OPTIMIZATION OF A GIVEN-PLANFORM LIFTING SYSTEM AT SUPERSONIC SPEED

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Abstract

The optimization problem of complex configurations is solved in the framework of the linear theory. Such variations of local incline angles of surface's elements are determined which lead to reduction of wave drag coefficient but keep the given magnitude of lift and pitching moment coefficients and also wing volume or areas of the wing cross-sections. For solving the problem of optimization the variation method is used. The Reverse Flow Theorem is lied in its base. The results showing the necessity to consider the optimization problem not only for wing separately, but also for complete configuration containing all lifting surfaces and fuselage are presented. The numerical analysis of thick wing-body configuration confirmed the correctness in the estimations of the induced-wave drag reduction, which was obtained by the linear theory. Also it confirmed the opportunity to determine an optimum fuselage axis deformation on the basis of the linear theory.

Nomenclature

φ	- velocity potential
V_∞	- freestream velocity
M	- Mach number
β	$=\sqrt{M_\infty^2-1}$
C_L	- lift coefficient
C_D	- induced-wave drag coefficient
C_m	- pitching moment coefficient
C_p	- upper surface pressure coefficient
C_p^*	- upper surface pressure coefficient (reverse flow)
Λ	- Lagrangian
$\lambda_1,\ \lambda_2$	- Lagrange multipliers
μ_i	- step size at *i*-th iteration
δ	$=\dfrac{C_D^0-C_D^{opt}}{C_D^0}(\%)$, relative reduction of induced-wave drag coefficient
Λ_{LE}	- leading-edge sweep angle
c	- local wing chord
b	- wing span
x, y, z	- Cartesian coordinates
$\bar{X}$	$= x/c$
$\bar{Y}$	$= y/c$
$\bar{Z}$	$= z/b$
α	- local angle between meanline and *x*-axis
ϕ_{FA}	- incidence angle of fuselage axis
ϕ	- twist
δS	$=\dfrac{S_i^{opt}-S_i^0}{S_i^0}(\%)$, - relative changing of wing cross-section area
S_i	- area of *i*-th wing cross-section

Superscripts

0	- initial geometry conditions
opt	- final design conditions

Introduction

The reduction of wave drag is one of the most important problems for modern aircraft with supersonic cruise speed. The streamlining conditions of the supersonic passenger aircraft in cruising mode ($M \sim 2$, $C_L \sim 0.1$) allow to use widely of the linear theory.

As a rule, gradient and variation methods are used great number of varying parameters (incidence angles of surface's elements) in the wing optimization [1], [2]. They can be easily adapted to optimization problems for complex configurations and allow taking into account different restrictions. It should be noted that in spite of success achieved during the development of optimization methods on the basis of Euler equations, this problem makes high requirements to computer capabilities [3], [4]. So, the paper [4] shows that solution depends on dimension of calculation grid, even for a wing-alone configuration. The assessments demonstrated a great role of fuselage axis deformation in optimization problems. But even while estimating simple wing–body and wing–canard configuration the number of estimated cells increases in several times, thus these procedures can't be widely used in practice.

Due to this at present time, the development of high-operative optimization methods based on the linear theory is still an actual task. The most perfect of them is the version of the variation method based on consequence of the Reverse Flow Theorem [1]. Use of the mentioned method allows to do mass parametric calculations not only for wing separately but for complex configurations: wing-canard, wing-fuselage, wing-canard–fuselage and so on. Similar algorithms and programs permit to solve problems of aerodynamic designing quickly and can be easily included to CAD system.

Optimization problem

The supersonic flow over a thin wing is considered within the framework of the linear theory based on the linear equation of velocity potential:

$$\beta^2 \cdot \frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \varphi}{\partial y^2} - \frac{\partial^2 \varphi}{\partial z^2} = 0.$$

The velocity potential on the upper wing surface, which is in XOZ plane, is determined by using the local incidence angles of the surface $\alpha(x_1, z_1) = -\dfrac{V_y(x_1, z_1)}{V_\infty}$ according to equation:

$$\varphi(x,z) = \frac{V_\infty}{\pi} \cdot \iint_S \frac{\alpha(x_1, z_1) dx_1 dz_1}{\sqrt{(x - x_1) - \beta^2 (z - z_1)}}.$$

The following restrictions are considered during optimization:

a) lift coefficient: C_L,
b) pitching moment coefficient: C_m,
c) limitation of the value of local incidence angles: $|\alpha| < \alpha_{max}$,
d) volume or wing cross-section areas,
e) limitation of the wing cross-section thickness.

