OPTIMUM DESIGN OF STIFFENED PANEL USING THE METHOD OF MATHEMATICAL PROGRAMMING

Doc. Ing. Antonín Píštěk, CSc.

Ing. Přemysl Hobza

Institute of Aerospace Engineering, Faculty of Mechanical Engineering,
Brno University of Technology, Technická 2, 616 69, Czech Republic

Keywords: *optimization, panel structure*

Abstract

The optimum design problem is one of the inverse tasks, that means the structure parameters are calculated according to a known criteria of the selection, mathematically described as a cost function.

The factor determining the development of optimum design besides the hardware equipment is the developing of the new methods and new mathematical programming algorithms. The optimum design of a skin stringer panel using the method of non-linear programming of Himelblaua [2] is presented in this paper.

The cost function is the mass of the panel. The constrains has to be chosen so that our panel should satisfied the requirements for local and global panel buckling, local and global stringer buckling and it should satisfy as well as the static resistance demands.

1 Design parameters, basic dimensions and the loading of the panel

Optimum design of our panel will be defined by following design parameters:

$t \;\; = X_1$ - skin thickness ,
$n_v = X_2$ - number of stringers , (1)
$t_v \;\; = X_3$ - reference stringer thickness ,
$h_{vc} = X_4$ - reference stringer height,

Panel skin is loaded by the normal stresses σ_x, σ_y and shear stress $\tau_{x,y}$. The following notation will be introduced for the continuous normal edge loading and the shear loading :

$$N_x = \sigma_x \cdot t \,, N_y = \sigma_y \cdot t \,, N_{x,y} = \tau_{xy} \cdot t \qquad (2)$$

Supposing that the stringers are loaded by the force corresponding with the σ_x stress then

$$N_v = F_v \cdot \sigma_x \qquad (3)$$

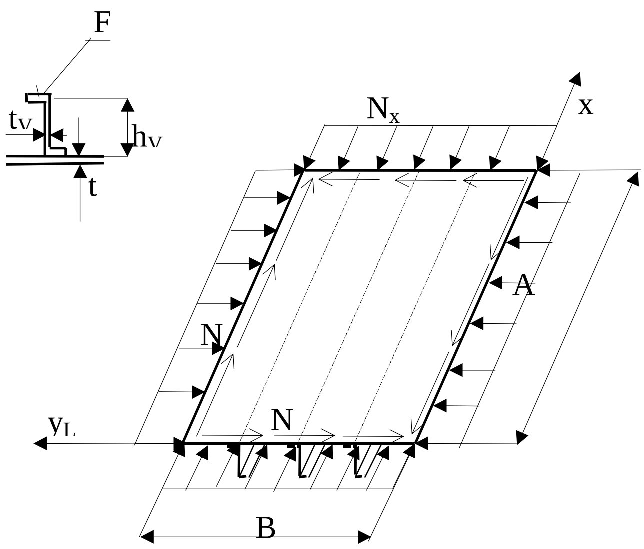

Figure 1. Basic dimensions and the positive orientation of the panel loading.

Continuous loading N_x, N_y a $N_{x,y}$ are constant through the panel edge and the normal stresses generated in stringers are the same. By the stringer thickness we will understand the characteristic dimension of a general section. The real section area and the moments of inertia will be determined using the dimensionless coefficients:

$$F_v = K_{FV} \cdot h_v \cdot t_v, \; I_v = K_I \cdot \frac{1}{12} \cdot t_v \cdot h_v^3 \qquad (4)$$

Coefficients K_{FV} a K_I are for chosen section constant.

