PARAMETER ESTIMATION FLIGHT TEST MANEUVER OPTIMIZATION CONSIDERING MEASUREMENT COLORED RESIDUALS

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Abstract

This work deals with the optimization of flight test maneuvers for aerodynamic parameter estimation considering that the measurements are contaminated with colored residuals. The colored residuals consideration is important to give the direct and realistic assessment of the parameter estimation uncertainty levels prior to flight tests. The optimization technique is based on the concept of flight test data information content and Cramer-Rao lower bound. The discrete autocorrelation matrix of the measurement noise is used in order to compose the optimization criteria considering colored residuals. Some results of a flight test campaign of the CEA-205 CB.9 Curumim aircraft are discussed. The advantages and disadvantages of the proposed maneuvers optimization technique are presented, stressing the easiness of implementation of the signals and the strong improvement in the estimation procedures made possible with the application of the optimized maneuver signals.

1 Introduction

The problem of experiment design for aircraft parameter estimation has been mainly treated since the 70's. Marchand [1] dealt with this problem in the frequency domain by shaping the excitation signals in order to maximize the power spectrum density near the natural frequencies of the dynamic model under investigation. In this case, the signals considered are traditional bang-bang square waves such as the doublet, the 2-1-1 and the 3-2-1-1, and the objective is to maximize the output sensitivities to the parameters of interest by maximizing the dynamic excitation in the frequencies near the system modes, and indirectly reduce the Cramer-Rao lower bound. In the present work, the maneuvers performed with these excitation signals will be called conventional maneuvers.

On the other hand, Stepner and Mehra [2], Mulder and Breeman [3] and Gupta and Hall [4] directly dealt with the minimization of the Cramer-Rao lower bound for experiment design. These authors used some norm of the Information Matrix, M. In some cases the maximization of the trace or of the determinant of M was used. In other cases, the minimization of the trace or of the determinant of the Dispersion Matrix was used. Some authors in this context also considered the utilization of weighting matrices to provide high assessment to most important parameters. The first author to deals with the optimization of the maneuver time was Chen [5]. Afterwards, Morelli [6] also treated the minimization of the time, but considering explicit objectives for the parameter Cramer-Rao lower bound. In this way, there is no necessity to use weighting matrices.

Regarding the constraints imposed to the maneuver design, the previous woks normally considered input variables limitation. This approach provides indirectly assessment to the output variables. Morelli also consider directly output variables constraints application, providing strong assessment to flight safety and mathematical models constraints limitation. The indirectly input constraints imposition through the excitation signal amplitude or energy facilitates and decreases the computational cost of the maneuver design procedure. In this way, however, the procedure depends on a posteriori analysis to verify whether the output variables respect the operational and mathematical constraints. Another concerns is about the excitation signal type. In [2] [4] design its optimized excitation signals as continuous functions, normally sine functions. Other work [6] presented techniques regarding bang-bang square waves excitation signals. This kind of excitation signals provides advantages concerning the implementation by pilot. Besides, the square waves provided richer frequency spectrum than the sine waves.

Regarding the measurement noise modelling, all previous works assumed that the measurements are contaminated with a Gaussian white noise with zero mean. The parameter estimation Cramer-Rao lower bound assessment considering white measurement residuals can be very optimistic. Iliff and Maine [7] proposed a rule-of-thumb index to correct the Cramer-Rao lower bound computed from the conventional Fisher Information Matrix considering Gaussian zero mean white noise. The same authors in [8][**?**][9] concluded that the discrepancies between the Cramer-Rao lower bound computed considering Gaussian zero mean with measurement noise was due to the presence of colored residuals on the flight test data measurements. Afterwards, Morelli and Klein [10] proposed an approach to determine de accuracy of likelihood parameter estimation on the presence of colored measurement residuals.

This approach is firstly used in the present work to design the parameter estimation flight test maneuvers in order to provide reliable assessment to the accuracy of the parameter estimation during the flight test campaign planning. As mentioned before, the Cramer-Rao lower bound computed through the Fisher's Information Matrix do not take into account colored residuals on measurements. Normally, dealing with the aircraft parameter estimation problem with real flight test data considering this approach to assess the uncertainty modelling results in very optimistic analysis. To design experiments in such a way that reliable assessment to the expected estimates variance is provided prior to flight tests is very important in parameter estimation flight test campaigns management, increasing the process efficiency and decreasing the costs. The technique to correct the variance of the estimates proposed by Morelli and Klein is being used in this work as the optimization criterion to compose the cost function of the parameter estimation flight test maneuver design algorithms taking the tremendous advantages of getting realistic assessment to the uncertainties levels of the estimates during the flight test campaign planning.

2 Problem Formulation

Prior to deal with the experiment design problem and Cramer-Rao Inequality, it is important to establish some concepts about the maximum likelihood estimator, MLE. The MLE has been widely applied to aircraft parameter estimation [11][12][13]. The dynamic models which describe aircraft dynamics for parameter estimation can be defined by

$$\dot{x}(t) = f(x(t), u(t), \theta) \tag{1}$$

with $x(0) = 0$ and x is the state vector. In addition u is the control input vector.