As the flow is governed by the linear potential equation the optimization problem can be subdivided into two tasks: optimization of the middle surface of aircraft configuration and optimization of the wing cross-section shape.

Optimization of middle surface

The first task (the task of middle surface optimization) is formulated in the following manner: to find local incidence angles of middle surface's elements for a given-planform lifting system, which has minimum of induced-wave drag with restrictions: C_L, C_m, α_{max}.

To solve this task the method of Lagrangian multipliers is used. Its solution is reduced to finding of Lagrangian minimum:

$$L = C_D + \lambda_1 \cdot C_L + \lambda_2 \cdot C_m. \qquad (1)$$

Using consequence of the Reverse Flow Theorem [1], variation of the objective function L can be written as:

$$\delta L = \frac{2}{S} \cdot \iint_S (C_p(\alpha) - C_p^*(\alpha) - \lambda_1 \cdot C_p^*(\alpha = 1) +$$

$$+\lambda_2 \cdot C_p^*(\alpha = \frac{x}{b})) \cdot \delta\alpha \cdot dS \,. \quad (2)$$

And then, the local incidence angle variation leading to reduction of Lagrangian and hence reduction of the induced-wave drag coefficient will be:

$$\delta\alpha = -\mu \cdot (C_p(\alpha) - C_p^*(\alpha) - \lambda_1 \cdot C_p^*(\alpha = 1) +$$

$$+\lambda_2 \cdot C_p^*(\alpha = \frac{x}{b}))\,. \quad (3)$$

So, with small $\mu > 0$, such changing in α provides the reduction of induced-wave drag coefficient C_D without changing of the coefficients C_L and C_m. Value μ_{opt} providing maximal reduction of the induced-wave drag is determined by the condition: $\frac{\partial}{\partial\mu}(\delta L) = 0$. Lagrangian multipliers are found from equations $\delta C_L = \delta C_m = 0$ with $\alpha = \delta\alpha + \alpha^0$, where α^0 is previous incidence angle distribution of the surface.

The optimum incidence angles distribution of the surface is determined by the iterative procedure. Values μ_i, λ_1, λ_2 are determined at the each iteration step then variation $\delta\alpha^i$ is found according to the formula (3), so $\alpha^{i+1} = \delta\alpha^i + \alpha^i$ is known.

40-50 iterations are enough in order to obtain convergence solution even for complex configurations. Primary magnitudes of the lift and pitching moment coefficients are keeping during the iterative procedure. So, the initial geometry of wing middle surface, which provides the given magnitudes of coefficients C_L and C_m, must be determined before the iterative procedure.

Optimization of the wing cross-section shape

The second task of the optimization is formulated in the following manner: to determine the cross-section shape of the wing, which has minimum of zero-lift drag with restriction of the wing volume (or restrictions of the wing cross-section areas).

The number of limitations for the task of the wing cross-section shape optimization is increased. Additional restrictions are required in order to obtain an airfoil which is reasonably thick and closed at the trailing edge.

The cross-section shape optimization task is solved with the same variation method, which was taken for solving the middle surface optimization. Lagrangian multipliers are found from equations system. In spite of a lot of additional restrictions, there aren't difficulties in its solving. Since the matrix of the system has arrow–form (restriction of the wing volume) or is easily transformed to linearly independent (2x2) matrixes (as for restriction of the cross-section area).

Numerical procedure

In order to obtain of the discrete equations, integrated area is divided into a finite number of elements with equal intervals. Grid lines are parallel to Mach lines. It is assumed that $\alpha(x,z) = \alpha^*_{ij}$ is constant for each element. Fractional elements are considered while the influence of elements belonging to leading and trailing edges are calculated. The form of these elements corresponds to actual geometry. In the case of the subsonic edge (the task of the middle surface optimization) fractional elements aren't used.

