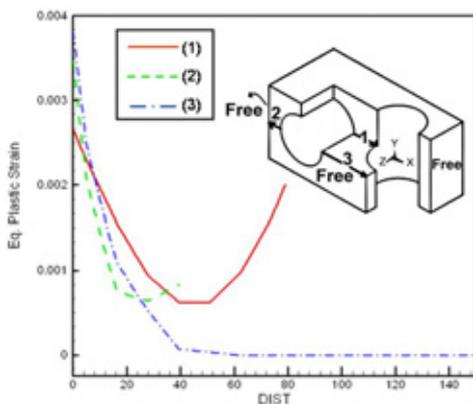


Fig. 7(a). The plastic strain on the free surface (paths 2 & 3) and inside the laminate.

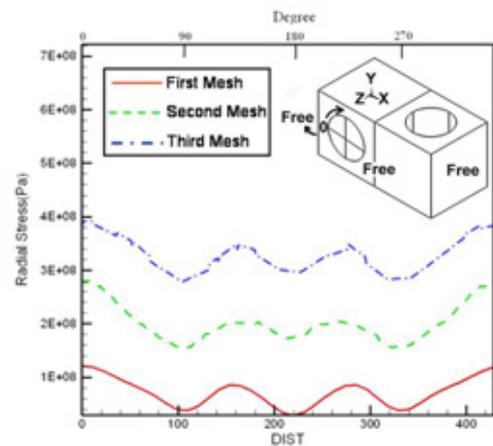


Fig. 8(b). The radial stress distribution around the fiber circumference on the free surface for three mesh sizes.

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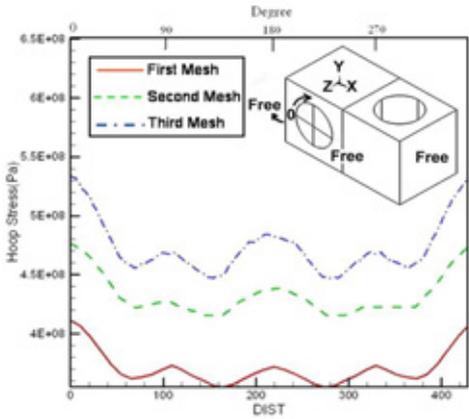


Fig. 8(c). The hoop stresses distribution around the fiber circumference on the free surface for three mesh sizes

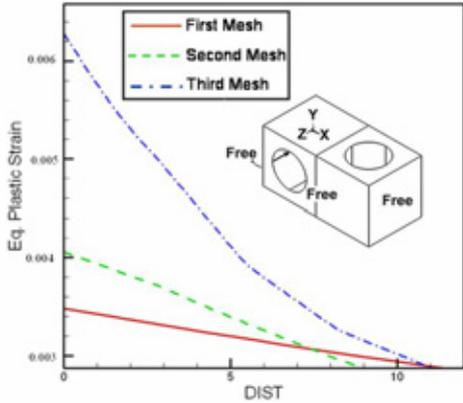


Fig. 9. Changes in plastic strain with the mesh size at the fiber end

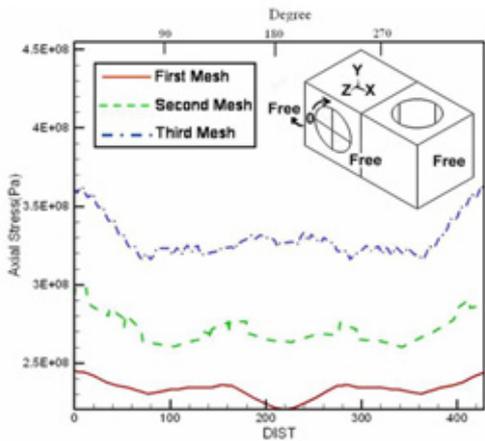


Fig. 8(d). The axial stresses distribution around the fiber circumference on the free surface for three mesh sizes

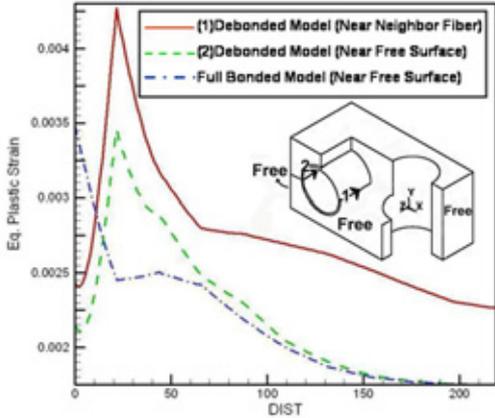


Fig. 10(a). The plastic deformation in presence of fiber/matrix debonding at the fiber end

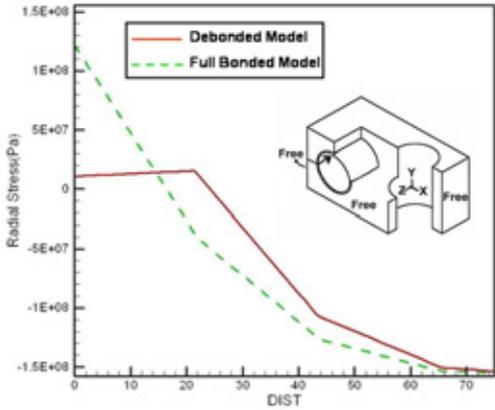


Fig. 10(b). The radial stress values in presence of fiber/matrix debonding at the fiber end