USE OF NAVIGATION SATELLITE SYSTEMS FOR AIRCRAFT OPERATIONS : SENSITIVITY TO ERRORS

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Abstract

Satellite navigation systems (GPS actually and GNSS in the future) are more and more often called as positioning means and help to aeronautical navigation for all the phases of flight. In geodesy, the use of GPS carrier phase in a differential mode give centimeter precision for localisation. These performances allow to think about a localisation of this type in particular for precision landings.

Carrier phase measurement accuracy is of the millimeter order but this measure presents a constant ambiguity corresponding to a whole number of cycles between the satellite and the receiver. To be converted into a distance satellite-receiver, its associated ambiguity has to be determined. Moreover the measures are affected by signal propagation biases and phase noises. Accuracy of orbits and satellite geometry play also a role in the determination of the position.

By differencing the measures with the ones coming from a reference station, it is possible to reduce significantly these errors but their levels are still important compared to the wavelength of the signal (around 20 cm).

Ambiguity resolution on the fly (AROF) procedures are inspired by methods used in geodesy. For their use on the fly, the short time spans and the necessity of real time do not allow to compensate for all the errors. Depending on the nature and the level of these errors, the methods do not always permit the determination of the good ambiguities, which has an incidence on the integrity and the continuity of service.

We propose to give first a quick review of all those errors. Then we'll study the effects of the most important of them on the positioning given by different methods and we'll check the utility to fix the ambiguities to integer values.

1 Introduction

The accuracy of the position obtained from satellites positioning systems (GPS, GLONASS, Galileo in the future) vary from tens of meters to centimeters according to the receiver capabilities and the need to know the position in real time or not. We are interested in the use of these systems for aeronautical navigation and in particular for precision approaches.

Two types of measurements can be used from the code and the carrier phase. The carrier is modulated by pseudorandom noise (PRN) codes. The code measurement is the difference between the code received from the satellite and the code generated by the receiver, the difference corresponding to the signal transit time from the satellite to the receiver regardless of clocks biases. It provides instantaneous range (pseudorange in fact because of clocks biases) to the satellite. The carrier phase measurement, more precise than the last one, is the difference between the phase of the signal received from the satellite and the phase of the signal generated by the receiver. It provides an accurate estimate of the change in the pseudorange. It is usually not used for the positioning itself but can be related to the signal transit time.

These measurements are affected by biases and errors, some of them being correlated in space or in time. To reduce or even cancel them, it is possible to use corrections sent by a reference station which perfectly knows its position, or directly the measurements of the reference station. The positioning accuracy obtained from GPS after differential corrections is less than 10 m in real time with the code (Differential GPS : DGPS), and two orders less after a few minutes with the carrier phase.

Requirements of civil aviation (accuracy, integrity, alarm treshold...) depend on altitude of decision (see Table 1). CATI requirements for precision approach have been reached recently with DGPS [1]. To reach CATII and CATIII, code measurements are no more sufficient, the carrier phase measurement must be considered. It implies to make in place techniques of ambiguity resolution in real-time, the measurement being known at a whole number of wavelength.

	Horizontal error,95%	Vertical error,95%	Decision height
CATI	18.7 m	5.4 m	200 feet
CATII	6.3 m	2.5 m	100 feet
CATIII	4.6 m	0.8 m	50 feet

Table 1 Requirements in precision of position for precision approaches

Section 2 describes the model for positioning with carrier phase and errors on this measurement. Section 3 presents different ambiguity resolution on-the-fly (AROF) procedures. In section 4 and 5 these procedures are compared on simulations of landings.

