NUMERICAL MODEL OF SINGLE MAIN ROTOR HELICOPTER DYNAMICS

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Abstract

Helicopters have six degrees of freedom, so for the simplification it is assumed that motion can be separated into longitudinal and lateral motion and that they can be investigated independently. It is noted that the mathematical model of the helicopter is regarded to helicopter forward motion at velocity W. A mathematical model that would incorporate all helicopter motion, all together with takeoff and landing, would be far complicated. Influence of resonance and vibration is also ignored. The blade through back is also ignored in this paper, because otherwise the blade angle velocity in the plane of rotation would no longer be constant. Separate study of individual motions of blades is a great simplification, because there is an interdependency of all blade motions. If the motions are not separated, then it is necessary to analyze the stability of all the motions of the blade.

The choice of the coordinate origin in the center of inertia enables elimination of certain moments of inertia so the Euler equations can be simplified. Viewing the rotor as a whole eliminates the need for investigating individual blade motion. A great assistance to this is introducing the rotor disc axis and the control axis.

Determination of aerodynamic derivatives is related to series of approximations. It should be noted that, besides assumptions in modeling, also mathematical simplifications were made (for example, omittance of higher order small values in equations) which couldn't have been derived in the form of an assumption due to their meaning which is tightly related to a specific equation.

It is possible to determine projections of position vector with respect to the Earth bound coordinate system instead of using projections of helicopter velocity with respect to the moveable coordinate system as the output characteristics. Projecting the helicopter velocity onto an Earth bound coordinate system and then integrating velocity projections by time with initial conditions may solve this problem.

A further analysis of the mathematical model can be made in order to investigate the dynamic and static properties, and to determine control that would guaranty the object execution of the required dynamic behavior.

This model has been used for the calculations of the Mi-8 helicopter with the new composite main rotor blades (designed at Belgrade Faculty of Mechanical Engineering), also with the new airfoils optimized for high subsonic speeds at the blade tips.

1. Motion of supporting rotor blades

To understand the flight dynamics of helicopters and determine dynamic moments and forces that act upon the helicopter, it is necessary to preinvestigate the motion of supporting rotor blades. From a vast number of different types of helicopters, the single rotor helicopter has been chosen, with blades coupled with the main rotor by hinges about which they can freely move. It should be noted that there are also rotors that have blades with rigid connection to the hub.

1.1 Equations of blade fluttering

Rotor blades are regarded as rigid bodies. The horizontal hinges are placed at length eR from the rotation axes. The shaft rotates at angular velocity Ω=const, and the blade flutters at angular speed dβ/dt. The blade axis is parallel to it's axis of inertia and passes through the hinge (Fig. 1).

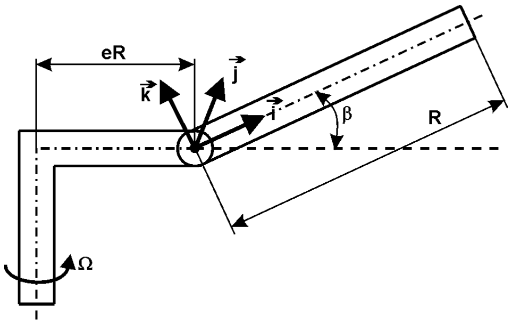

Fig. 1. Blade fluttering

In Figure 1, R represents the length of the blade and β is the flutter angle of the blade. After a rather complex calculus, the equations for blade fluttering are obtained in the form:

$$\ddot{\beta}+\Omega^2(1+\varepsilon)\beta=M_{Ay}/J_y \qquad (1.1)$$

$$J_x\,\dot{\beta}\,\Omega\cos\beta+J_x(-\dot{\beta})\Omega\sin\beta=0 \qquad (1.2)$$

$$-2J_y\,\Omega\,\dot{\beta}\sin\beta=M_z \qquad (1.3)$$

1.2 Equations of blade throwback

It is assumed that β=0 and that the blade is moving forward in relation to the vertical hinge by the throwback angle amount ξ. The vertical hinge is placed at distance eR from the shaft axis. The coordinate system is placed as in the previous case. Figure 2 presents a simplified scheme for determining the blade throwback.

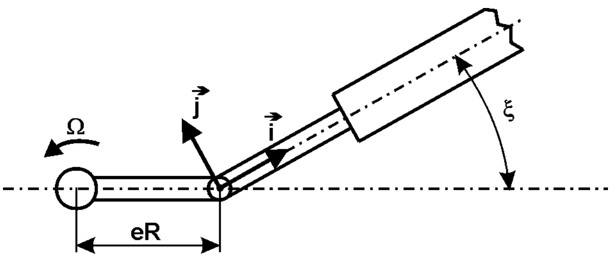

Fig. 2. Blade throwback

From this follows the equation for blade throw-back:

$$\ddot{\xi}+\Omega^2\varepsilon\,\xi-2\Omega\,\beta\,\dot{\beta}=M_z/J_z \qquad (1.4)$$

If the azimuth angle is described as ψ=Ωt, then:

$$\frac{d^2\xi}{d\psi^2}+\varepsilon\,\xi-2\beta\frac{d\beta}{d\psi}=\frac{M_z}{J_z\Omega^2} \qquad (1.5)$$

1.3 Equations of blade climb

It is assumed that flutter and throwback angles are equal zero. The blade pitch is the angle between the blade cross section chord and the plane of the hub, designated as θ_k. Figure 3 shows the coordinate system attached to the blade.

