LINEAR, PARAMETER-VARYING CONTROL AND ITS APPLICATION TO AEROSPACE SYSTEMS

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Abstract

This paper describes parameter-dependent control design methods for aerospace systems. Parameter-dependent systems are linear systems, whose state-space descriptions are known functions of time-varying parameters. The time variation of each of the parameters is not known in advance, but is assumed to be measurable in real-time. Three linear, parameter-varying (LPV) approaches to control design are discussed. The first method is based on linear fractional transformations which relies on the small gain theorem for bounds on performance and robustness. The other methods make use of either a single (SQLF) or parameter-dependent (PDQLF) quadratic Lyapunov function to bound the achievable level of performance. A summary of the application of LPV techniques to aerospace control problems is presented

1 Introduction

A variety of modern, multivariable control techniques have been applied to the design of feedback controllers for aerospace systems. These controllers are often designed at various operating points using linearized models of the system dynamics and are scheduled as a function of a parameter or parameters for operation at intermediate conditions. Various ad hoc methods have been used for controller scheduling but these techniques do not guarantee acceptable performance or even stability other than at the design points. Scheduling multivariable controllers can be very tedious and time consuming task. In this paper, linear, parameter-varying (LPV) techniques are described for the synthesis of automatically scheduled multivariable controller.

An outline of the paper is as follows. Section 2 provides a background on parameter varying systems and Section 3 provides information regarding linear, parameter varying systems. Control of LPV systems is presented in Section 4. The application of LPV control design techniques to aerospace systems is described in Section 5. The results of this paper are summarized in Section 6.

2 Parameter Varying Systems

Gain-scheduled control methods are based on interpolated, linear controllers that are scheduled as a function of one or several variables. Traditional gain-scheduling methods are inherently *ad hoc* and the resulting scheduled controller provide no stability or performance guarantee for rapid changes in the scheduling variables [1]. These issues were the main motivation for the current research on multivariable gain-scheduled control techniques which have become known as control of linear, parameter varying (LPV) systems [4, 5, 6, 14, 15, 20, 21].

Parameter-dependent systems are linear systems, whose state-space descriptions are known functions of time-varying parameters. The time variation of each of the parameters is not known in advance, but is assumed to be measurable in real-time. This type of system is called linear, parameter varying (LPV). The controller is restricted to be a linear system, whose state-space entries depend causally on the parameter's history. The goals of feedback include stabilization and performance improvement. Stabilization and more realistic problems involving closed-loop performance objectives while exploiting known bounds on the parameter's rate of variation can be posed in this framework. These problems are solved by reformulating the control design into

- finite-dimensional, convex feasibility problems which can be solved exactly, and
- infinite-dimensional convex feasibility problems which can be solved approximately.

This formulation constitutes a type of *gain-scheduling problem*.

In this section, some results are presented about effective control strategies for systems whose state-space model depends on parameters which are measurable in real-time. This is different from standard linear, time-varying optimal control theory, a time-variation which is not known in advance is considered, but is only known in real-time.

2.1 Parameter-dependent systems

In modeling a large, complex physical system with a non-linear, finite-dimensional state-space model, one chooses a collection of state variables for which the underlying dynamic evolution is understood, giving rise to the state equations. If this is a modestly sized state-space description, then large parts of the dynamic evolution of the "true" states are not represented. The state-space description "true" states are not represented. The state-space description will involve other variables, called *exogenous*, which have the certain properties:

- the dynamic evolutionary rules for the exogenous variables behavior is not understood, or is too complicated to be modeled;
- the values of the exogenous variables affect (in a known manner) the evolutionary rules which govern the dynamics of the state variables;
- the values of the exogenous variables change with time, but are measurable in real-time using sensors

If a large number of sensors are used, some of these sensors measure outputs in the system theoretic sense (ie., known, explicit nonlinear functions of the modeled states and time), while other sensors are accurate estimates of the exogenous variables. Hence, the model will be a time-varying, nonlinear system, with the future time-variation *unknown*, but measured by the sensors in real-time.