For optimum panel design we will suppose the loading which is described on figure 1 and we will consider these panel failures:

1. local skin buckling between two stringers
2. stringer buckling
3. global buckling of the panel
4. reaching the UTS or YTS of the used material

2 Local skin buckling between two stringers

The skin between two stringer as well as whole panel can be loaded by the linear forces N_x, N_y a N_{xy}. The buckling criteria of the local buckling of the skin is described by following equation:

$$\frac{N_x}{t \cdot \sigma^m_{xkr}} + \frac{N_y}{t \cdot \sigma^m_{ykr}} + \left(\frac{N_{xy}}{t \cdot \tau^m_{xykr}}\right)^2 \leq 1, \qquad (5)$$

where $\sigma^m_{xkr}, \sigma^m_{ykr}$ a τ^m_{xykr} are the critical stresses of the local buckling for each single loading The critical stress will be of the following form

$$\sigma^m_{ikr} = k^m_i \cdot \sigma^m_E, \qquad (i = x, y, xy), \qquad (6)$$

where k^m_x, k^m_y, k^m_{xy} are the coefficients of stability a σ^m_E is so called Euler's critical stress which will be of the following form

$$\sigma^m_E = \frac{\pi^2 . E}{12(1-\mu^2)} \cdot \left(\frac{t}{B}\right)^2 \cdot (n_v + 1)^2 = A_1 \cdot t^2 \cdot (n_v + 1)^2. \qquad (7)$$

The stability coefficients k^m_x, k^m_y a k^m_{xy} depend on the edge boundary conditions and dimensions. If the coefficients k^m_i almost does not change and it is possible to consider them as independent on the dimensions $(A \cdot b_v)$ then edge ratio is $A / b_v \rangle 1$ for the considered wing panels.

3 Stringer buckling

By the stringer design parameters t_v a h_v is understood the characteristic stringer dimensions , The stringer can be a general arbitrary section. For dimension computing following assumption will be considered:

1. the normal stringer stress has the same value with σ_x in the skin.
2. the normal stringer stress has to be equal to the critical buckling stringer stress.

$$\sigma_v = \sigma_x \leq \sigma_{vkr}. \qquad (8)$$

Since for the skin and the stringer the condition of the same deformation $(\varepsilon_x = \varepsilon_v)$ is valid, we can obtain the stringer normal stress using just the Hook's law,

$$\sigma_v = E \cdot \varepsilon_x = \frac{N_x}{t} - \mu \frac{N_y}{t}. \qquad (9)$$

Critical stringer buckling stress will be described using the identical form with the critical skin stress

$$\sigma_{vkr} = k_v \cdot \sigma_{vE}, \qquad (10)$$

where

$$\sigma_{vE} = \frac{\pi^2 \cdot E}{12(1-\mu^2)} \cdot \left(\frac{t_v}{h_v}\right)^2. \qquad (11)$$

Stringer coefficient of stability k_v is a constant for chosen stringer.

4 Global buckling of the panel

The global buckling of the panel is such a shape when the big deformation in several half waves of our panel supported by two spars and two ribs arrives. Supposing that the investigated panel is an orthotrophic plate simply supported on each edge. The loading is described in figure 1. The panel is loaded by the continuous normal load and the shear flow on each edge. The reference thickness will be

$$\bar{t} = t + \frac{F_v \cdot n_v}{B} = t + \frac{K_{FV} \cdot n_v \cdot t_v \cdot h_v}{B}. \qquad (12)$$

Similarly like in case of local skin buckling we can use for global buckling the same equation

$$\frac{N_x}{t \cdot \sigma^c_{xkr}} + \frac{N_y}{t \cdot \sigma^c_{ykr}} + \left(\frac{N_{xy}}{t \cdot \tau^c_{xykr}}\right)^2 \leq 1, \qquad (13)$$

where $\sigma^c_{xkr}, \sigma^c_{ykr}$ a τ^c_{xykr} are the critical global buckling stresses for each simple loading. For the applied loads in the stringer direction the differential stability equation becomes to

$$D_1 \frac{\partial^4 \cdot w}{\partial x^4} + 2D \frac{\partial^4 \cdot w}{\partial x^2 \cdot \partial y^2} + D \frac{\partial^4 \cdot w}{\partial y^4} + N_x \frac{\partial^2 \cdot w}{\partial x^2} = 0 \qquad (14)$$