The parameter estimations are computed from the output variables. The system parameters are connected to the output variables through the observation equations, given by

$$y(t) = h(x(t), u(t), \theta) \tag{2}$$

where y is the observation vector.

Besides, it is necessary to consider the measuring noise to compose the measuring variables vector y_m, defined by

$$y_m(i) = y(i) + \upsilon(i) \tag{3}$$

where $i = 1, 2, 3, \ldots, N$ and N is the measuring vector dimension. $\upsilon(i)$ is the measuring noise. Traditionally, the MLE formulation assumes that the measurement noise is a Gaussian zero mean withe noise, with covariance matrix defined by R.

Under suitable assumptions, the Maximum Likelihood Estimates are defined as the parameter vector θ that maximizes the probability density function of the occurrence of y_m, $p(y_m|\theta)$. Considering that $p(y_m|\theta)$ exists and that the maximum likelihood estimator is asymptotically unbiased, the Cramer-Rao inequality defines the lower bounds for the parameter estimation variances as the inverse of the Fisher's Information Matrix [14],

$$\operatorname{cov}\left(\tilde{\theta}\right) \geq M(\theta)^{-1} \tag{4}$$

where M is the Information Matrix and $\tilde{\theta}$ is the parameter estimates error. The inequality in equation (4) is true just for asymptotically unbiased estimators, such as maximum likelihood. In addition, by definition for the existence of the equality in (4) the estimator must be efficient. It is possible o demonstrate that the maximum likelihood is a efficient estimator for large number of data points. It is important to point out that for the system described in equation (1) to (3), and taking into account that y_m is not a function of θ, the Fisher's Information Matrix can be computed by [6]:

$$M = \left[\sum_{i=1}^{N} \frac{\partial y(i)}{\partial \theta}^{*} R^{-1} \frac{\partial y(i)}{\partial \theta}\right] \tag{5}$$

2.1 Parameter Estimation Variance with Measurement Colored Residuals

The assessment to the uncertainties levels of likelihood parameter estimations, traditionally, is obtained through the Fisher's Information Matrix as in equation (5) and applying the Cramer-Rao inequality. In previous works [7], as quoted above, the authors described some discomfort with this approach when dealing with real flight test data because of it is normally too optimistic.

Basically, the information theory does not deal with colored residuals and the Cramer-Rao lower bound computed through the Information Matrix become too optimistic when treating real flight test data, which normally contains colored residuals on measurements. The technique developed by Morelli and Klein [10] proposes the introduction of a term that consider the temporal correlation of the measurements residuals to account for colored residuals, by defining the variances as

$$\operatorname{cov}\left(\tilde{\theta}\right) = D\left[\sum_{i=1}^{N}\sum_{j=1}^{N} S(i)^{T} R^{-1} E\left\{\upsilon(i)\upsilon(j)^{T}\right\} R^{-1} S(j)\right] D \tag{6}$$

The equation (6) is the function proposed by Moreli and Klein to correct the uncertainties levels of parameter estimation from flight test data. This technique is being firstly used in this work as the optimization criterion to compose the cost function of the parameter estimation flight test maneuver design algorithms. Tremendous advantages are taken, getting realistic assessment to the uncertainties levels of the estimates during the flight test campaign planning. Therefore, the cost function to design optimized maneuvers is given by

$$J = \operatorname{tr}\left[D\left[\sum_{i=1}^{N} S(i)^{T} R^{-1} \sum_{j=1}^{N} R_{vv}(i-j) R^{-1} S(j)\right] D\right] \tag{7}$$

In equation (7), the autocorrelation matrix R_{vv} is obtained by previous measuring system noise modelling, what is suitable to be consider. The present work is not focused on the optimization algorithm used to minimize (7). The algorithm used is a Genetic Optimization Algorithm and more details about it should be taken in [15].

3 Experimental Setup

A dedicated flight test campaign for excitation signal design analysis was performed. The CEA 205 CB.9 Curumim aircraft dynamics under analysis was the longitudinal short period. Some maneuvers specified through the spectral analysis and optimized maneuvers specified through the techniques presented in this work were flown. The main objective was to compare the approaches and make some conclusions about the advantages of the optimized maneuvers considering colored residuals.

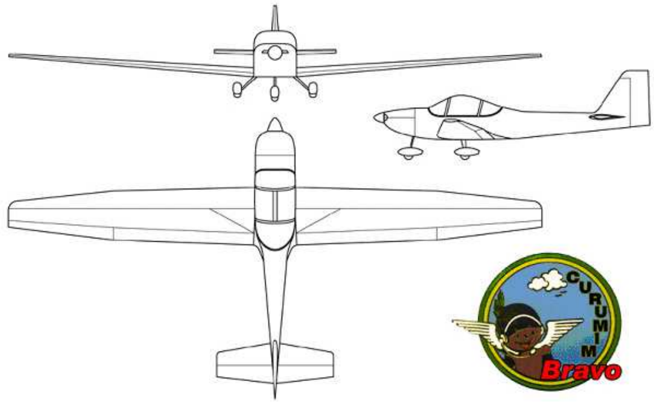
Firstly, during a pre-campaign some data compatibility check flights were performed in order to calibrate the flight test data acquisition system and to model the measuring residuals. Secondly, after the specification of the conventional and optimized maneuvers considering the a priori knowledge about the CEA 205 CB.9 Curumim aircraft dynamics, the flights for signal excitation valuation were performed.