In this case, if $\rho(t)$ denotes the exogenous variable vector, and $x(t)$ denotes the modeled state, then the state equations for the system have the form

$$\dot{x}(t) = f(x(t), \rho(t), \dot{\rho}(t), u(t)) \tag{1}$$

where $u(t)$ is the input (control). The entire trajectory ρ is not known, though the value of $\rho(t)$ is known at time t, and hence may be used in any control strategy.

If in (1) f is linear in the pair $[x, u]$, then the system will be called *linear parameter-varying* (LPV). Several control synthesis methods have been developed for LPV systems. These methods are briefly described in section 4.1, and more thoroughly in [12, 4, 15].

Next, it is shown how some nonlinear systems can be converted into LPV's with a modest amount of conservatism.

3 LPV systems

A few situations which lead (possibly conservatively) to LPV models are explored.

3.1 Quasi-LPV systems

An LPV may arise by considering state transformations on a class of nonlinear systems called "quasi-LPV," introduced in [1]. Consider a nonlinear system of the form

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} f_1(x_1) \\ f_2(x_1) \end{bmatrix} + \begin{bmatrix} A_{11}(x_1) & A_{12}(x_1) \\ A_{21}(x_1) & A_{22}(x_1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + B(x_1)u \\ y &= x_1 \end{aligned}$$

The system is a nonlinear system, but the nonlinearity is entirely contained in the output variable,

$y = x_1$. Now, assume that there exist differentiable functions $x_{2\text{eq}}$ and u_{eq} such that for every x_1,

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} f_1(x_1) \\ f_2(x_1) \end{bmatrix} + A(x_1) \begin{bmatrix} x_1 \\ x_{2\text{eq}}(x_1) \end{bmatrix} + B(x_1)u_{\text{eq}}(x_1)$$

Then, by defining new states and inputs

$$\begin{array}{rcl} \xi_1 & := & x_1 \\ \xi_2 & := & x_2 - x_{2\text{eq}}(x_1) \\ v & := & u - u_{\text{eq}}(x_1) \\ \tilde{A}_{22}(\xi_1) & := & A_{22}(\xi_1) - \left.\frac{dx_{2\text{eq}}}{dx_1}\right|_{x_1=\xi_1} A_{12}(\xi_1) \\ \tilde{B}_2(\xi_1) & := & B_2(\xi_1) - \left.\frac{dx_{2\text{eq}}}{dx_1}\right|_{x_1=\xi_1} B_1(\xi_1) \end{array}$$

one obtains a “quasi-LPV” system

$$\begin{array}{rcl} \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} & = & \begin{bmatrix} 0 & A_{12}(\xi_1) \\ 0 & \tilde{A}_{22}(\xi_1) \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} B_1(\xi_1) \\ \tilde{B}_2(\xi_1) \end{bmatrix} v \\ y & = & \xi_1 \end{array} \tag{2}$$

There are two important points to notice. First, although the system representation in (2) has a linearized appearance, it still exactly represents the original nonlinear system. The LPV form **is not** equivalent to a Jacobian linearization about an operating point. Secondly, the representation is called “quasi-LPV” since the so-called exogenous parameter ξ_1 is actually a state, and in fact is not exogenous at all. However, by applying the theory of LPV systems to this system one can obtain output-feedback controllers that achieve stabilization and tracking for subsets of initial conditions and reference inputs.