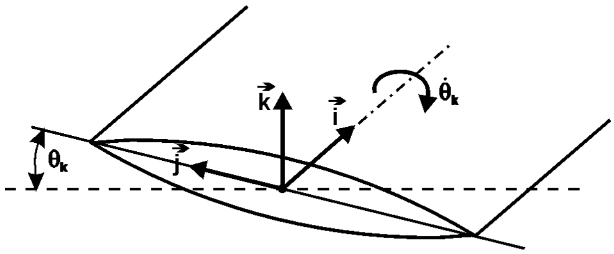

Fig. 3. Coordinate system linked to blade cross section

Equations of blade motion about longitudinal axis are:

$$\ddot{\theta}_k+\Omega^2\,\theta_k=M_x/J_x \qquad (1.6)$$

$$-2J_z\,\Omega\,\dot{\theta}_k\sin\theta_k=M_z \qquad (1.7)$$

$$J_y\,\Omega\,\dot{\theta}_k\cos\theta_k-J_y\,\dot{\theta}_k\cos\theta_k=0 \qquad (1.8)$$

2. Rotor forces

To resolve total forces into components, the following mayor axes can be used:

- The control axis,
- The rotor disc axis, which is normal to the rotor plane, i.e. the plane defined by blade tips trajectory, and
- The shaft axis.

Once the mayor axis is chosen, the remaining axis of the coordinate system will be normal to it and pointed lateral and towards the tail of the helicopter. Customarily, the force component along the chosen axis is called the **tow force**, the force component pointed towards the tail is the **H force**, and the force component pointed lateral called the **Y force**. If the force components are designated without subscripts, it is assumed they are determined relative to control axis, whereas subscripts "D" and "S" are used when relating to rotor axis and the shaft axis.

Since flutter and mount angles are usually small (amounts greater than 10° are considered extreme), the relation between these components can be obtained as:

$$T \approx T_D \approx T_S$$
$$H \approx H_D + T_D\, a_1 \cong H_S + T_S\, B_1$$

2.1 Longitudinal equilibrium of forces

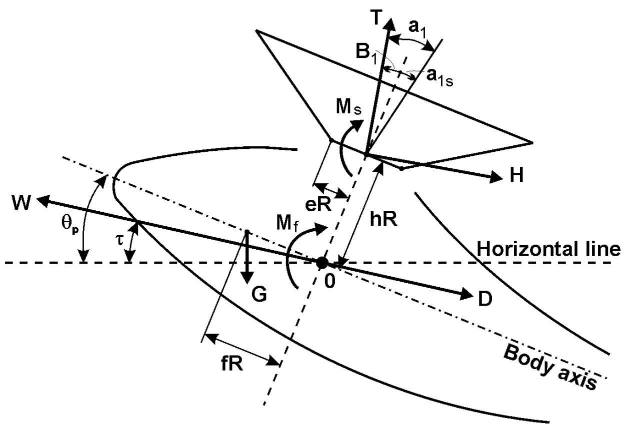

Fig. 4. Drawing for determining longitudinal equilibrium of forces

Angle B_1 is the longitudinal amplitude of a cyclic change of the blade pitch; angle a_{1s} is the angle between shaft and axis of rotor disc. After extensive calculus the expression for longitudinal amplitude of cyclic change in blade step is obtained as:

$$B_1 = \frac{M_f - G \cdot fR + H \cdot hR + M_S \cdot a_1}{T \cdot hR + M_S} \tag{2.1}$$

For e=0, it can be adopted that M_s=0 and M_f=0, and since T=G, follows:

$$B_1 = -\frac{f}{h} + \frac{H}{G} \tag{2.2}$$

$$\theta_p = -\frac{D}{G}\cos\tau - \frac{f}{h} + \frac{M_f}{G \cdot hR} \tag{2.3}$$

Equation (2.2) has a simple physical interpretation: the amplitude of the longitudinal cyclic control must have such a value in order to position the direction of the resultant rotor force through the center of mass.