2 Positioning with carrier phase, model and errors

In the ideal case of error-free measurements with synchronized clocks, the distance between a satellite and the receiver is equal to the measured fractional cycle plus an unknown number of whole cycles. This number is called the initial integer ambiguity or simply ambiguity and has to be determined. The phase is tracked since the first epoch (see figure 1), keeping the initial ambiguity constant, the change in the phase corresponding to the change in the satellite-user range. The carrier phase measurement can be written in unit of distance

$$\phi_r^s(t) = \phi_r(t) - \phi^s(t-\tau) + \lambda N$$

where $\phi_r(t)$ is the phase of the signal generated by the receiver at time t, $\phi^s(t-\tau)$ is the phase of the signal received from the satellite at t, τ is the signal transit time,λ the wavelength and N the initial ambiguity. Or in term of distance,

$$\phi_r^s(t) = \rho(t, t-\tau) + \lambda N$$

where $\rho(t, t-\tau)$ is the geometric distance between the receiver at time t and the satellite at time $t-\tau$.

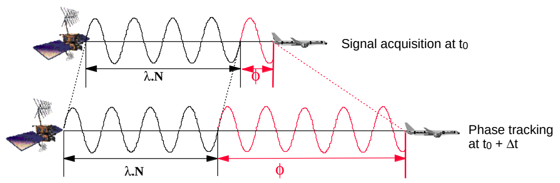

Fig. 1 Relation phase-range

GPS satellites transmit signals at two frequencies but only the $L_1 = 1575.42$ MHz signal is available to civil users. Its corresponding wavelength is $\lambda \simeq 19$ cm. To determine the good ambiguity, the measurement errors have to be lower than this length. We can group these errors and biases by their place of occurence : satellite, propagation medium, receiver. Their detailed description can be found in [2].

-satellite related errors : Errors appear in the satellite position and in the time broadcast in the navigation message. These errors can be intentional as a result of the selective availability (SA) wished by the US government. For authorized users or when the SA is turned off, the ephemeris and time errors can still reach 3m and 5ns rms respectively. The SA should be removed on the

next generation of satellites (blocks IIF) and the remaining errors are expected to be reduced.

-propagation errors : The atmosphere is a refractive medium. This implies changes in direction of propagation of the signal - the signal is no more direct - and variations in the phase speed. Effects depend upon refraction index of the medium and vary greatly following the atmospheric layer. It is generally decomposed in two parts : the ionosphere and the troposphere.
The ionospheric effect is proportional to the total electronic content along the signal path and so depends on the solar activity, the latitude, the local time and the terrestrial magnetic field. It is also inversely proportional to the frequency and can then be determined when two frequencies are available. The ionospheric error can reach 20 m at the zenith and 60 m for a satellite near the horizon. Only 50 % of this error can be corrected by the coefficients of correction broadcast by the satellites.

The tropospheric effect depends upon the temperature, pressure and humidity and can be divided into a dry and a wet components. The zenith delay for the dry component is about 2 m but can be predicted very accurately from surface pressure measurements. The wet component is less important but shows great variability both spatially and temporally and then is hardly predictable.

-receiver related errors : The error due to receiver noise is about 1 m rms for the code and 2 mm for the carrier phase.

The environment at the receiver antenna can affect considerably signals propagation. The most important effect is due to multipath. Multipath corresponds to the same signal arriving to the antenna from different directions with a small delay, the direct signal being reflected. The error can be reduced by multi-correlators or narrow correlation techniques, but can still reach a quarter of wavelength with one reflected ray and the double with two reflected rays.

The receiver clock error is greater but is considered as an unknown and is determined.

Most of the errors and biases are correlated in space and in time. By differencing measurements taken at the same instant from two receivers closed enough ($<$ 20 km), these errors will be greatly reduced. To eliminate the receiver clock error which is common to all satellites, we use also difference of measurements with respect to a reference satellite. We can also apply atmospheric corrections given by models of atmosphere in [2] and [7]. The resulting errors after double differences (difference with respect to a reference station and to a reference satellite) can be estimated in fonction of the baseline length d, distance from the receiver to the reference station (see Table 2).

Source of errors	Level (mm) ,with d in km
orbit	$0.25*$ d
ionospheric refraction	$1.5*$ d
tropospheric refraction	$1*$ d
multipath	<40
receivers noise	<4

Table 2 Levels of resulting noises after double differences

These estimations are valid if the two receivers are well synchronised and if no cycle slip occurs during the positioning. Origins of cycle slips are numerous: obstruction, weak signal for a satellite near the horizon, geomagnetic activity, ionospheric scintillation, multipath . . . If the position is already known, it is possible to detect and correct them, but if they appear during the ambiguity resolution, the procedure would have to be reinitialized, affecting the continuity of service of the system.