For statistical comparison purposes was planned that for each maneuver under analysis thirty flight test runs should be performed, all in the same flight conditions, namely

- **Pressure Altitude,** h**:** 1371 m;
- **True Airspeed,** V_{tas}**:** 31.3 m/s.

From the thirty runs for each maneuvers, the best fifteen were chosen for statistical analysis. The criterion for choosing the best runs was established in order to discard the samples which provide the fastest estimates from the medium value for each parameter.

The excitation signals were manually applied by the pilot. For each maneuver under analysis was performed one flight. For all flights the hazardous analysis was elaborated considering the emergency and the minimizing procedures, including the output constraints for the maneuvers optimization algorithms. All flight test maneuvers were specified in order to maintain a load factor between 1.6 and 0.4 G's.



CEA-205 CB.9 Curumim Characteristics		
Wing area	16.40	m^2
Wing span	14.00	m
Length	7.40	m
Mean aerodynamic chord	1.22	m
Aspect ratio	11.95	-
Weight *	560.00	kgf
Load factor limits (555 kgf)	+2.9/-1.5	g
Stall speed (clean, 555 kgf)	120.00	km/h
Maneuver speed	77.00	km/h
VNE	160.00	km/h

* (2 occupants, 70 lts fuel and flight test equipment)

Fig. 1 Three view of the CEA-205 CB.9 Curumim

3.1 Flight Testing Aircraft

The flight testing aircraft was the CEA 205 CB.9 Curumim. This is a light bi-place aircraft developed and built by Centro de Estudos Aeronáuticos of the Universidade Federal de Minas Gerais. The tree view of the Curumim and some characteristics of the aircraft are shown in figure 1.

The longitudinal short period a priori model considered during the maneuver design procedure was:

$$
\begin{aligned}
\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} &= \begin{bmatrix} Z_\alpha & 1+Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta e} \\ M_{\delta e} \end{bmatrix} \delta_e \\
\begin{bmatrix} \alpha_m \\ q_m \\ az_m \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ Z_\alpha \frac{31.3}{9.8} & Z_q \frac{31.3}{9.8} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ Z_{\delta e} \frac{31.3}{9.8} \end{bmatrix} \delta_e
\end{aligned} \qquad (8)
$$

where:

$$\begin{aligned} Z_\alpha &= -1.768; Z_q = 0.080 \\ M_\alpha &= -7.394; M_q = -1.934 \\ Z_{\delta e} &= -0.160; M_{\delta e} = -8.360 \end{aligned} \tag{9}$$

3.2 Conventional Maneuvers Specified by PSD Analysis

Some conventional maneuvers were analyzed to provide comparison basis with the optimization techniques developed in this work. The excitation signals analyzed were the *Doublet*, the 2-1-1 and the 3-2-1-1. The shaping os these signals was done by its power spectrum density analysis considering that the natural frequency of the mode of interest was 3.31 rad/s [15].

As cited above, thirty flight test runs were performed for each maneuvers under analysis. The Curumim aircraft short period dynamic response for the angle-of-attack, pitching angular rate and vertical acceleration for the best fifteen runs are presented in figures 4 to 6. A Maximum Likelihood Output-Error estimation procedure was performed for each flight test run. The parameter estimation results for conventional maneuvers are shown in figures 8 to 13 via statistical distribution computations.

3.3 Optimized Maneuvers

To evaluate the techniques proposed in this work some optimized flight test maneuvers considering the optimization criterion described by cost function in equation (7) and the colored noise medelling were designed. As well as for the conventional maneuvers design, the main constraint imposed was in the vertical acceleration variation:± 0.6 G. The elevator input excursion was also constrained between ± 10 degrees. Concerning the adequate excitation of the short period longitudinal mode the maximum time for each maneuver was established to be 15 seconds. In addition, for implementation by the pilot the minimum period between the commutation times was established to be 0.5 seconds.

In the present work one of the optimized maneuvers was chosen for detailed analysis. The excitation signal of this maneuver is shown in figure 3. In figure 7, the data for the fifteen best runs of the optimized maneuver is described. The aircraft response in angle-of-attack, pitch rate and vertical acceleration can be observed.

For comparison purposes the Cramer-Rao lower bound and the power spectral density of the optimized and conventional maneuvers excitation signals are described in table 1 and in figure 2, respectively.