Results

Results obtained for the wing-fuselage configuration and supersonic passenger aircraft configuration are presented to show the contribution of fuselage axis deformation in the induced-wave drag reduction. The optimization problem of the wing separately is also solved.

The lift, pitching moment and induced-wave drag coefficients are obtained for simplified configuration while the fuselage and the wing are approximated with thin curved surfaces.

The plate middle surface was chosen as an initial geometry.

The general view of configuration is shown in upper part of fig. 1. Optimization mode: $M = 1.8$, restriction of the lift coefficient - $C_L = 0.1$

The induced-wave drag reduction is estimated as $\delta = \dfrac{C_D^0 - C_D^{opt}}{C_D^0}(\%)$.

In fig. 1, we can see the computed results of the basic trapezoidal wing with supersonic leading edge ($\Lambda_{LE} = 45°$, $M = 1.8$). The reduction of the induced-wave drag for such wing shape is low (δ =2.6%).

But in the case of the wing-fuselage configuration (fig.2), the induced-wave drag reduction increases up to 26%. Considerable induced-wave drag reduction is due to changing the flow pattern over the wing with the optimum fuselage axis deformation.

To verify the obtained results marked induced-wave drag reduction was calculated ones more but taking into account of the thick fuselage. The discrepancy between obtained estimations was less than 1%.

The second configuration is presented in figs. 3 and 4. Optimization mode: $M = 2.0$, restriction of the lift coefficient $C_L = 0.1$. In the lower part of fig. 3 we can see the parameters of the wing middle surface, which obtained for separate wing. The induced-wave drag reduction is equal to 16%.

As for configuration with optimized wing and uncurved fuselage, the optimization effect is reduced. Computed parameters δ for this example are given in the table.

ϕ_{FA}	δ (%)
0	3.3
1.3°	7.1
2.4°	9.2
3.3°	9.3
3.7°	9.2
4.1°	9.0
4.8°	8.1

Maximum of the induced-wave drag reduction (δ =9.3%) was obtained when incidence angle of the fuselage axis was less than incidence angle of the optimal wing board section. So, maximum reduction of induced–wave drag for wing-fuselage configuration should be less than for the wing separately.

In the case of wing-fuselage configuration δ increases up to 24% (1.5 times more than for the wing separately). In fig. 4 the geometrical parameters of the wing middle surface and fuselage axis shape are given. In the central part, the wing section deformation increases in comparison with the deformation were obtained for the separate wing. The section shape changes less on the outer-part of the wing. Incidence angles of sections changed insignificantly.

In fig. 5 it’s shown the results of the cross-section shape optimization for separate wing in the second configuration. Optimization mode: $M = 2.0$, restriction of the wing volume. The parabolic profile with the 50%-chord maximum thickness position and 4% thickness was chosen as an initial shape. Additional restrictions for the thickness were introduced. It shouldn’t be more than 6% and less than 2%. The reduction of the lift-zero wave drag is 26%. In upper part of fig.5, wing span distribution of relative changing of the wing cross-section area is presented.

The wing cross-section shapes obtained during the optimization with the cross-section area restrictions are shown in fig.6. Initial geometry and additional restrictions of thickness of the cross-section shape were the same as in the first variant of optimization. The wave drag reduction was 8 %.

Number of grid elements in these calculations was 1500÷3000.

Conclusions

Variation method, based on the Reverse Flow Theorem and the linear theory, allows to solve quite effectively the optimization problems of the middle surface of lifting elements in complex configurations, to determine shape of the fuselage axis and cross-section shape of the wing.

The estimations of the induced-wave drag reduction determined by the linear theory which taking into account of the thick fuselage demonstrated the negligible discrepancy with suggested method.