2.2 Lateral equilibrium of forces

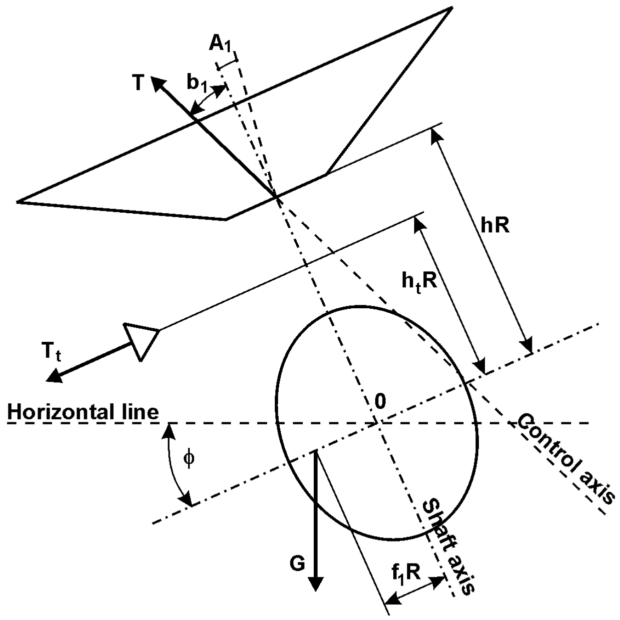

Fig. 5. Drawing for determining lateral equilibrium of forces

Angle A_1 presents the amplitude of lateral cyclic change in blade step of the supporting helicopter rotor:

$$A_1 = -\frac{G \cdot f_t R + M_S \cdot b_1 + T_t \cdot h_t R}{G \cdot hR + M_S} \tag{2.4}$$

By replacing value A_1 into corresponding equations, we obtain the value of angle ϕ, which determines the position of the fuselage.

$$\phi = -\frac{T_t}{G} + \frac{G \cdot f_t R + M_S \cdot b_1 + T_t \cdot h_t R}{G \cdot hR + M_S} \tag{2.5}$$

If $M_S = 0$ and $h_t = h$, which can often be assumed, then:

$$\phi \approx \frac{f_1}{h},$$

which means the rotor hub is positioned vertically above the center of mass. All values of these determined angles are the so called *trimmed values*.

3. Non-linear mathematical model of flight dynamics

Mathematical modeling of helicopter motion is a very complex task and, therefore, it is necessary to introduce series of assumptions and approximations. Knowledge of motion of individual helicopter blades is not necessary for investigating dynamic characteristics of the helicopter, except in a special case, but rather for defining forces and moments in a disturbed flight it is sufficient enough to view the rotor as whole. Because of a great number of different helicopters, in this paper a single rotor helicopter was studied, that has its blades connected to the hub by hinges. As mentioned before, the helicopter can perform different motions and it would be very difficult to make a mathematical model that would combine all those motions. It is assumed the helicopter is airborne and in straightforward flight. It is required that the helicopter, at straight forward flight, has following velocity components: W_x, W_y, and W_z, at nominal values, and angle of turn ψ, angle of roll ϕ, angle of climb θ, as long as the intensity of disturbance is in permitted limits. Figure 6 presents a general block diagram of the helicopter with moving coordinate system tied to its center of mass.

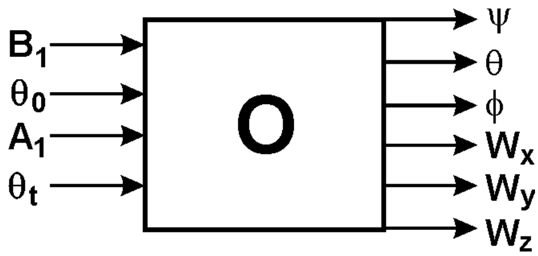

Fig. 6. General block diagram of helicopter

After introducing a series of assumptions we come to a non-linear mathematical model by deviations in the form:

$$\frac{d(\Delta W_x)}{dt} = \frac{1}{m}\left[f_1\left(\Delta W_x, \Delta W_z, \Delta\dot{\theta}, u_1, u_2\right) - (mg\cos\tau)\Delta\theta\right] \tag{3.1}$$

$$\frac{d(\Delta W_z)}{dt} = \frac{1}{m}\left[f_2\left(\Delta W_x, \Delta W_z, \Delta\dot{\theta}, u_1, u_2\right) + W_{zN} m\Delta\dot{\theta} - (mg\sin\tau)\Delta\theta\right] \tag{3.2}$$

$$\frac{d(\Delta\theta)}{dt} = \Delta\dot{\theta} \tag{3.3}$$

$$\frac{d(\Delta\dot{\theta})}{dt} = \frac{1}{J_y} f_3\left(\Delta W_x, \Delta W_z, \Delta\dot{W}_z, \Delta\dot{\theta}, u_1, u_2\right) \tag{3.4}$$

$$\frac{d(\Delta W_y)}{dt} = \frac{1}{m}\left[f_4\left(\Delta W_y, \Delta\dot{\phi}, \Delta\dot{\psi}, u_3, u_4\right) + W_{zN} m\Delta\dot{\psi} + mg\cos\tau\Delta\theta + mg\sin\tau\Delta\psi\right] \tag{3.5}$$

$$\frac{d(\Delta\phi)}{dt} = \Delta\dot{\phi} \tag{3.6}$$

$$\frac{d(\Delta\dot{\phi})}{dt} = \frac{1}{J_x}\left[f_5\left(\Delta W_y, \Delta\dot{\phi}, \Delta\dot{\psi}, u_3, u_4\right) + J_{xz}\Delta\dot{\psi}\right] \tag{3.7}$$

$$\frac{d(\Delta\psi)}{dt} = \Delta\dot{\psi} \tag{3.8}$$

$$\frac{d(\Delta\dot{\psi})}{dt} = \frac{1}{J_z}\left[f_6\left(\Delta W_y, \Delta\dot{\phi}, \Delta\dot{\psi}, u_3, u_4\right) + J_{xz}\Delta\dot{\phi}\right] \tag{3.9}$$

where:

- $U_1 = \Delta B_1$ – is the amplitude of cyclic change in pitch in the longitudinal direction (regarding to longitudinal motion),
- $U_2 = \Delta\theta_0$ – is the change of collective pitch of the helicopter rotor blade (regarding to longitudinal motion),
- $U_3 = \Delta A_1$ – is the amplitude of cyclic change in pitch in the lateral direction (regarding to lateral motion), and
- $U_4 = \Delta\theta_t$ – is the change of collective pitch of the tail rotor (regarding to lateral motion).

Block diagrams are presented in figures 7 and 8.

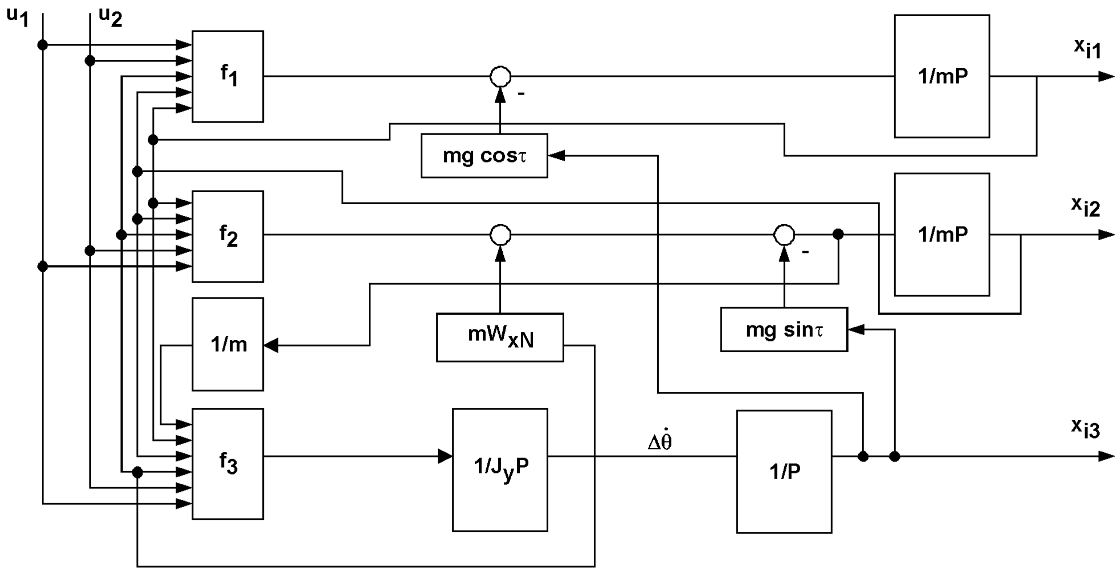

Fig. 7. Block diagram for longitudinal motion

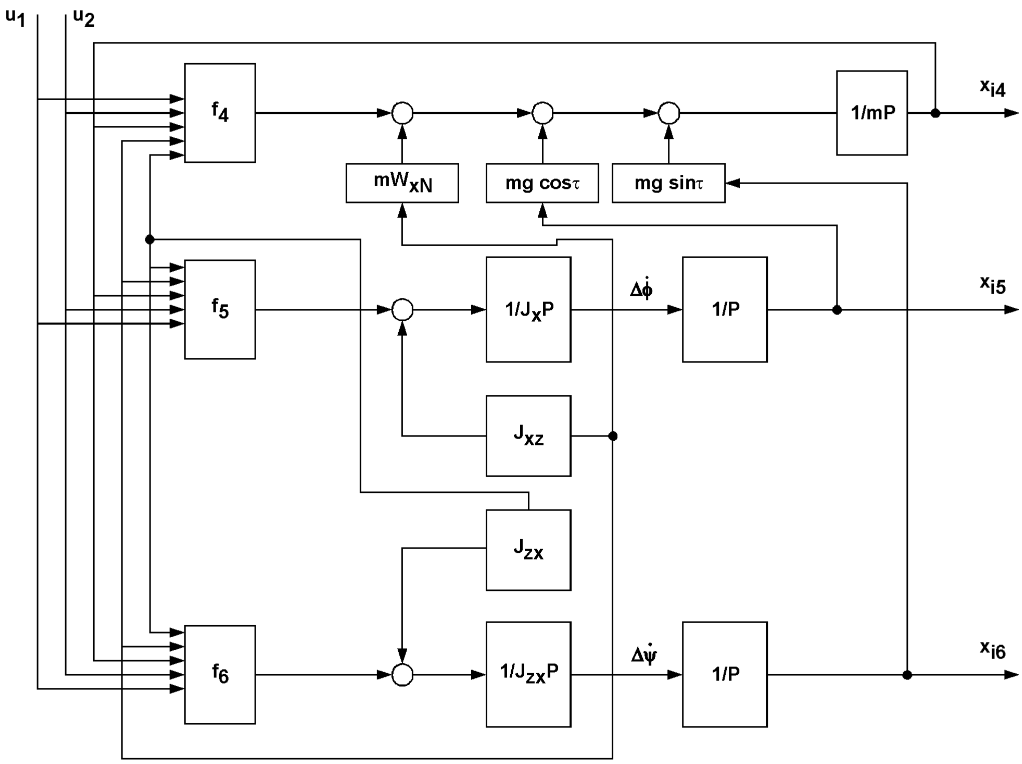

Fig. 8. Block diagram for lateral motion

4. Linearized mathematical model of flight dynamics

In technical applications it has been shown that, with an acceptable accuracy, linearized mathematical models may be used under the condition that deviations of physical quantities from their nominal values are small. Non-linear mathematical model of helicopter flight dynamics is inadequate for finding general solutions in an analytical form, even though the problem is solved in aid of modern computer technology.

The outcome of adopted presumptions is that the output values, input values, and the vector of state for both longitudinal and lateral motion will be:

$$\underline{X}=(X_1 \ldots X_9)^T \qquad \underline{X}_i=(X_{i1} \ldots X_{i6})^T \qquad \underline{u}=(u_1 \ldots u_4)^T \tag{4.1}$$

The matrix equation of state for the linearized mathematical model with non-dimensional quantities, deviations and quantities of state is shown by 4.2. The 4.3. presents the equation at exit.

$$\dot{\underline{X}}=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} & 0 & a_{58} & a_{59} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{75} & 0 & a_{77} & 0 & a_{79} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & a_{95} & 0 & a_{97} & 0 & a_{99} \end{bmatrix}\underline{X}+\begin{bmatrix} b_{11} & b_{12} & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b_{41} & b_{42} & 0 & 0 \\ 0 & 0 & b_{53} & b_{54} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_{73} & b_{74} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_{93} & b_{94} \end{bmatrix}\underline{u} \tag{4.2}$$