Finally we can obtain the following equation for the global buckling critical stress

$$\sigma^c_{xkr} = \frac{\pi^2 \cdot D_1}{A^2 \cdot t}\left[A + 2\frac{D}{D_1} \cdot \left(\frac{A}{B}\right)^2 + \frac{D}{D_1}\left(\frac{A}{B}\right)^4\right]. \qquad (15)$$

Where:

$$D = \frac{E \cdot t^3}{12(1-\mu^2)} \qquad (16)$$

skin cylindrical stiffness of an unit width,

$$D_1 = D + \frac{E \cdot I}{B} \qquad (17)$$

global longitudinal panel cylindrical stiffness,

and

$$I = I_x - \frac{B \cdot t^3}{12} \qquad (18)$$

I_x is the moment of inertia of the entire section in the centre of the gravity of the panel section.

The differential equation for the critical stress of global buckling of the panel loaded perpendicularly to the stringers becomes

$$D_1 \frac{\partial^4 . w}{\partial x^4} + 2D \frac{\partial^4 . w}{\partial x^2 . \partial y^2} + D \frac{\partial^4 \cdot w}{\partial y^4} + N_y \frac{\partial^2 \cdot w}{\partial y^2} = 0 \qquad (19)$$

and
then

$$\sigma^c_{ykr} = \frac{\pi^2 \cdot D}{A^2 \cdot t} \cdot 2\left(1 + \sqrt{\frac{D_1}{D}}\right) = \frac{\pi^2 \cdot E \cdot t^3}{A^2 \cdot t \cdot 12(1-\mu^2)} \cdot 2\left(1 + \sqrt{\frac{D + \frac{E \cdot I}{B}}{D}}\right) \qquad (20)$$

The critical global buckling stress of the panel loaded by the shear flow is described by the following equation

$$D_1 \frac{\partial^4 \cdot w}{\partial x^4} + 2D \frac{\partial^4 \cdot w}{\partial x^2 \cdot \partial y^2} + D \frac{\partial^4 \cdot w}{\partial y^4} + 2N_{xy} \frac{\partial^2 \cdot w}{\partial x \cdot \partial y} = 0 \qquad (21)$$

For the plates with the considerable anisotropy, when the stiffness in one direction is much higher then the one in other direction, the following equation is valid for the panel loaded with the single shear

$$\tau^c_{xykr} = \frac{2\pi^2 \cdot D\sqrt{1 + n_v \cdot \kappa \cdot \gamma}}{A^2 \cdot t} \cdot \sqrt{4 + 3\sqrt{1 + n_v \cdot \kappa \cdot \gamma} + \frac{1}{\sqrt{1 + n_v \cdot \kappa \cdot \gamma}}} \qquad (22)$$

5 Cost function and the function of the panel constrains from the point of view of the mathematical programming.

For optimum design of a stiffened panel one variant of the Himmelblau and Paviani numerical optimisation method was developed, which is published in literature [2], from where we have also used the modified code developed in FORTRAN. The code is able to find the extreme of the function $W(X)$ in case of the equality constrains

$h_i(X) = 0,\ i = 1,...,m$

and the inequality constrains

$g_i(X) \geq 0,\quad i =$ m+1,...,p,

where $X = (X_1,\dots,X_n)^T$ is the vector of design variables.

The following functions define the problem of the optimum design of a stiffened panel:

Equality constrain is not prescribed, m=0.