The double difference measurement can then be written, in absence of cycle slip:

$$\Delta\nabla\phi(t) = \Delta\nabla\rho(t) + \lambda.\Delta\nabla N + \Delta\nabla\varepsilon_\phi \quad (1)$$

or after linearization with (n+1) visible satellites:

$$\varphi = C.\delta X + \lambda.N + B_\varphi \quad (2)$$

where φ is the n-vector of double differences of measurements minus the estimated range, C the matrix of difference of direction cosines, δX the position error, N the n-vector of double difference of ambiguities, B_Φ the n-vector of resulting noises. The double difference term will be implicit in the rest of the paper.

Ambiguity Resolution On-the-Fly (AROF) procedures are based on this mathematical model and assumed that the noises B_Φ are gaussian, with zero-mean and time independant, which is not the case as we have just seen before.

3 AROF procedures

AROF procedures are used in geodesy since a long time, but this domain of application is very different than precision approach for civil aviation. Geodesists use them in general for static or pseudo-kinematic receivers with long periods of measurements and post process the results which enables to use precise ephemeris and more complex atmospheric models. They make precaution about multipath, and even in multipath environment, the long period allow the errors to compensate themselves. The change in the satellites-receiver geometry playing an important role, the assumption of white gaussian noise is then acceptable in this case.

There exist two families of ambiguity resolution methods, based on either decision or estimation theory, but they all try to do the same thing: find the best intersection in the surfaces of constant ambiguities considering the measurements errors.

3.1 Decision techniques

Decision techniques - Ambiguity Function Method (AFM [3]), Least Square Ambigutity Search Technique (LSAST [4, 5]), Maximum A Posteriory Ambiguity Search (MAPAS [6, 7]) - are based on multiple hypotheses tests. Considering that only four satellites are necessary to determine the position and the clock error, four between the visible ones are chosen. After double differences, intersections of surface of constant ambiguities constitute a grid in the 3-dimensional space, each point of this grid being a possible solution for the receiver position (see figure 2).

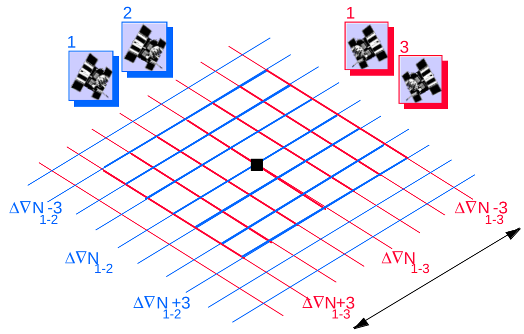

Fig. 2 Ambiguity search space in two dimensions (three satellites)

Those points are then tested over time. In AFM, tests are processed in the physical search space. In the two other methods presented in this paper, the mathematical ambiguity search space is considered.

AFM was developped for static receivers. It can be used for kinematic applications but the computational burden is far worse than the other techniques.

In LSAST and MAPAS, the remaining satellites, which are redundant, are used to accelerate the convergence toward the solution. For each combination of ambiguities defined by the four (primary) satellites, the ambiguities corresponding to the remaining (secondary) satellites are estimated as real values and then rounded to their nearest integer to fulfill the integer constraint. The difference between these two methods comes from the rejection criterion :

- in LSAST, the measurement residuals are bounded by a χ^2-distribution. A primary combination $[abc]$ is then conserved if

$$\| \varphi - \hat{\varphi}_{abc} \| < \chi^2$$

where $\hat{\varphi}_{abc}$ is the n-vector of estimated measurements for $[abc]$.

- MAPAS uses only the predicted measurements of the secondary system. The error of prediction is supposed to have a normal probability density function. The a posteriori probability of a combination knowing the prediction error is then computed and the point is rejected if this probability is lower than a treshold which is progressively increased during time. If the probability is greater than an other predefined treshold, the point is accepted as the good one.