$$\underline{X}_i=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}\underline{X} \tag{4.3}$$

Beside the way this is presented, in a form of common matrix, also should be noted that longitudinal and lateral motions are separated, because this was the condition for deriving this mathematical model. In equations 4.2 and 4.3, equations for longitudinal motion are presented within the first for rows of the matrices, while the remaining five rows present the equation of state and equation of lateral motion. Designations used in 4.2 are:

$$C^* = \frac{1}{1-i_{xz}^2/i_x i_z} \qquad a_{11}=x_u \qquad a_{12}=x_w \qquad a_{13}=-m_c\cos\tau$$

$$a_{14}=x_q \qquad a_{21}=z_u \qquad a_{22}=z_w \qquad a_{23}=-m_c\sin\tau$$

$$a_{24} = \hat{W}_{xN} + z_q \quad a_{55} = y_v \quad a_{56} = m_c \cos\tau \quad a_{58} = m_c \sin\tau$$

$$a_{59} = \hat{W}_{xN} \quad a_{41} = m_u + m_{\dot{w}} z_u \quad a_{42} = m_w + m_{\dot{w}} z_w \quad a_{43} = -m_{\dot{w}} m_c \sin\tau$$

$$a_{44} = m_q + m_{\dot{w}}\left(\hat{W}_{xN} + z_q\right) \quad b_{11} = x_{B_1} \quad b_{12} = z_{B_1} \quad b_{21} = x_{\theta_0}$$

$$b_{22} = z_{\theta_0} \quad b_{41} = m_{\dot{w}} z_{B_1} + m_{B_1} \quad b_{42} = m_{\dot{w}} z_{\theta_0} + m_{\theta_0} \quad b_{53} = y_{A_1}$$

$$b_{54} = y_{\theta_t} \quad b_{73} = \left(l_{A_1} + n_{A_1} i_{xz} / i_x\right) C^* \quad b_{74} = \left(l_{\theta_t} + n_{\theta_t} i_{xz} / i_x\right) C^*$$

$$b_{94} = \left(n_{\theta_t} + l_{\theta_t} i_{xz} / i_z\right) C^* \quad a_{75} = \left(n_v i_{xz} / i_x + l_v\right) C^* \quad a_{77} = \left(l_p + n_p i_{xz} / i_x\right) C^*$$

$$a_{95} = \left(n_v + l_v i_{xz} / i_z\right) C^* \quad a_{97} = \left(n_p + l_p i_{xz} / i_z\right) C^* \quad a_{99} = \left(n_r + l_r i_{xz} / i_z\right) C^*$$

$$b_{93} = \left(n_{A_1} + l_{A_1} i_{xz} / i_z\right) C^* \quad a_{79} = \left(l_r + n_r i_{xz} / i_x\right) C^*$$

Figures 9 and 10 represent the block diagrams of linearized mathematical models of longitudinal and lateral motions.