Table 1 Relative Cramer-Rao lower bound prior to flight test - (%)

	Z_α	Z_q	$Z_{\delta e}$	M_α	M_q	$M_{\delta e}$
Optimized	8.0	62.4	71.4	5.1	7.3	5.0
3-2-1-1	12.1	91.2	110.1	9.5	11.1	7.4
2-1-1	16.3	101.6	95.8	11.7	11.3	10.5
Doublet	14.0	89.3	79.5	9.5	12.4	9.8

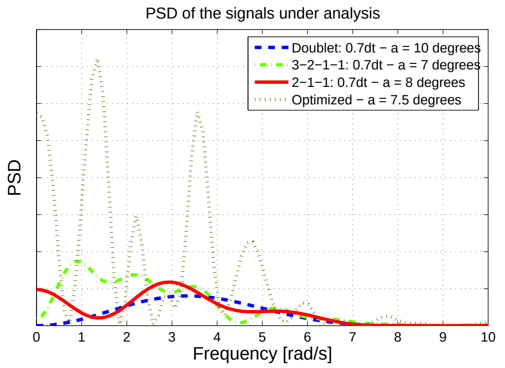


Fig. 2 Comparison between power spectral density of the signals under analysis

4 Results Analysis

In table 1 can be observed that the optimized signal reduce the estimates variance for all parameters of the model under considering. Observing the power spectral densities in figure 2 it is observed that the optimized signal provide an increase in the excitation energy probably due to its longer time of excitation. It is observed that the optimized signal offers large power spectral densities in other frequencies than 3.31 rad/s, resulting in excitation of other unknown modes of

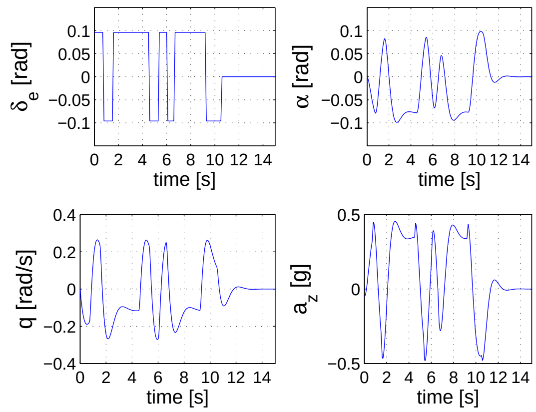


Fig. 3 Planning simulation for the optimized maneuver

the system may resulting in better parameter estimates accuracy.

The figures 8 to 13 show the dispersions and the probability density function for the six parameter estimates for the best fifteen runs of each maneuver under analysis. The observed tendency is that the optimized maneuver decrease the variance of the estimates for all parameters, increasing its reliability. These results are those expected when analyzing the variance of the estimates and the power spectral density of the excitation signals. In addition, concerning that the natural frequency of the system nodes is a function just of the system matrix, the signals specified as a function of its power spectral density do not take into account parameters outside this matrix.

5 Conclusions

A new technique for flight test maneuver optimization for parameter estimation with colored residuals was proposed in this work. Extensive experimental data analysis was done, including a dedicated flight test campaign which provideed statistical basis for conclusions and validation of the technique.

This work seems to be the first initiative to aircraft parameter estimation experiment design regarding colored residuals on measurements to reliably access the parameter estimates variance prior to flight testing. The following points about the technique presented in this work should be highlighted:

1. it provides the implementation of practical constraints on input and output variables resulting in a efficient way to specify optimized flight test maneuvers that respect the operational envelope of the aircraft.

2. it provides conditions to develop excitation signals for parameter estimation flight test maneuvers that are passive for practical implementation by the pilot.

3. it provides the reliable assessment to the parameter estimates variance and to the adequate evaluation of the resultant excitation through the colored residuals consideration.

Therefore, the main contribution of this work is the practical formulation, realistic and directly applicable to the parameter estimation flight test campaign planning and execution. It became evident its viability, necessity and advantages through the decrease in the parameter estimates uncertainties and its capacity to support flight test campaigns safety assessment and cost reduction.

For further improvements, the implementation of parameter estimation maneuver design for non-linear, unstable and closed loop dynamics should be considered. In addition, the implementation of this technique in-flight and in real time with direct interfaces with the parameter estimation algorithm is desired to provide highly accurate parameter estimates.

References

[1] Marchand, M. *Untersuchung der Bestimmbarkeit der Flugmechanischen Derivative des CCV-Versuchsträgers F-104 G*. Braunschweig:DFVLR, 1977

[2] Stepner, D.E. and Mehra, R.K.. *Maximum Likelihood Identification and Optimal Input Design for Identifying Aircraft Stability and Control Derivatives*, NASA CR 2200. Palo Alto, Calif., USA, 1973.

[3] Mulder, J.A. and Breeman, J.H.. *Identification of Dynamic Systems - Applications to Aircraft Part 2: Nonlinear Analysis and Manoeuvre Design*, AGARD AG-300. 1994.

[4] Gupta, N.K. and Hall W.E. Jr.. *Input Design for Identification of Aircraft Stability and Control Derivatives*, NASA CR 2493. Palo Alto, Calif., USA, 1975.

[5] Chen, R.T.N.. *Input Design for Aircraft Parameter Identification: Using Time-Optimal Control Formulation*, AGARD-CP-172. 1975.