Inequality constrains:

1. Local skin buckling between two stringers is given by the equation (5), which becomes:

$$g_1(X) = 1 - \frac{N_x}{A_2 \cdot k_x^m} - \frac{N_y}{A_2 \cdot k_y^m}\left(\frac{N_{xy}}{A_2 . k_{xy}^m}\right)^2,$$

where

$$A_2 = A_1 \cdot \left(X_2 + 1\right) .X_1^3 \text{ and}$$

$$A_1 = \frac{\pi^2 \cdot E}{12\left(1-\mu^2\right)} \cdot \frac{1}{B^2}$$

2. Modifying the equations (8) a (9) we can obtain the following equation for the local stringer buckling

$$g_2(X) = \frac{\mu \cdot N_y}{X_1} - \frac{N_x}{X_1} + A_v\left(\frac{X_3}{X_4}\right)^2,$$

$$A_v = \frac{\pi^2 \cdot E \cdot k_v}{12(1-\mu^2)}.$$

3. The global panel buckling is given by inequality (13), which we modify to obtain:

$$g_3(X) = 1 - \frac{N_x}{S_x} - \frac{N_y}{S_y} - \left(\frac{N_{xy}}{S_{xy}}\right),$$

when

$$S_x = \frac{A \cdot X_1^3\left[\left(1+\alpha^2\right)^2 + G\right]}{\alpha^2\left(1+\tilde{t}\right)},$$

$$S_y = \frac{A_1 \cdot . X_1^3 \cdot 2 \cdot (1+\sqrt{1+G})}{\alpha^2},$$

$$S_{xy} = \frac{A_1 \cdot X_1^3 \cdot 2 \cdot \sqrt[4]{1+G} \cdot \sqrt{4\sqrt{1+G} + 3(1+G) + 1}}{\alpha^2},$$

$$G = \frac{X_2 \cdot K_1 \cdot X_3 \cdot X_4^3 (1-\mu^2).\kappa}{B \cdot X_1^3}$$

$$\tilde{t} = \frac{K_{Fv} \cdot X_3 \cdot X_4 \cdot X_2}{B \cdot X_1}$$

4. The strength is included in the constrain

$$g_4(X) = R_m - \frac{1}{X_1}\sqrt{N_x^2 + N_y^2 - N_x N_y + 3N_{xy}^2}.$$

5. The limitation of the number of stringers is one of the design-technological limitations:

$$g_5(X) = n_{v\max} - X_2,$$

where $n_{v\max}$ engaged maximum number of stringers

6. Limitation of the minimum skin thickness is given by the function:

$$g_6(X) = X_1 - t_{\min}.$$

7. The maximum skin thickness is limited similarly

$$g_7(X) = t_{\max} - X_1.$$

The thickness $t_{\min}$ and $t_{\max}$ are the input data.

8. The stringer height is defined by the function

$$g_8(X) = h_{v\max} - X_4 .$$

9. Limitation in equation

$$g_9(X) = X_3 - X_1$$

assures the regularity of the asymptotic values of coefficients of stability k_x^m.

Than the cost function is obtained as follows

$$W(X) = \rho \cdot (A \cdot B \cdot X_1 + K_{Fv} \cdot X_2 \cdot X_3 \cdot X_4 \cdot A) .$$

6 Examples

As an example we take a wing panel for a small transport aircraft.

The input data are:

A	= 400	[mm]	panel length (rib spacing)
B	= 532	[mm]	panel width
N_x	= 645	[N/mm]	load in the stringer direction
N_y	= 0	[N/mm]	traverse load
N_{xy}	= 75	[N/mm]	shear load
K_{Fv}	= 2,543	[1]	stringer area coefficient
K_1	= 4,926	[1]	stringer moment of inertia coefficient
K_V	= 1,162	[1]	stringer stability coefficient
κ	= 3	[1]	influence of stringer asymmetry
R_m	= 380	[MPa]	allowed tension stress