We have tested also an improvement of these two methods by processing a search over the secondary ambiguities, in the ellipsoid defined by their real values and their covariance matrix, rather than just rounding them to their nearest integer.

3.2 Estimation techniques

In estimation techniques - Least square AMBiguity Decorrelation Adjustment (LAMBDA [8, 9]), Fast Ambiguity Search Filter (FASF [10]), Direct integer Ambiguity Search (DIAS[11]), Choleski decomposition ([12]) - there is no separation between primary and secondary satellites. Ambiguities are first estimated by resolving the system (2) with real values, and then a search is performed in the space defined by the real values and their covariance matrix. This ambiguity search space is smaller than for decision techniques but of greater dimension. Moreover system (2) is underdeterminated, a few epochs of measurements will be necessary to inverse it, depending on the number of visible satellites. The change in the user-satellites geometry is slow, to avoid numerical stability problems during the inversion, a larger number of epochs would have to be considered or a larger period of measurement.

Ambiguities are highly correlated and can not be obtained by rounding the real values to their nearest integer independantly from each other. The search can be conducted by fixing the first ambiguity in the interval defined by the real value and its variance. The possible second ambiguities are then calculated, conditioned on the first, and so on until the full vector of integer ambiguities $\hat{N}$ is obtained. The vector $\hat{N}$ which minimize $\| \tilde{N} - \hat{N} \|_{cov\tilde{N}}$, where $\tilde{N}$ and $cov\tilde{N}$ are respectively the vector of real values and its covariance matrix, is chosen as the solution. This search is long in term of computing time. These methods are implemented in a way to reduce it.

- In LAMBDA, a decorrelation step is first applied leaving a smallest ellipsoidal search space, more sphere-like.

4 Description of the simulations

Simulations of landings were performed in order to compare the performance and the sensitivity to errors of LSAST and LAMBDA. LSAST is used without and with the improvement (LSAST and LSAST+ respectively).

We have simulated approaches and landings at Toulouse airport every 2 mn during 24 hours. Planes begin their descent from 1500 ft with a slope of 3^o and a speed of 76 m/s. The reference station is located on the airport at about 8,5 km from the start. Phase measurements are available every 1s which correspond to 113 epochs from the begining to the CATIII treshold.

Different levels of noise were used to consider errors on these measurements : a gaussian noise with standard deviation $\sigma_\varphi = 0.4$ cm for thermal noise of the receivers, $\sigma_\varphi = 1$ cm for tropospheric delay, $\sigma_\varphi = 2$ cm for tropospheric plus ionospheric delays, a gaussian noise with standard deviation $\sigma_\varphi = 2$ cm + a bias of 2 cm corresponding to a multipath generated by a single reflected ray on only one signal.

Different constellations of satellites were also used to estimate the influence of the number of satellites : 24 GPS, 24 GPS + 2 GEO, 24 GPS + 2 GEO + 24 Glonass. The full constellation of Glonass satellites has been considered even if there are only 11 satellites left. The russian federation may be not able to rebuilt its positioning system but this number of satellites should be reached by 2010 with the european system Galileo. The two geostationary satellites are the ones of Egnos, first component of Galileo.

5 Sensitivity to measurement errors

5.1 Time of convergence

In the LAMBDA method, theoretically the resolution is feasible when the system (2) is overdetermined. With the frequency of measurements and the number of visible satellites, it is overdetermined in a few seconds, but the geometry receiver-satellites have not really changed. Direction cosines at each epoch are quasi-identical and the system becomes unstable during the inversion. To avoid these problems of numerical stability, we have decided to wait for 10 times the minimal number of epochs necessary for the inversion. The time of observation is, in all cases, less than 60 s and then lower than the time necessary to reach the CATIII treshold.

In the LSAST method, the time of convergence depends on the rejection criterion and so, on the a priori error. Figures 3 and 4 show the number of points still contained in the primary space with the time of observation, for different levels of noise, for one simulation of approach.