We can generalize this “quasi-LPV” idea to a class of nonlinear parameter-dependent systems of the form

$$\begin{array}{l} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1,p) \\ f_2(x_2,p) \end{bmatrix} \\ \quad + \begin{bmatrix} A_{11}(x_1,p) & A_{12}(x_1,p) \\ A_{21}(x_1,p) & A_{22}(x_1,p) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + B(x_1,p)u \\ y = x_1 \end{array}$$

where $p(t)$ is the exogenous parameter vector. Again, assume that there exist differentiable functions $x_{2\text{eq}}$ and u_{eq} such that for every x_1 and every $p \in P$

$$\begin{array}{l} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} f_1(x_1,p) \\ f_2(x_{2\text{eq}}(x_1,p),p) \end{bmatrix} \\ \quad + A(x_1,p) \begin{bmatrix} x_1 \\ x_{2\text{eq}}(x_1,p) \end{bmatrix} + B(x_1,p)u_{\text{eq}}(x_1,p) \end{array}$$

Then, the parameter-dependent, time-varying, coordinate change and notational definitions

$$\begin{array}{rcl} \xi_1(t) & := & x_1(t) \\ \xi_2(t) & := & x_2(t) - x_{2\text{eq}}(x_1(t),p(t)) \\ v(t) & := & u(t) - u_{\text{eq}}(x_1(t),p(t)) \\ \tilde{A}_{22}(\xi_1) & := & A_{22}(\xi_1) - \left.\frac{dx_{2\text{eq}}}{dx_1}\right|_{x_1=\xi_1} A_{12}(\xi_1) \\ \tilde{B}_2(\xi_1) & := & B_2(\xi_1) - \left.\frac{dx_{2\text{eq}}}{dx_1}\right|_{x_1=\xi_1} B_1(\xi_1) \\ E_2(\xi_1,p) & = & -\left.\frac{dx_{2\text{eq}}}{dp}\right|_{x_1=\xi_1} \end{array}$$

results in the following “quasi-LPV” system

$$\begin{array}{rcl} \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} & = & \begin{bmatrix} 0 & A_{12}(\xi_1) \\ 0 & \tilde{A}_{22}(\xi_1) \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \\ & & \begin{bmatrix} B_1(\xi_1) \\ \tilde{B}_2(\xi_1) \end{bmatrix} v + \begin{bmatrix} 0 \\ E_2(\xi_1,p) \end{bmatrix} \dot{p}(t) \\ y & = & \xi_1 \end{array} . \tag{3}$$

This quasi-LPV system includes additional terms, which have been expressed as an additional “input” vector $\dot{p}$. Under closed-loop control, it is likely that the controller would have access to a noisy measurement of $\dot{p}$ (the noise model appropriate for the $\dot{p}$ measurement would be problem dependent). If no reliable $\dot{p}$ measurement is available, then $\dot{p}$ can simply be treated as a disturbance which must be rejected. Based on engineering judgment (problem dependent) some expected properties of $\dot{p}$ could be assumed (for instance, a member of L_2 or L_∞, with $\|\dot{p}\| \leq 1$ for all operating conditions) and control strategies based on rejecting these types of disturbances be used.

3.2 Jacobian Linearizations

Often in industrial settings, a finite collection of linear models is used to describe the behavior of a system throughout an operating envelope. The linearized models describe the small signal behavior of the system at a specific operating point,

The collection is parametrized by one or more physical variables whose values represent the operating point. If the state variables have physical meaning that is invariant across all of the model collection, then it makes sense to develop (usually with least squares) polynomial fits of the state-space matrices to get a continuous parameterization of the operating envelope. At that point, any of the methods described can be used to design controllers for the resulting LPV, with nonlinear simulations being the ultimate test of the suitability of the controller.

4 Control

In this section, methods for controlling LPV systems, starting with stabilization, and then performance enhancement are presented. In section 4.1, the stabilization problem for linear parameter-dependent systems, which includes a Youla-parameterization of all quadratically stabilizing, linear, parameter dependent controllers is discussed. In section 4.2, the control of systems with general parameter-dependence is discussed, using an (input/output gain (ie., induced norm) performance objective. The analysis method bounds the square-integral (L_2) gain using a single quadratic Lyapunov function. In section 4.3, methods using parameter-dependent Lyapunov functions, which allow the controller to exploit known bounds on the rate-of-variation of the parameter are presented. These are the most computationally demanding of all the methods, but are also the most flexible and least conservative. Section 4.4 covers the L_2 control of systems whose parameter dependence is linear fractional (LFT) using controllers with linear fractional dependence. The analysis tool in the LFT approach is the scaled-small gain theorem. This approach is more conservative than the single Lyapunov function approach, but leads to algorithms which are computationally less demanding.