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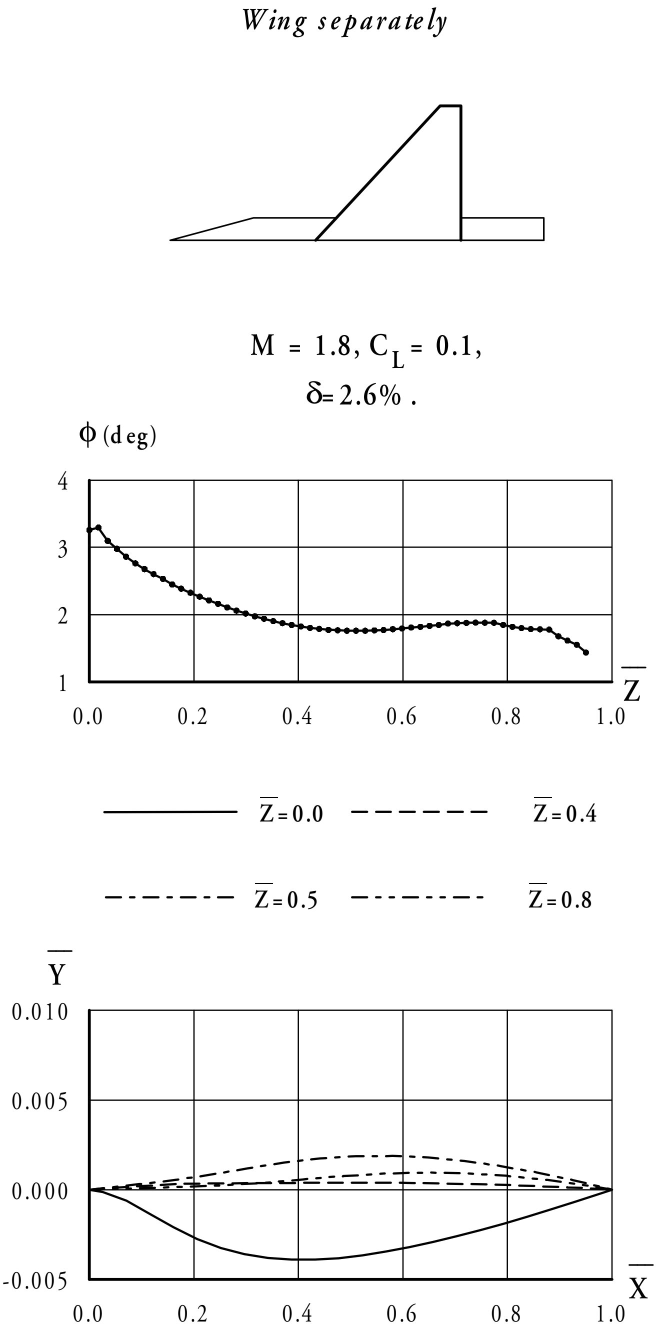
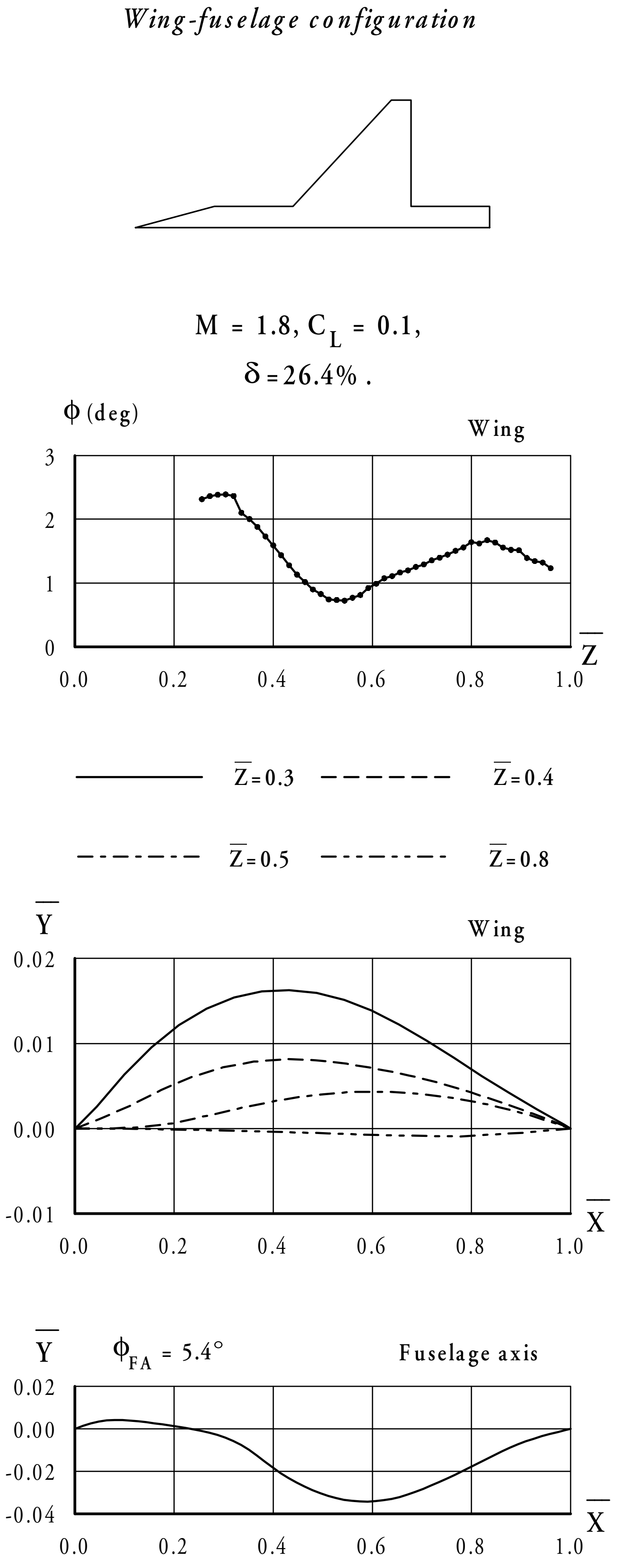