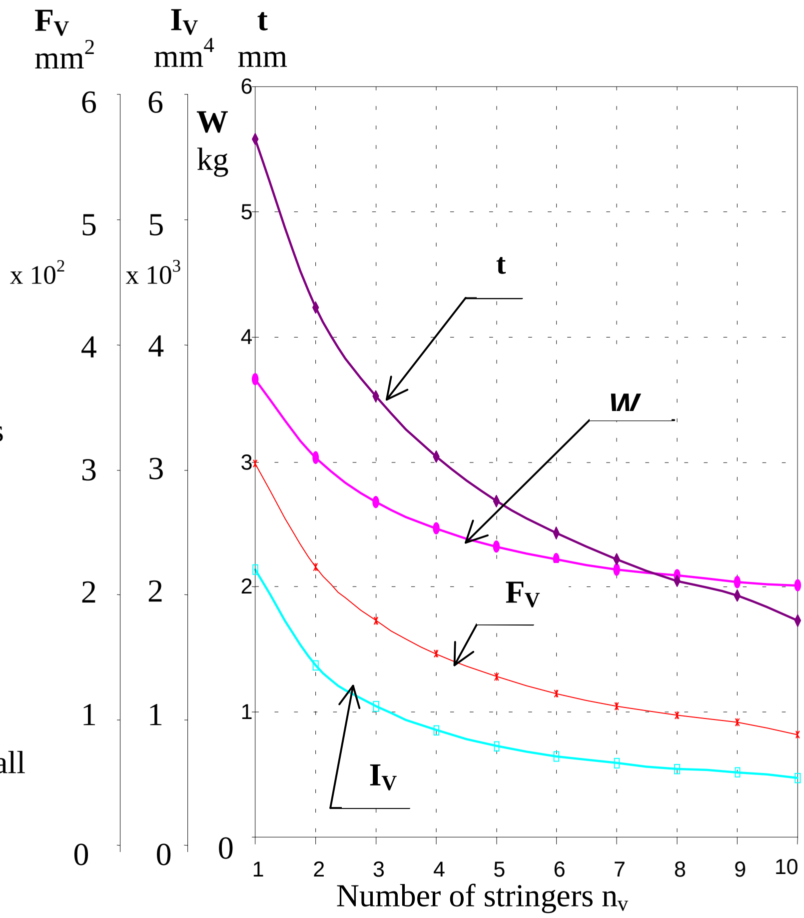

Figure 2.
Relation between the panel optimum dimensions and the number of the stringers.

The characteristics of the Z stringer correspond with the profile PR 105 and the computations from literature [4].

The method presented in this paper was used to design optimum number of stringers for a wing skin-stringer panel using the same input data. Figure 2 shows the results for the different numbers of stringers varying from 1 to 10. For the technologically favorable thickness t = 2.5 mm optimum number of stringers is 6.

Note that all the designs satisfy the redistribution load requirements however mass difference between the optimum panel and one stringer panel is almost double.

7 Conclusion

Application of Mathematical - Programming Method to the optimum structural design of a stiffened panel is attempt to class the optimisation calculation to general structure design using the numerical methods. Despite of some shortage the results are valuable for the designers because they provide them in advance important information about basic design parameters. Based on calculation into which is necessary to add experience and technological special requirements it is possible to choose the wing conception so that the global mass effective structure of the plane could be reached. The calculation enables to follow up design parameters sensitivity of the mass effective skin-stringer structures.

8 References

[1] Lipin,E.K.: Optimalnoje projektorovanije podkreplenych panelej při kombinirovannom nagruženii. Učonnyje zapisky CAGI, IX, 1978.

[2] Himmelblau,D.M.: Prikladnoje nelinejnoje projektirovanije, Mir, Moskva, 1975.

[3] Timoshenko,S.: Theory of Elastic Stability. McGraw-Hill Book Company, New York, 1936.

[4] Píštěk,A.: Kandidátská disertační práce, Brno, 1980.

[5] Birjuk,B.I., Lipin,E.K., Frolov,V.M.: Metody projektirovanija konstrukcij samoletov. Mašinostrojenije, 1977.

[6] Gallagher,R.H., Zienkiewicz,O.C.: Optimum Structural Design, John Wiley and Sons, London, 1973.