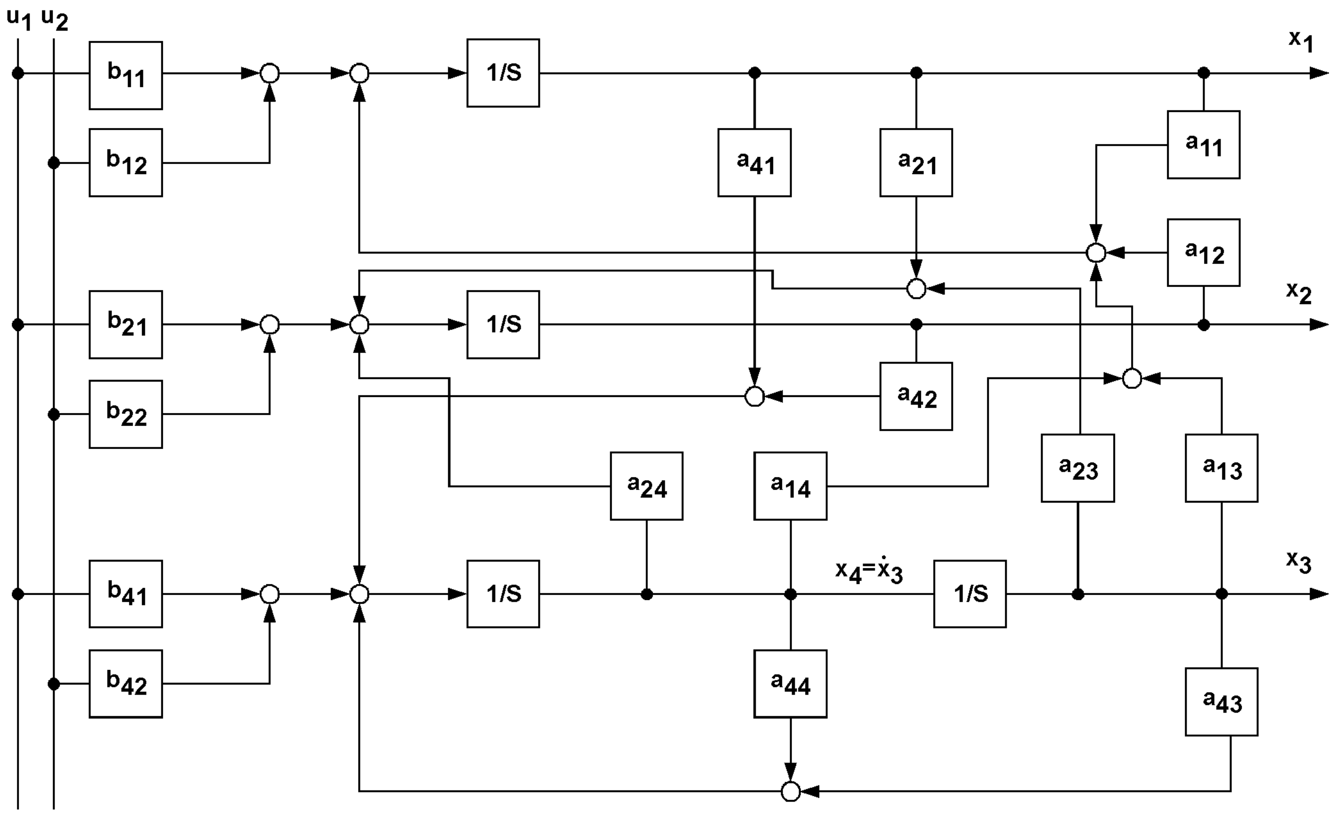

Fig. 9. Block diagram of the linearized mathematical model of longitudinal motion