Wing separately
M = 1.8, C_L = 0.1,
δ= 2.6% .
φ (deg)
Z
Z = 0.0
Z = 0.4
Z = 0.5
Z = 0.8
Y
X

Fig. 1



Wing-fuselage configuration
M = 1.8, C_L = 0.1,
δ = 26.4% .
φ (deg)
Wing
Z
Z = 0.3
Z = 0.4
Z = 0.5
Z = 0.8
Y
Wing
X
Y
φ_FA = 5.4°
Fuselage axis
X

Fig. 2

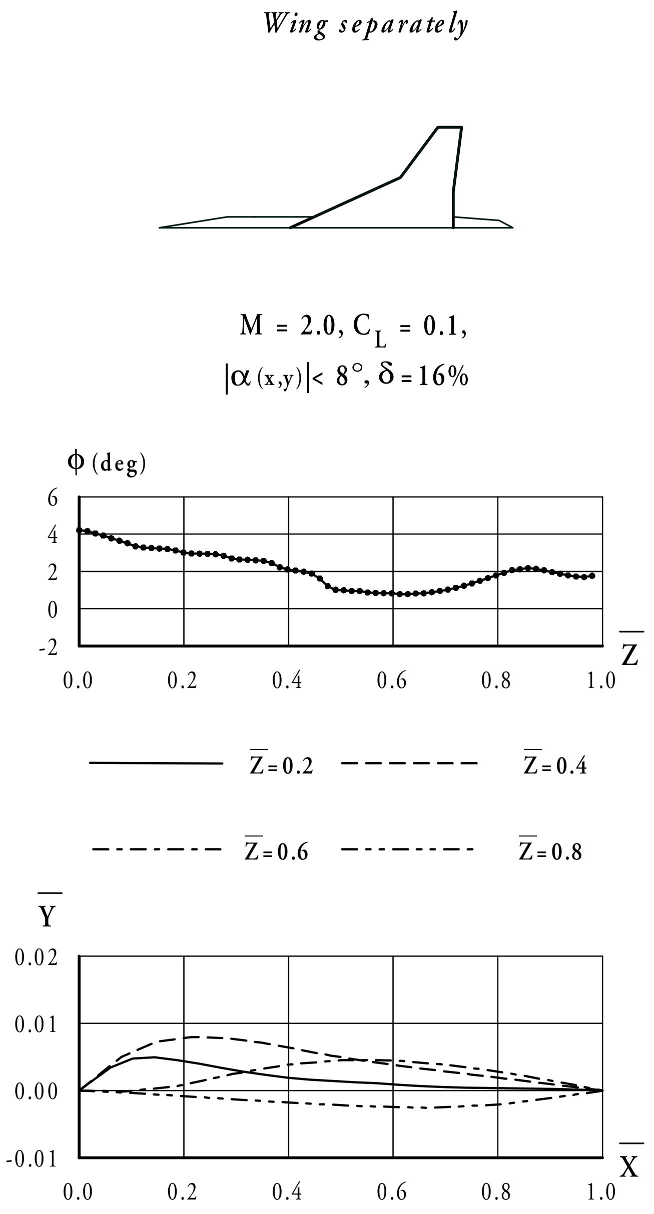