[6] Morelli, E.A.. *Practical Input Optimization for Aircraft Parameter Estimation Experiments*, NASA RP 1138. Hampton, Virginia: The George Washington Univerity, 1990.

[7] Iliff, K.W. and Maine, R.E.. *Practical Aspects of Using a Maximum Likelihood Estimation Metohd to Extract Stability and Control Derivatives from Flight Data*, NASA TND 8209. Edwards, Calif., USA, 1976.

[8] Iliff, K.W. and Maine, R.E.. *Further Observations on Maximum Likelihood Estimates of Stability and Control Characteristics Obtained from Flight Data*,Atmospheric Flight Mechanics Conference,100-112, AIAA paper 77-1133. Hollywood, Fla., 1977.

[9] Maine, R.E. and Iliff, K.W.. *The Theory and Practice of Estimating the Accuracy of Dynamic Flight-Determined Coefficients*, NASA RP 1077. Edwards, Calif., USA, 1981.

[10] Morelli, E.A.. *Determining the Accuracy of Maximum Likelihood Parameter Estimates With Colored Residuals*, NASA CR 194893. Hampton, Virginia, USA, 1994.

[11] Maine, R.E. and Iliff, K.W.. *Application of Parameter Estimation to Aircraft Stability and Control: The Output-Error Approach*, NASA RP 1168.Edwards, Calif., USA, 1986.

[12] Jéther, L.H.V.. *Identificação Paramétrica de Derivadas de Estabilidade e Controle Longotudinais da Aeronave Xavante AT-26: uma aplicação do método da verossimilhança*. São José dos Campos - SP - Brasil:ITA, 2002.

[13] Jategaonkar, R. and Plaetschke, E.. *Maximum Likelihood Parameter Estimation from Flight Test Data for General Non-Linear Systems*. Braunschweig:DLR, 1983.

[14] Goodwin, G.C. and Payne, R.L.. *Dynamic System Identification: Experiment Design and Data Analysis*. New York:Academic Press Inc., 1977.

[15] Salis Brasil, N.N.. *Desenvolvimento e Otimização de Manobras de Ensaios em Vôo Para Estimação de Derivadas de Estabilidade e Controle de Aeronaves*. São José dos Campos - SP - Brasil:ITA, 2005.

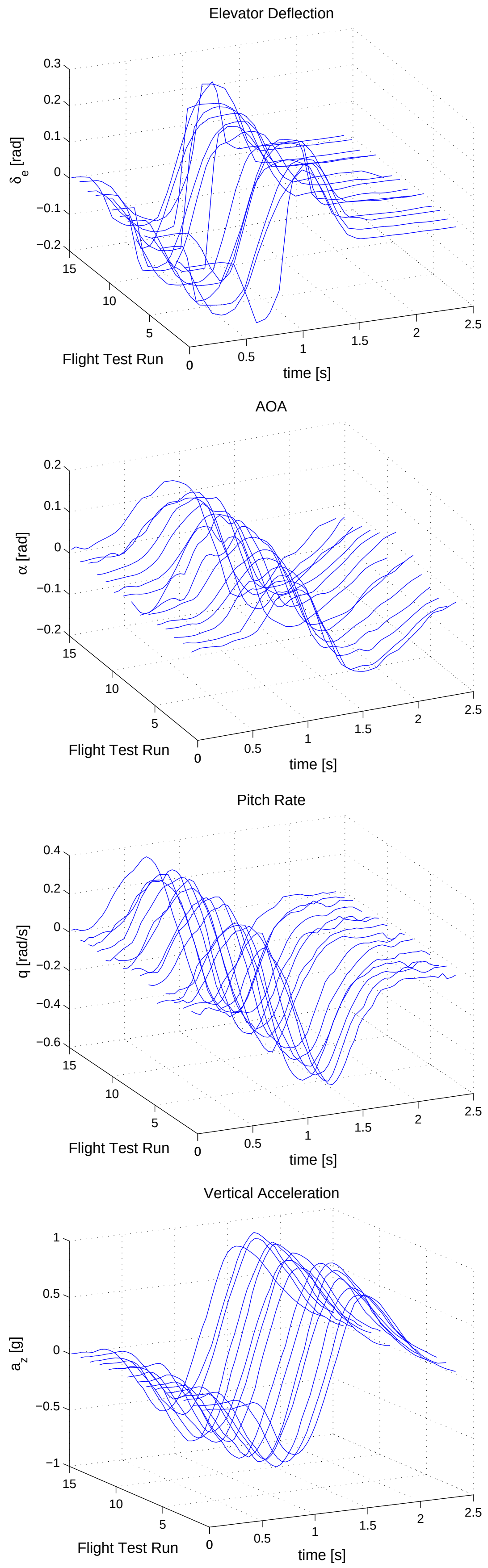
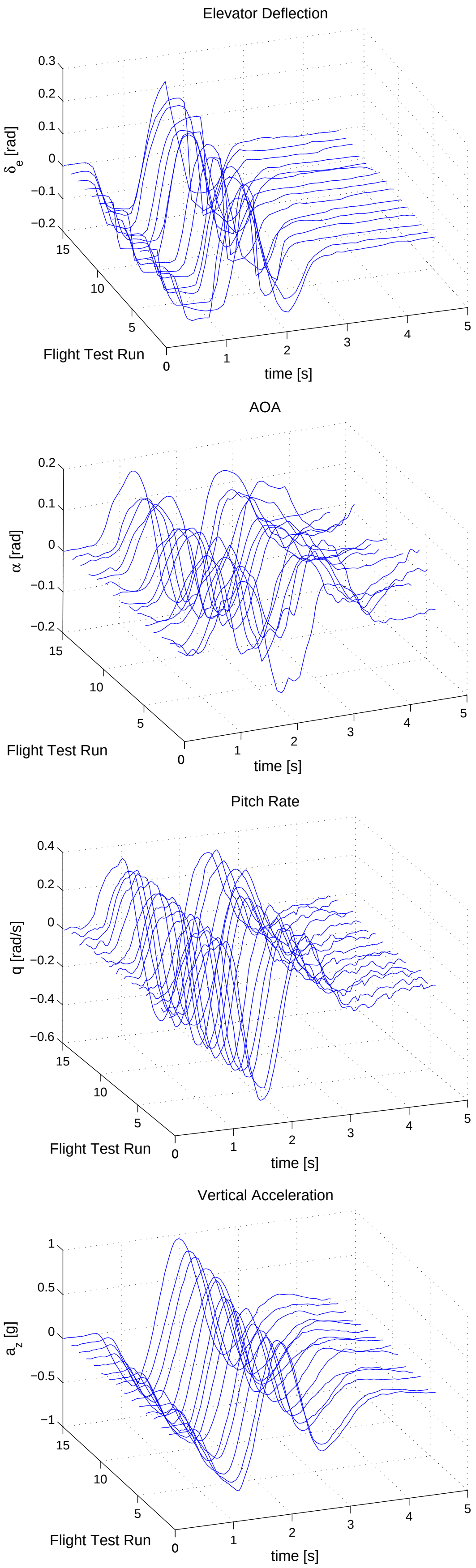