4.1 Stabilization and Performance

If one adopts a quadratic Lyapunov approach to guarantee closed-loop stability and performance, then many interesting and useful results about parameter-dependent systems are easily derivable. In order to motivate the quadratic Lyapunov approach, consider the linear, time-invariant system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \qquad (4)$$

and recall two main facts:

Theorem: (Exponential Stability) *The system in (4) is internally exponentially stable if and only if there exists a matrix $X = X^T > 0$ such that $A^T X + XA < 0$.*

Theorem: (Bounded Real Lemma) *The system in (4) is internally exponentially stable, and*

$$\left\| C(sI - A)^{-1} B \right\|_\infty < 1$$

if and only if there exists a matrix $X = X^T > 0$ such that

$$A^T X + XA + XBB^T X + C^T C < 0.$$

Hence, for linear, time-invariant (LTI) systems, stability and induced L_2 norm performance (H_∞) can be exactly characterized using quadratic Lyapunov functions. By applying these theorems to closed-loop systems, well-known, computable necessary and sufficient conditions for feedback stabilization and H_∞ optimization can be derived.

Now, consider a linear parameter-varying system

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x + B(\rho(t))u(t) \\ y(t) &= C(\rho(t))x(t) \end{aligned} \qquad (5)$$

where $\rho(t)$ is an exogenous variable, known to take on values in a prescribed connected, compact set P. For ease of notation, suppose that $P \subset \mathbf{R}$.

Lemma: (Quadratic Stability) *If there exists a matrix $X = X^T > 0$ such that*

$$A^T(\rho)X + XA(\rho) < 0$$

for all $\rho \in P$, then the system in (5) is exponentially stable for any trajectory $\rho(\cdot)$ which satisfies $\rho(t) \in P$.

Using this lemma as a starting point for analysis, parameter-dependent synthesis techniques can be derived. Specifically, synthesis problems can be posed which ask if there exists a linear, parameter-dependent controller such that the analysis tests described in theses Lemmas hold for the closed-loop system (which is, of course, parameter-dependent).

The stabilization problem for this parameter-dependent system is as follows: Do there exist continuous functions $A_K(\rho), B_K(\rho), C_K(\rho)$ and $D_K(\rho)$, and a matrix $X = X^T > 0$ such that the symmetric matrix

$$\begin{bmatrix} A(\rho) + B(\rho)D_K(\rho)C(\rho) & B(\rho)C_K(\rho) \\ B_K(\rho)C(\rho) & A_K(\rho) \end{bmatrix}^T X + X \begin{bmatrix} A(\rho) + B(\rho)D_K(\rho)C(\rho) & B(\rho)C_K(\rho) \\ B_K(\rho)C(\rho) & A_K(\rho) \end{bmatrix} < 0$$

for all $\rho \in \mathcal{P}$? Under this condition, the closed-loop system (using the parameter dependent controller $A_K(\rho), B_K(\rho), C_K(\rho), D_K(\rho)$) will be exponentially stable for all time-varying trajectories $\rho(\cdot)$, as long as $\rho(t) \in \mathcal{P}$ for all t.

So, by using a single, quadratic Lyapunov function to establish stability, exponential stability with time varying parameters is guaranteed. Following along the lines of [13, 11, 10], it is possible to characterize *all* quadratically stabilizing, linear, parameter dependent controllers, [9], as:

Theorem 4.1 *Let A, B, C be continuous functions, on a compact set $\mathcal{P} \in \mathbf{R}^p$. There exists a quadratically stabilizing controller (as above) if and only if there exists continuous functions F and L on $\mathcal{P}$, and matrices $X_F = X_F^T > 0, X_L = X_L^T > 0$ such that*

$$(A(\rho) + B(\rho)F(\rho))^T X_F + X_F (A(\rho) + B(\rho)F(\rho)) < 0$$

and

$$(A(\rho) + L(\rho)C(\rho))^T X_L + X_L (A(\rho) + L(\rho)C(\rho)) < 0$$

for all $\rho \in \mathcal{P}$.