Wing separately
M = 2.0, C_L = 0.1,
|α (x,y)|< 8°, δ = 16%
ϕ (deg)
6
4
2
0
-2
Z
0.0
0.2
0.4
0.6
0.8
1.0
Z = 0.2
Z = 0.4
Z = 0.6
Z = 0.8
Y
0.02
0.01
0.00
-0.01
X

Fig. 3

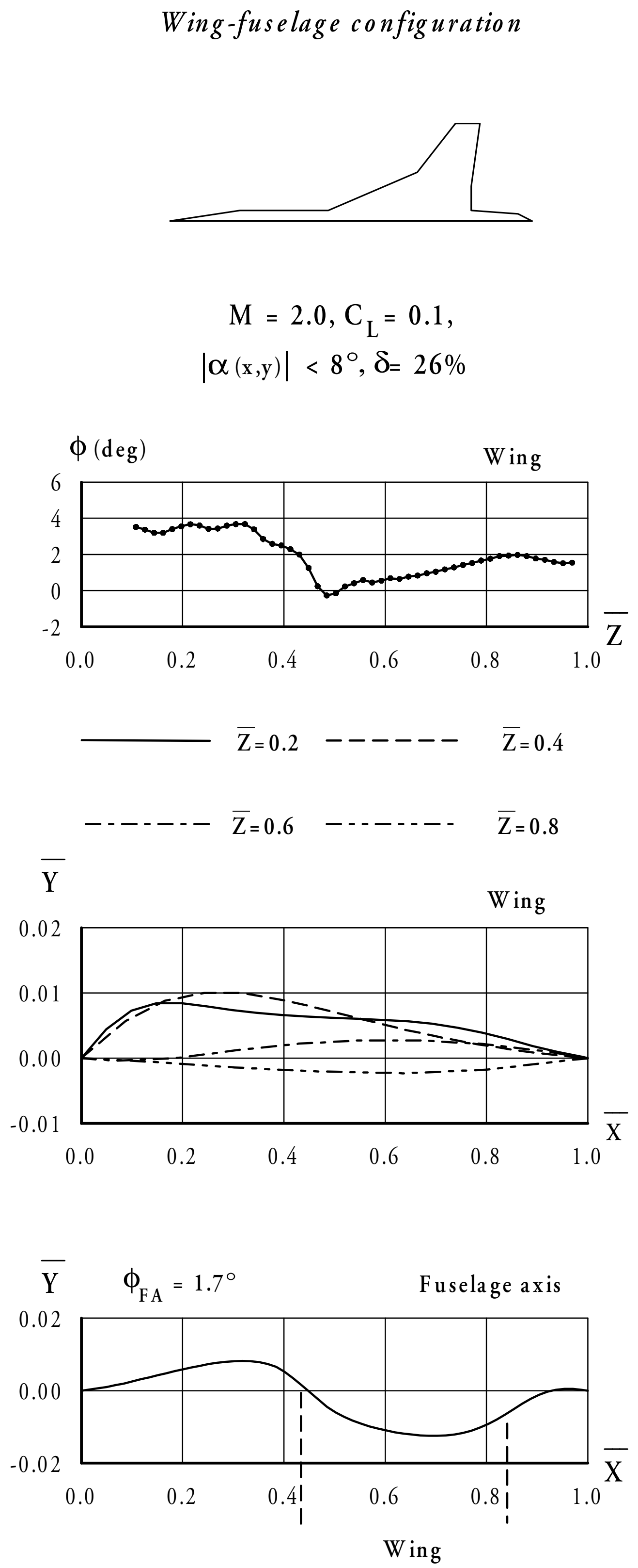
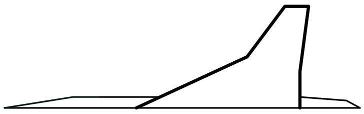
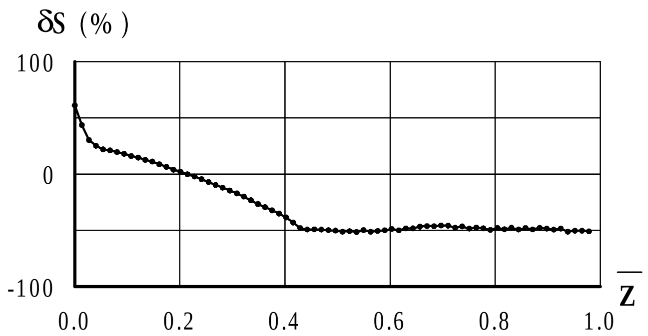