Fig. 4 CEA-208 CB. 9 Curumim dynamic response for the fifteen best runs for the *Doublet* maneuver



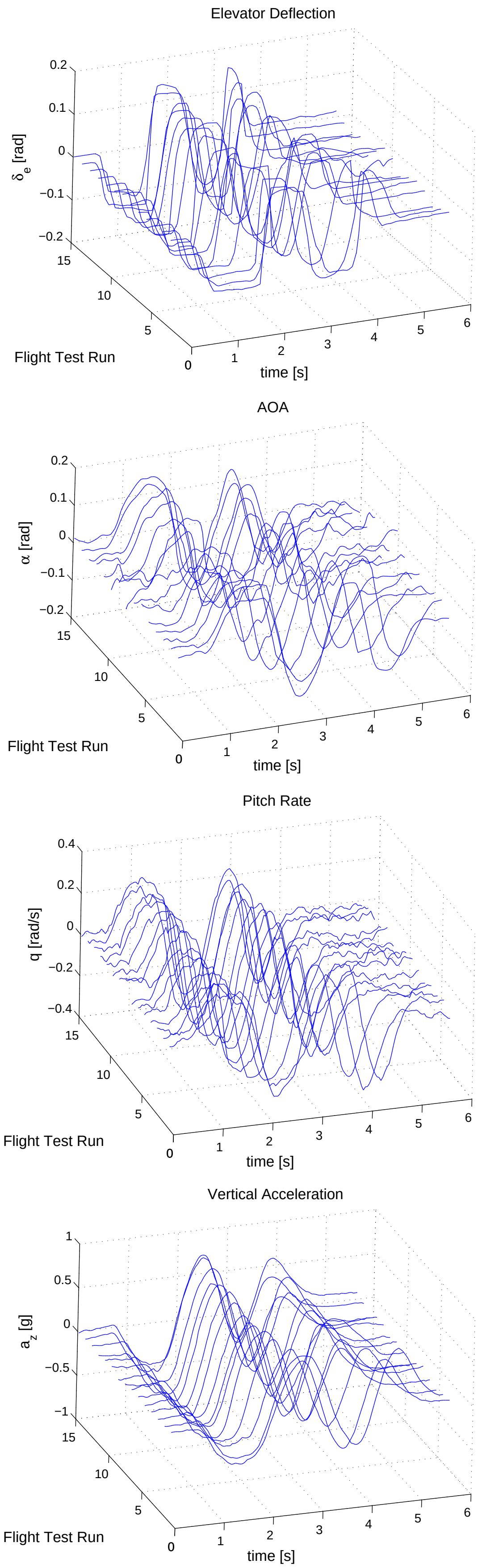


Fig. 5 CEA-208 CB. 9 Curumim dynamic response for the fifteen best runs for the 2-1-1 maneuver

Fig. 6 CEA-208 CB. 9 Curumim dynamic response for the fifteen best runs for the 3-2-1-1 maneuver