Then, the input/output behavior of all linear, finite dimensional output-feedback, parametrically-dependent controllers achieving quadratic stability over $\mathcal{P}$ is parametrized as

$$\begin{bmatrix} \dot\eta_1 \\ \dot\eta_2 \\ u \end{bmatrix} = \begin{bmatrix} A+BF+LC+BD_QC & BC_Q & -L-BD_Q \\ B_QC & A_Q & -B_Q \\ F+D_QC & C_Q & -D_Q \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ y \end{bmatrix} \tag{6}$$

where the matrices A_Q, B_Q, C_Q and D_Q are continuous functions on $\mathcal{P}$, and A_Q itself is quadratically stable on $\mathcal{P}$.

In essence, this theorem implies that it is very natural to gain-schedule a controller K by actually gain-scheduling the free Q parameter. It is only necessary to first compute quadratically stabilizing state feedbacks ($F(\rho)$) and output injections ($L(\rho)$), and then follow the standard Youla parameterization.

4.2 Closed-loop performance

In order to address closed-loop performance (beyond stability) suppose the parameter-dependent plant G, has additional inputs (d) and outputs (e) along with the measurements (y) and controls (u),

$$\begin{bmatrix} \dot x \\ e \\ y \end{bmatrix} = \begin{bmatrix} A(p) & B_1(p) & B_2(p) \\ C_1(p) & D_{11}(p) & D_{12}(p) \\ C_2(p) & D_{21}(p) & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix}. \tag{7}$$

Choose F and L to quadratically stabilize the pairs (A, B_2) and (A, C_2). It is then possible to define quadratically stable, parameter dependent, state-space systems T_1, T_2, and T_3 (depending only on G, F, and L) such that the closed-loop operator from d to e, using quadratically stabilizing, parametrically dependent controllers from Theorem 4.1, is of the form

$$T_1 + T_2 Q T_3$$

where Q is the quadratically stable, parameter-dependent Youla operator used in the controller parameterization. Hence, for all parametrically-dependent quadratically stabilizing controllers, the zero-state, closed-loop operator from d to e is an affine function of the free parameter Q. This means that any convex functional evaluated on the achievable closed-loop operators will be a convex function of the free variable Q. This convexity property could be exploited to give some procedures for tailoring the parameter-dependent Q operator to achieve performance objectives that go beyond mere stabilization, [8, 7, 9]. Currently though, the problem of choosing an appropriate operator Q for performance remains a big challenge to the LPV control problem.

Another approach to attack the "performance" problem relies on a modification of the well-known Bounded Real Lemma theorem. The inequality

$$A^T X + XA + XBB^T X + C^T C < 0.$$

can also be written as

$$\begin{bmatrix} A^T X + XA & XB & C^T \\ B^T X & -I & 0 \\ C & 0 & -I \end{bmatrix} < 0$$

which is called a *linear matrix inequality* (LMI) in the variable X. It represents a convex constraint on the variable X, and determining the existence of such an X can then be cast as a convex feasibility problem.

The input/output L_2 gain of a parameter-dependent system can be (conservatively) bounded using the same technique.

Lemma: *Given a parameter-dependent linear system with state-space matrices $A(\rho), B(\rho)$ and $C(\rho)$. Suppose there exists a matrix $X = X^T > 0$ such that*

$$\begin{bmatrix} A^T(\rho)X + XA(\rho) & XB(\rho) & C^T(\rho) \\ B^T(\rho)X & -I & 0 \\ C(\rho) & 0 & -I \end{bmatrix} < 0$$

for all $\rho \in \mathcal{P}$. Then for every parameter trajectory $\rho(\cdot)$ which satisfies $\rho(t) \in \mathcal{P}$ for all t, the system is exponentially stable, and for zeros initial conditions, $\|e\|_2 \leq \|d\|_2$. (the induced norm from d to e is less than 1).