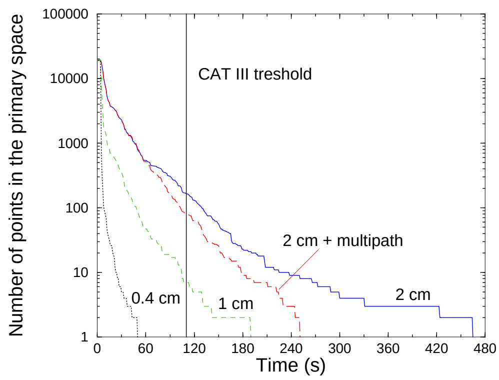

Fig. 3 Time of convergence of LSAST for different levels of noise.

The methods converged before the CATIII treshold only for the measurements with a standard deviation of 0.4 cm. For noises more important, there is still more than one solution. We can chose as solution the one which gives the lower residual of measurements.
In the case of errors with multipath, the convergence is faster than without the multipath, the gaussian noises being the same. In fact, the rejection criterion is not suited to that kind of error and some points have been rejected altough the gaussian part of the noise was low. In the test with LSAST, the exact solution has been rejected and it converged toward a bad solution which would have disappeared if we had waited for a longer time. This is not the case with LSAST+.
On figure 5, we can see the impact of the number of satellites on the convergence, in the case of gaussian noises with multipath (the number of satellites on the figure is the number of visible satellites during this simulation).

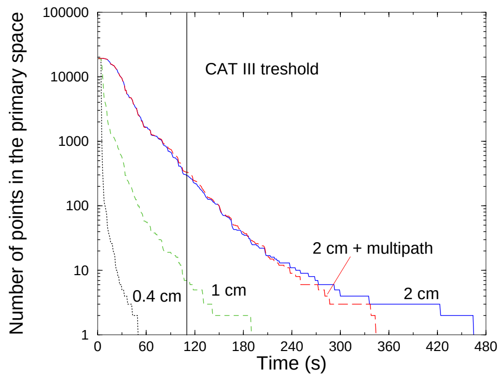

Fig. 4 Time of convergence of LSAST+ for different levels of noise.

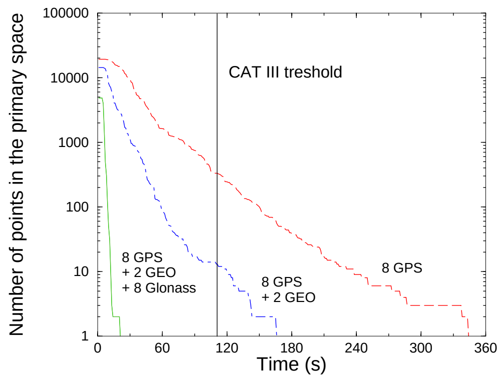

Fig. 5 Time of convergence of LSAST+ for different constellations.

The time of convergence decrease inversely with the number of satellites, this for two reasons.

Firstly, if the number of measurements increase, the precision on the initial position is better, and then, the search space is smaller. Less points have to be tested over time. Secondly, we have more redundant measurements and the bad solutions are rejected more quickly.
With the two geostationary satellites, the time of convergence has been divided by two, and with the GLONASS, this time became lower than the time of observation of the LAMBDA method.

5.2 Availability and success of the procedure

The system is said to be available if we can give a position from carrier phase measurements at the CATIII treshold. The procedure is a success if the good ambiguities have been found, that is, if the position is the good one.
In the LAMBDA method, there is always a solution, the time of convergence being lower than the travel time. In LSAST, we can have no solution if all the points have been rejected.
In tables 3 to 5 are grouped together the results in function of the level of noise (standard deviation), for the three constellations of satellites. For each method, the first line represent the percentage of availability, and the second one, the percentage of success when a solution is available. They have been calculated from the 720 simulations of approach over 24 H.

For the three methods, the success decrease when the noise increase. For a given noise, the most interesting method depends on the satellites constellation considered.