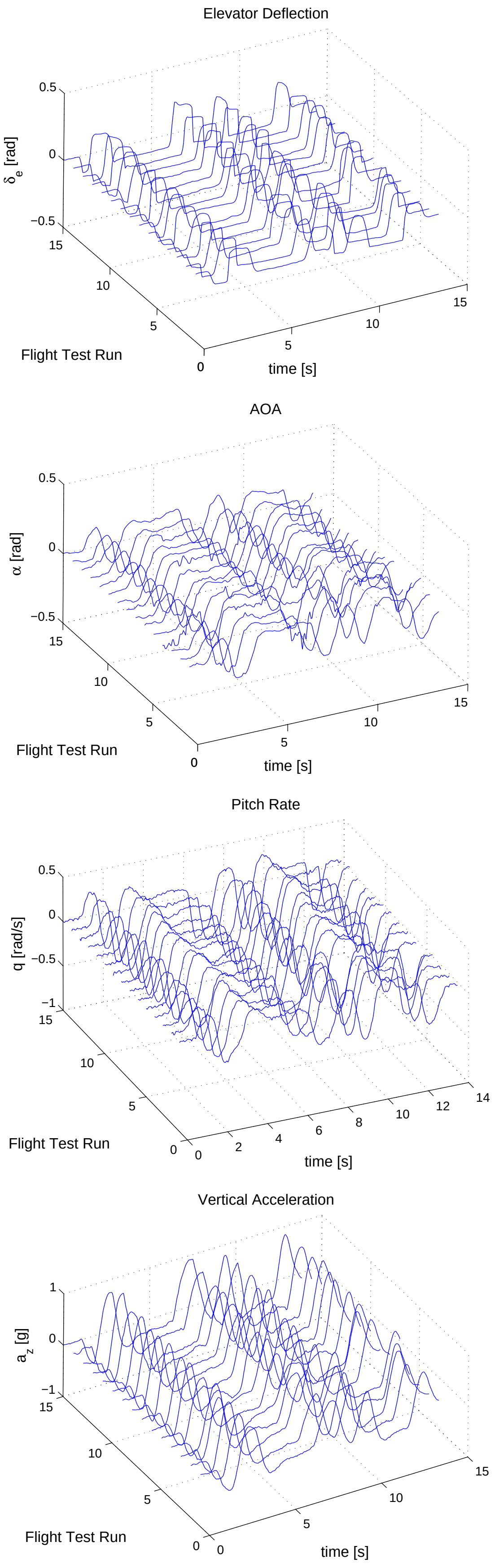


Fig. 7 CEA-208 CB. 9 Curumim dynamic response for the fifteen best runs for the optimized maneuver