Using this lemma as a starting point for analysis, a synthesis problem can be posed which asks if there exists a linear, parameter-dependent controller such that the analysis test described in the Lemma holds for the closed-loop system, hence guaranteeing the performance of the closed-loop system. This theoretical problem forms the foundation of a references [6, 12, 3, 17]. Reference [6] solves the problem and contains all of the derivations, along with discussions of computational complexity and small, but nontrivial examples. Specifically, the existence of such a controller is expressed in terms of a convex feasibility problem. The convex constraints are made up of 3 distinct parts: a "full-information" (or state-feedback) condition, a "full-control" (or filtering) condition, and the spectral radius condition. These are all generalizations of the well-known H_∞ synthesis results and reduce to the standard conditions when one considers LTI plant with LTI controllers.

4.3 Parameter Rate-variation Bounds

The problem formulation discussed in the previous section has a significant drawback – since stability (and possibly performance) is guaranteed for *every* parameter trajectory $\rho(\cdot)$ satisfying $\rho(t) \in \mathcal{P}$ for all t, design techniques based on this characterization tend to be overly conservative. In fact, there are entire classes of problems for which this strong type of stabilization is simply not possible. Reference, [2], is a generalization to the work in [6]. It allows one to exploit a-priori known bounds on the parameter's rate-of-variation, at the expense of more complicated convex feasibility programs. A result pertaining to L_2 gain from d to e is as follows.

Lemma: (Parameter-dependent Performance) *Suppose that $\nu > 0$. If there exists a continuously differentiable function $X(\rho)$ such that $X(\rho) > 0$ and the two inequalities*

$$\begin{bmatrix} \nu\frac{dX}{d\rho} + A^T(\rho)X(\rho) + X(\rho)A(\rho) & X(\rho)B(\rho) & C^T(\rho) \\ B^T(\rho)X(\rho) & -I & 0 \\ C(\rho) & 0 & -I \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} -\nu\frac{dX}{d\rho} + A^T(\rho)X(\rho) + X(\rho)A(\rho) & X(\rho)B(\rho) & C^T(\rho) \\ B^T(\rho)X(\rho) & -I & 0 \\ C(\rho) & 0 & -I \end{bmatrix} < 0 \tag{9}$$

are satisfied for all $\rho \in \mathcal{P}$, then the system in (5) is exponentially stable for any trajectory $\rho(\cdot)$ satisfying $\rho(t) \in \mathcal{P}$, $|\dot{\rho}(t)| \leq \nu$, and for $x(0) = 0$, we have $\|e\|_2 < \|d\|_2$. Note that these inequalities represent convex constraints on the variable X, but that now X is itself a matrix function of the parameter ρ. Hence, the unknown, X, is an element of a function space, so that the conditions in (8) and (9) are really infinite-dimensional linear matrix inequalities.

All of the controller synthesis results described earlier (stabilization, parameterization of all stabilizing controllers, dependencies of closed-loop on the free Youla "Q" parameter, and minimization of the induced-norm from d to e, for both state-feedback and output-feedback cases) can be generalized to handle this setting. These extensions are the main results of references [2, 15]. These methods allow the exploitation of a-priori known bounds on the parameter's rate-of-variation. The design equations are infinite-dimensional Affine Matrix inequalities. In designing controllers, one can search for feasibility over a finite dimensional subspace, which if successful, yields controller's with guaranteed properties. However, if the finite-dimensional subspace if appropriate the optimization may not yield useful results, even though a controller may exist (and be found with a different initial choice of finite-dimensional subspace). It has been our experience that suitable basis functions can be chosen based on the parameter-dependence of the plant itself.

The LPV controllers provide local stability and performance guarantees near equilibrium point models used for design. At non-equilibrium points, these controllers are linear interpolated based on the nearest equilibrium. LPV methods provided for systematic design of gain-scheduled multivariable controllers that includes performance and robustness objectives in the design process. Simulation-based testing is still critical to ensure these characteristics are valid on the full, nonlinear system.