Wing-fuselage configuration
M = 2.0, C_L = 0.1,
|α (x,y)| < 8°, δ= 26%
ϕ (deg)
Wing
6
4
2
0
-2
Z
0.0
0.2
0.4
0.6
0.8
1.0
Z = 0.2
Z = 0.4
Z = 0.6
Z = 0.8
Y
Wing
0.02
0.01
0.00
-0.01
X
Y
ϕ_FA = 1.7°
Fuselage axis
0.02
0.00
-0.02
X
Wing

Fig. 4

Volume restriction



M = 2.0



baseline

Z=0.0 Z=0.3 Z=0.6

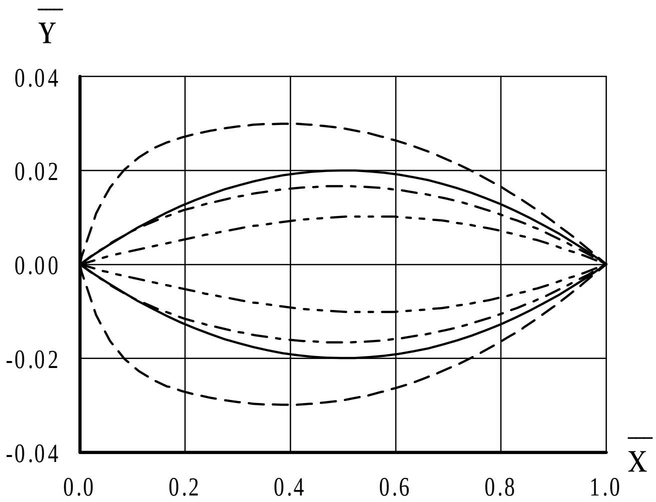
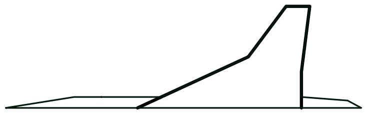


Fig. 5

Cross-section area restriction



M = 2.0

baseline

Z=0.0 Z=0.3 Z=0.6

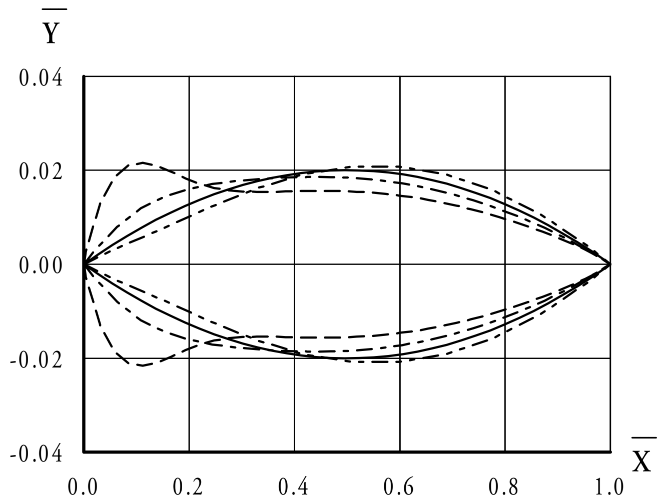


Fig. 6