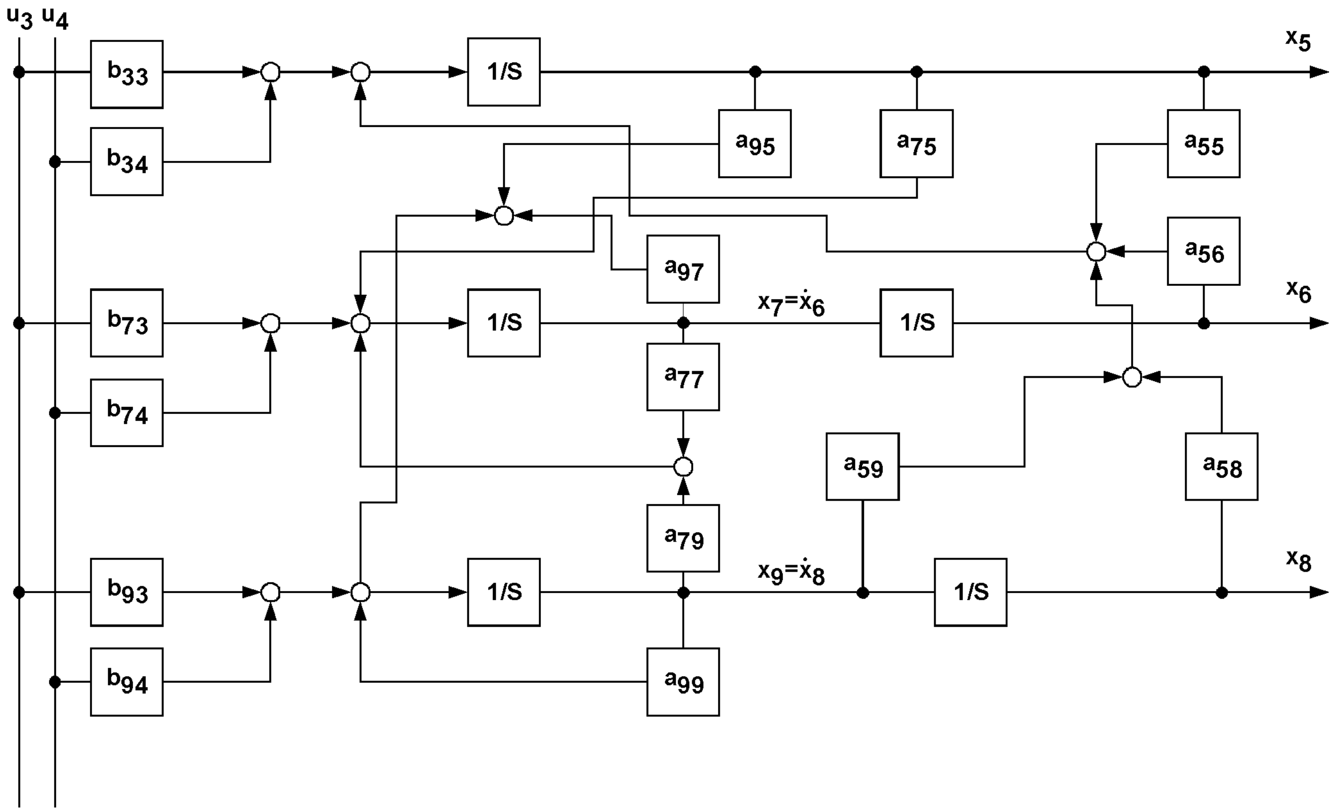

Fig. 10. Block diagram of the linearized mathematical model of lateral motion

5. Program results

The program was tested on the example of a single rotor helicopter which main rotor blades are tied to the hub over hinges. The helicopter is described by the following input data: Helicopter weight G=45000N, Rotor abundance degree s=0.058, Rotor radius R=8m, Hub height coefficient h=0.25, Drag coefficient δ=0.013, Number of blades of the main rotor b=4, Blade mass m=74.7kg, Rotor operate mode coefficient μ=0.3, Gradient of lift a=5.7, Velocity of blade top ΩR=208m/s, Distance of blade mass center coefficient x_g=0.45, Distance of hinge from shaft eR=0,04R, and Air density at flight altitude (100m) ρ=1.215 kg/m^3.

This program took into account, to the highest allowable extent, the benefits of the usage of the three airfoils deigned at Belgrade Faculty of Mechanical Engineering, for advanced helicopter rotor blades. The 12% thick R12M reaches up to 0.70R, while the thickness progressively drops to 9% at 0.85R (R9M) and 6% (R6M) at the tip (Fig. 11).

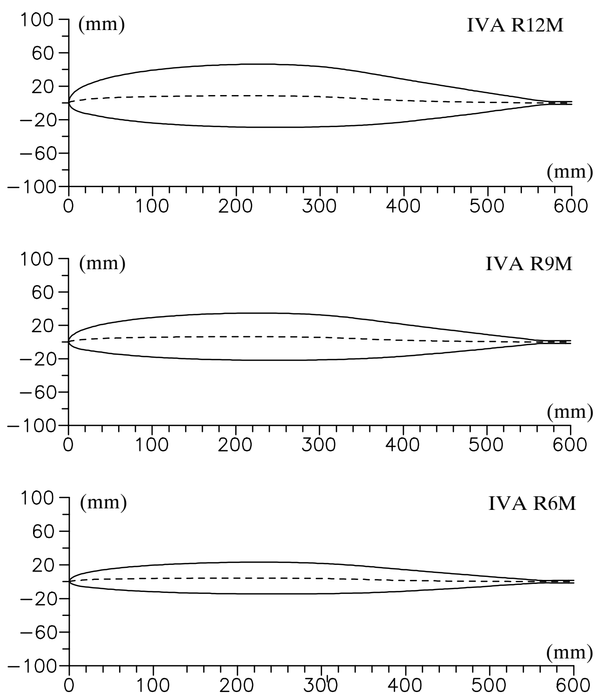

Fig. 11. Helicopter rotor blade airfoils

Some of their aerodynamic characteristics are shown in Fig.'s 12. (a) - (c).