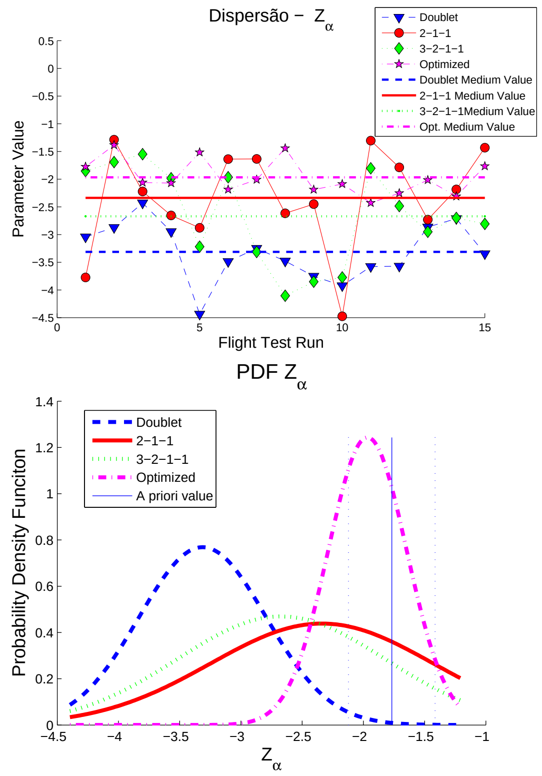


Fig. 8 Maneuver Comparison - Estimation Results for Z_α

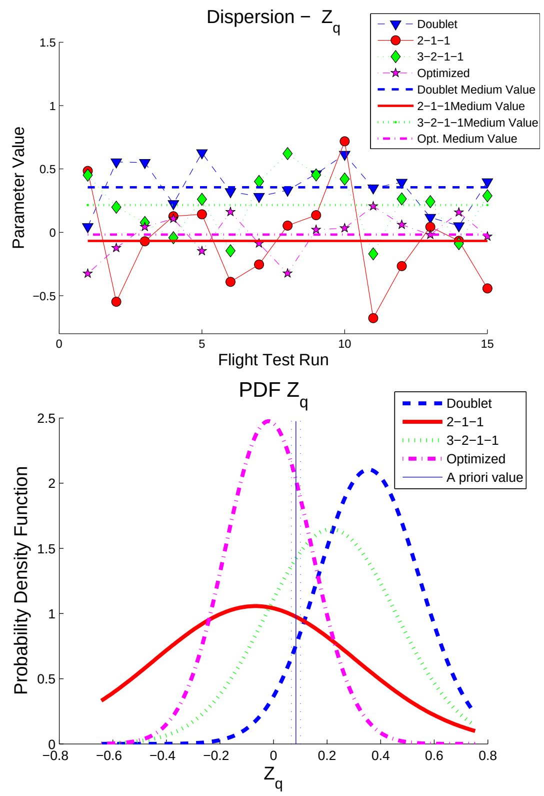


Fig. 9 Maneuver Comparison - Estimation Results for Z_q

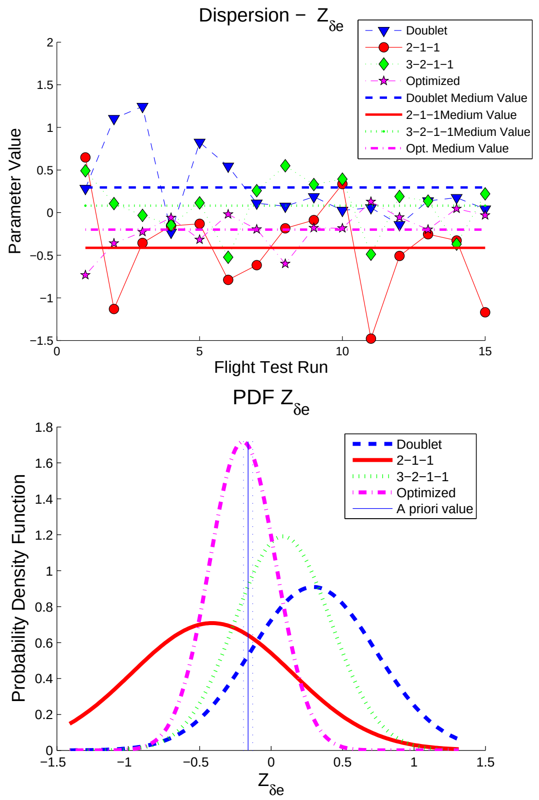


Fig. 10 Maneuver Comparison - Estimation Results for $Z_{\delta e}$

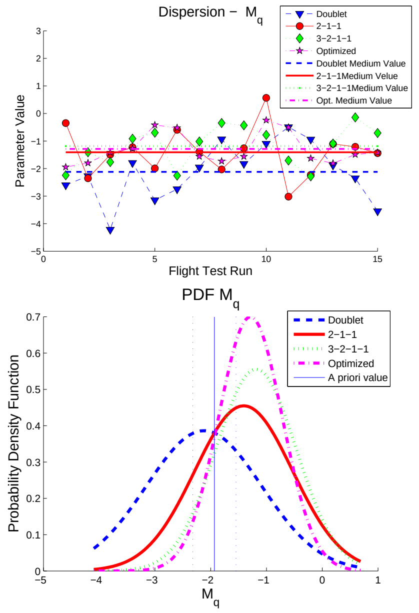


Fig. 12 Maneuver Comparison - Estimation Results for M_q

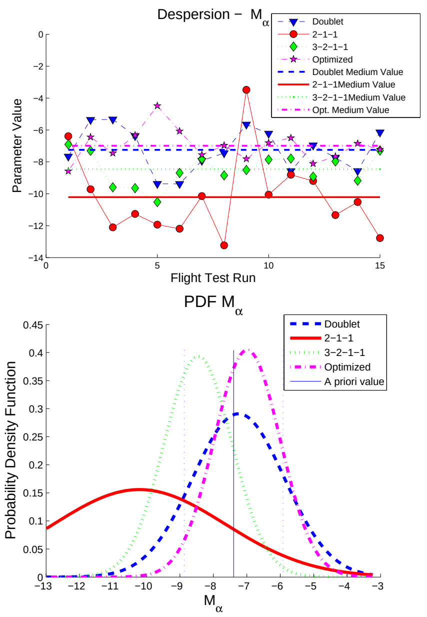


Fig. 11 Maneuver Comparison - Estimation Results for M_α

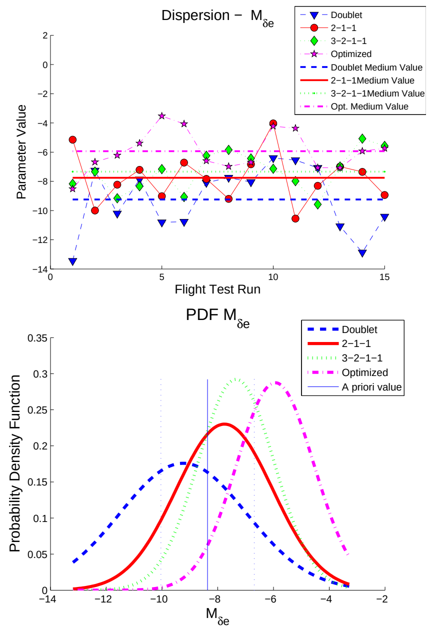


Fig. 13 Maneuver Comparison - Estimation Results for $M_{\delta e}$