LSAST+ gives results identical to the LSAST ones for weak noises, but is clearly more robust for higher noises, whatever the constellation. When the number of satellites increase, the availability decrease. We can find two reasons to this. First, decision techniques converge more quickly when the number of visible satellites is more important; with GPS we can have considered in some cases that the system was available because there was a solution at the 113th epoch, but this solution would have disappeared after a few minutes. Also, the probability to have an error greater than expected on one measurement is most important.
The success increase with the number of satellites. For the totale constellation, GPS + GEO + GLONASS, when a solution is available, we can be sure that it is the good one, for LSAST as for LSAST+, LSAST+ being more often available.

	0.4 cm	1 cm	2 cm	2 cm + multi
LSAST	100	99.7	99.7	99.6
	100	99.2	82.0	27.6
LSAST+	100	100	100	99.7
	100	99.6	90.8	30.4
LAMBDA	100	100	100	100
	99.4	98.3	84.6	72.1

Table 3 Availability and success of AROF procedures for GPS constellation.

	0.4 cm	1 cm	2 cm	2 cm + multi
LSAST	99.9	99.3	95.3	94.3
	100	100	92.9	77.9
LSAST+	99.9	99.9	99.9	98.6
	100	100	99.9	88.6
LAMBDA	100	100	100	100
	100	100	99.8	99.0

Table 4 Availability & success of AROF procedures for GPS + GEO constellation.

	0.4 cm	1 cm	2 cm	2 cm + multi
LSAST	99.4	97.9	81.4	73.4
	100	100	100	100
LSAST+	99.4	99.3	99.0	97.4
	100	100	100	100
LAMBDA	100	100	100	100
	99.8	99.8	99.8	99.8

Table 5 Availability for GPS + GEO + GLONASS constellation.

If we compare LSAST+ and LAMBDA for the GPS constellation, LSAST+ is more efficient for gaussian noises but much more sensitive to biases correlated in time, as the simulated multipath. Moreover LAMBDA has a time of convergence very lower than to the one of LSAST+.

For the totale constellation, LAMBDA find the good combination of ambiguities most often than LSAST+, but LSAST+ has a lower time of convergence and a better integrity.

To sum up, in a real environment susceptible to create multipath hard to evaluate, receiving signals on only one frequency which allows not to eliminate the ionospheric delays, considering exclusively the GPS constellation, it is better to use an estimation method such as LAMBDA with adding integrity tests. For the totale constellation, GPS + GEO + GLONASS, the LSAST method with improvement is more interesting, it converges more quickly and has in a way its own integrity test, since, in a few seconds, it rejects all the points or keeps the good one.

When the good ambiguities are determined, the position accuracy is lower than 5 cm. If a wrong combination is chosen, we can be at a few meters of the real position.

6 Conclusion

We want to use satellite navigation systems (GPS, Glonass, Galileo) for precision approach and guidance on the taxiway. The precision required makes necessary to use the carrier phase, which implies to resolve ambiguities about twenty centimeters.

Measurement errors being not inconsiderable, it leads us to make differences of measurements with respect to a reference station closed enough, in order to reduce them.

Ambiguity resolution procedures are numerous but they are not properly suited to this situation : short time spans, short wavelength, noises important and badly known.

To evaluate the sensitivity of these procedures to measurements errors, we made simulations of approach. We then compared the results of two methods : an estimation technique LAMBDA, and a decision technique LSAST. The last one has been improved in a way to leave it more robust (LSAST+).

For a weak number of satellites, LAMBDA turned out to be more robust to higher errors and not scheduled but it gives no criterion of integrity. If we considere not only the GPS satellites but all the ones available in the future (geostationaries, Galileo), the number of visible satellites is multiplied by two and LSAST+ becomes faster than LAMBDA. Moreover, in a few seconds, either it finds the good position or reject all the possible points.

The CATIII requirements will be difficult to reach with the GPS alone because of the too important level of noise. Nevertheless, it should be possible with the future GNSS : GPS and Galileo will have two civil frequencies, which will allow to reduce the noise, and there will be twice the number of satellites as now.

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