4.4 Linear Fractional Plants

Consider a problem-specific set of block diagonal matrices

$$\left\{\operatorname{diag}\left[\delta_1 I_{s_1}, \ldots, \delta_f I_{s_f}\right] =: D_S(\delta) : \delta_i \in \mathbf{R}\right\} \subset \mathbf{R}^{s \times s}$$

associated with integers $S := (s_1, s_2, \ldots, s_f)$ and the constant matrix $M \in \mathbf{R}^{(n+n_e+s)\times(n+n_d+s)}$, partitioned as

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

The dimension n corresponds to the number of states, n_e corresponds to the number of output errors, n_d corresponds to the number of disturbance inputs and s corresponds to the dimension of S.

The LFT system G_δ is described by the equations

$$\begin{bmatrix} \dot{x}(t) \\ e(t) \end{bmatrix} = \left[\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} + \begin{bmatrix} M_{13} \\ M_{23} \end{bmatrix} \Delta(t) \left[I - M_{33}\Delta(t)\right]^{-1} \begin{bmatrix} M_{31} & M_{32} \end{bmatrix}\right] \begin{bmatrix} x(t) \\ d(t) \end{bmatrix}$$

where piecewise continuous Δ trajectories satisfy

$$|\delta_i(t)| \leq 1, \qquad \Delta(t) = D_S(\delta(t))$$

are called *allowable*. This is just a very special form of an LPV system. Note that $F_L(M, \Delta(t))$ denotes a parameter-dependent state-space matrix. Hence the parameter-dependent, LFT plant model and controller can be thought of as the block diagram shown in Figure 1.

For LFT parameter-dependent systems, parameterization of all stabilizing controllers is based on the structured small-gain theorem. This is in contrast to the previous discussion of LPV systems where the results were based on parameter-dependent Lyapunov functions. For control design, first define a set of parameter-dependent, LFT plant models that depend in an LFT manner on scheduling parameters $D_R(\delta)$.

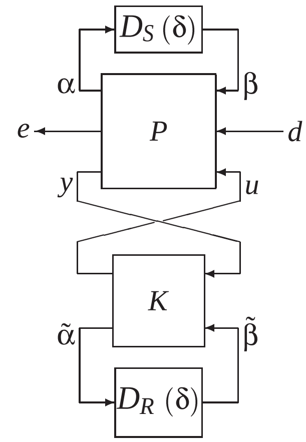

Fig. 1 Block Diagram of LFT Parameter Varying Plant and Controller

Then there is an LFT controller such that the closed-loop system has a performance level less than 1 if and only if there are positive definite matrices X and Y that satisfy a set of matrix inequalities as defined in reference [4]. In general, the dependence of the gain–scheduled LFT controller on the parameters δ_i is no more complex than the plant. The LTI part of LFT controller, K, is reconstructed from X and Y and implemented directly (see [4] for details).

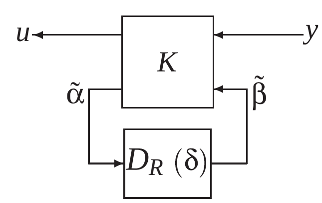

Fig. 2 LFT Parameter Varying Controller

5 Application to aerospace systems

Linear, parameter-varying control techniques are being applied extensively in the aerospace area. The synthesis of LPV controllers for aircraft and missiles represents a majority of the applications [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 14, 42, 43]. Other application areas include control of turbofan engines [41] and active flutter suppression [39, 40].

6 Summary

This paper describes parameter-dependent control design methods for aerospace systems. LPV systems are a very special class of nonlinear systems which appears to be well suited for control of aerospace systems. In general, LPV techniques provide a systematic design procedure for gain-scheduled multivariable controllers. This methodologies allow performance, robustness and bandwidth limitations to be incorporated into a unified framework. It is important to note that simulation-based testing of these designs is still critical.

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