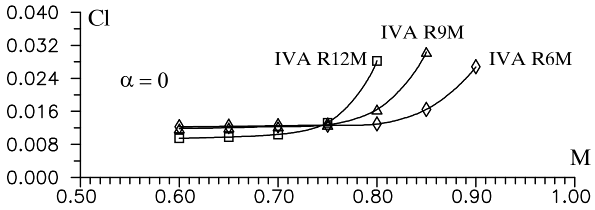

Fig. 12. (a)

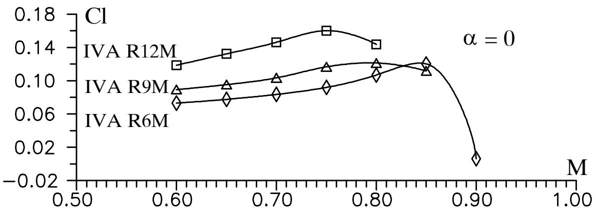

Fig. 12. (b)

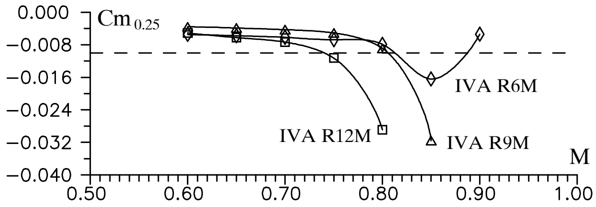

Fig. 12. (c) Some aerodynamic characteristics of the airfoils

The airfoils were designed by inverse approach, satisfying the requirements of close-to-zero moment coefficients, low profile and transonic drag and satisfactory lifting characteristics. Details of this process are out of the scope of this paper.

For longitudinal motion the mathematical model in matrix form is:

$$\dot{\underline{X}} = A\,\underline{X} + b\,\underline{u}$$

$$\dot{\underline{X}} = \begin{bmatrix} -0.0509 & 0.1323 & -0.0734 & 0.00263 \\ 0.1216 & -1.2525 & 0 & 0.3 \\ 0 & 0 & 0 & 1 \\ 6.512 & 12.1 & 0 & -0.844 \end{bmatrix} \underline{X} + \begin{bmatrix} 0.1344 & 0.066 \\ 0.3578 & -0.9477 \\ 0 & 0 \\ -28.329 & 17.88 \end{bmatrix} \underline{u}$$

$$\underline{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

It is obvious that matrix A is singular. From the point of automation control this means the system has unlimited number of equilibrium states.

6. Conclusion

The flight dynamics mathematical model of a helicopter, which would be strictly determined, would comprise of a system of non-linear, non-stationary, partial differential equations. To simplify these equations, we introduce a number of assumptions. Ignored are the elastic characteristics of the helicopter so the helicopter can be thought as a rigid body and, in that way, the dispersal of parameters is eliminated. Also, fuel consumption is disregarded and so the unsteadiness due to temporal change in helicopter mass is eliminated.

Because the helicopter has six degrees of freedom, for simplification it is assumed the motion can be separated into longitudinal and lateral motion and that they can be investigated independently. It is noted that the mathematical model of the helicopter is regarded to helicopter forward motion at velocity W. A mathematical model that would incorporate all helicopter motions, including take-off and landing, would be very complicated. Influence of resonance and vibration is also ignored. The blade throughback is also ignored in this paper, because if that wasn't the case the blade angle velocity in the plane of rotation would no longer be constant. Separate study of individual motions of blades is a great simplification, because there is an interdependency of all blade motions. If the motions are not separated, then it is necessary to analyze the stability of all the motions of the blade.

The choice of the coordinate origin in the center of inertia enables elimination of certain moments of inertia so the Euler equations can be simplified. Viewing the rotor as a whole eliminates the need for investigating individual blade motion. A great assistance to this is introducing the rotor disc axis and the control axis.

Determination of aerodynamic derivatives is related to series of approximations. It should be noted that, besides assumptions in modeling, also were made mathematical simplifications (for example, omittance of small values in equations) which couldn't have been derived in the form of an assumption due to their meaning which is tightly related to a specific equation.

It is possible to determine projections of position vector with respect to the non-moveable coordinate system tied to Earth instead of using projections of helicopter velocity in respect to the moveable coordinate system as the exit characteristics. Projecting the helicopter velocity onto a non-moveable coordinate system and then integrating velocity projections by time with initial conditions may solve this problem.

A further analysis of the mathematical model can be made in order to investigate the dynamic and static properties, and to determine control that would guaranty the object execution of the required dynamic behavior.

Subscripts

In this paper the following subscripts were used, where the first letter stands for:

- X – derivative of force function F_x,
- Y – derivative of force function F_y,
- Z – derivative of force function F_z,
- L – derivative of moment M_x,
- M – derivative of moment M_y,
- N – derivative of moment M_z.

If these letters are lower case, which means the considered derivative is non-dimensional. Lower case letters, besides these designations, characterize variables by which deriving was performed:

- u – by velocity W_x,
- v – by velocity W_y,
- w – by velocity W_z,
- p – by angular velocity,
- q – by angular velocity,
- r – by angular velocity,
- θ_0 – by collective step,
- B_1 – by angle B_1,
- A_1 – by angle A_1, and
- θ_{0t} – by collective step of the tail rotor.

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