NON LINEAR κ-ε TURBULENCE MODELING FOR INDUSTRIAL APPLICATIONS

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Abstract

The non linear eddy viscosity models are good candidates for the simulation of turbulent flows of industrial interest. In fact these models allow to avoid the complexity of the second-order closure models, and to capture more physics than the classical two equation turbulence models based on the Boussinesq hypothesis. Non linear κ-ε turbulence models have been developed by the authors, in the framework of the CEC BRITE-EURAM project AVTAC, and so far validated in case of a transonic flow around an airfoil and a wing mounted in a wind tunnel. The aim of this paper is to assess the behaviour of the developed non linear turbulence models in case of a typical complex 3-D aeronautical application such as the flow around a wing-body configuration.

1 Introduction

The simulation of turbulent industrial flows is currently undertaken by solving the Reynolds Averaged Navier Stokes (RANS) equations.
These equations need a closure to compute an unknown term which stems from the double correlation of the turbulent fluctuations : the Reynolds stress tensor.

$$\tau_{ij} = -\overline{\rho u_i' u_j'} \qquad (1)$$

A set of transport equations for the Reynolds stress tensor components can be directly derived by the Navier Stokes equations. Nevertheless, the difficult numerical handling and the high computational cost required to solve the transport equations system, makes this technique not suitable for the simulation of turbulent engineering flows.
A more used approach is to relate the unknown Reynolds stresses to the known mean flow quantities through a turbulence model.

1.1 Linear eddy viscosity models

The linear eddy viscosity turbulence models are based on the Boussinesq hypothesis which consists of an analogy between a laminar and a turbulent flow. By applying this assumption, the Reynolds stress tensor results linearly related, through the eddy viscosity, to the mean flow strain tensor :

$$\tau_{ij} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} \right) - \frac{2}{3}\rho\kappa \underline{\underline{I}} \qquad (2)$$

where κ is the turbulent kinetic energy and, due to its order of magnitude, is usually neglected in the computation of external flows.
The eddy viscosity μ_t depends on the velocity and the length scale of the turbulent eddies.

The simplest turbulence models are the algebraic models where the eddy viscosty is completely determined in terms of local mean flow variables. A well known and widely used algebraic model is the Baldwin-Lomax model [1].
In the one-equation turbulence models, only one or a combination of the turbulent scales is computed by solving a transport equation. A one-equation model widely used in aeronautical applications is the Spalart-Allmaras model [2].
The two-equations turbulence models are complete in the sense that two transport equations

for both the turbulent scales are solved, and the Reynolds stress tensor can be completely determined from the local state of the mean flow and of the mean turbulent quantities. The velocity scale is chosen to be the square root of the turbulent kinetic energy κ, while the length scale is usually determined from κ and an auxiliary quantity. Examples are the κ-ε models that use the turbulent dissipation rate ε [3], and the κ-ω models that make use of the specific dissipation rate ω [4].

1.2 Non linear eddy viscosity models

The non linear eddy viscosity models allow to take into account important stresses relaxation effects, and to improve the physics of the turbulence models in all the situations where the normal stresses anisotropy plays an important role, but avoiding the numerical complexity of the Reynolds stress transport models.
An anisotropic generalization of the eddy viscosity concept can be achieved considering the constitutive relation 2 as the leading term of a series expansion of functionals.

1.3 Motivations

Non linear κ-ε models have been developed by the authors, in the framework of the CEC BRITE-EURAM project AVTAC [5], coupling the second order Reynolds tensor as provided by Speziale [6] and the second and third order stress-strain relationship by Shih [7] to the Myong and Kasagi [8] κ-ε turbulence model.

These models have been first assessed predicting the transonic flow around the RAE2822 airfoil [9] [10].

The turbulent non linear stresses have shown (fig. 2) to be effective whereas the flow is separated or close to the separation, and to improve the shock-boundary layer interaction and the boundary layer recovery behind the shocks. The third order turbulent stresses have not provided, an appreciable improvement with respect to a second order constitutive relation and the best results have been achieved through the stress-strain relationship derived by Shih.

The same non linear eddy viscosity models, but employing only the constitutive relation by Shih, have been applied to RAE M2155 wing placed in a wind tunnel [9] [11], and the same type of results have been obtained (fig. 2) .

Aim of this paper is to apply the developed non linear models to a wing-body configuration in order to assess the behaviour of the non linear κ-ε models in case of a complex 3-D aeronautical application.

2 Theoretical aspects

The turbulence model used in the computations is made up by the Myong and Kasagi κ-ε [8] model coupled to the constitutive relation developed by Shih [7].

For sake of completeness, the Shih Reynolds stress tensor with second and third order terms is here written :

$$
\begin{aligned}
\tau_{ij} \;=\;& \mu_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right)-\frac{2}{3}\rho\kappa\delta_{ij}\\
&+\frac{A_3}{2}\rho\frac{\kappa^3}{\varepsilon^2}\left(\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}-\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}\right)\\
&+A_5\rho\frac{\kappa^4}{\varepsilon^3}\left[\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_p}\frac{\partial u_p}{\partial x_j}+\frac{\partial u_k}{\partial x_j}\frac{\partial u_k}{\partial x_p}\frac{\partial u_p}{\partial x_i}-\frac{2}{3}\Pi_3\delta_{ij}\right.\\
&-\frac{1}{2}\frac{\partial u_l}{\partial x_l}\left(\frac{\partial u_i}{\partial x_k}\frac{\partial u_k}{\partial x_j}+\frac{\partial u_j}{\partial x_k}\frac{\partial u_k}{\partial x_i}-\frac{2}{3}\Pi_1\delta_{ij}\right)\\
&\left.-\frac{1}{2}\frac{\partial u_l}{\partial x_l}\left(\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}+\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}-\frac{2}{3}\Pi_2\delta_{ij}\right)\right]
\end{aligned}
\tag{3}
$$

where the invariants Π_i are defined as :

$$
\Pi_1=\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}\quad,\quad \Pi_2=\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j} \tag{4}
$$

$$
\Pi_3=\frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_p}\frac{\partial u_p}{\partial x_k}
$$

The functions A_3 and A_5, in front of the quadratic and cubic term respectively, depend on the turbulent variables and on the main strain and rotation

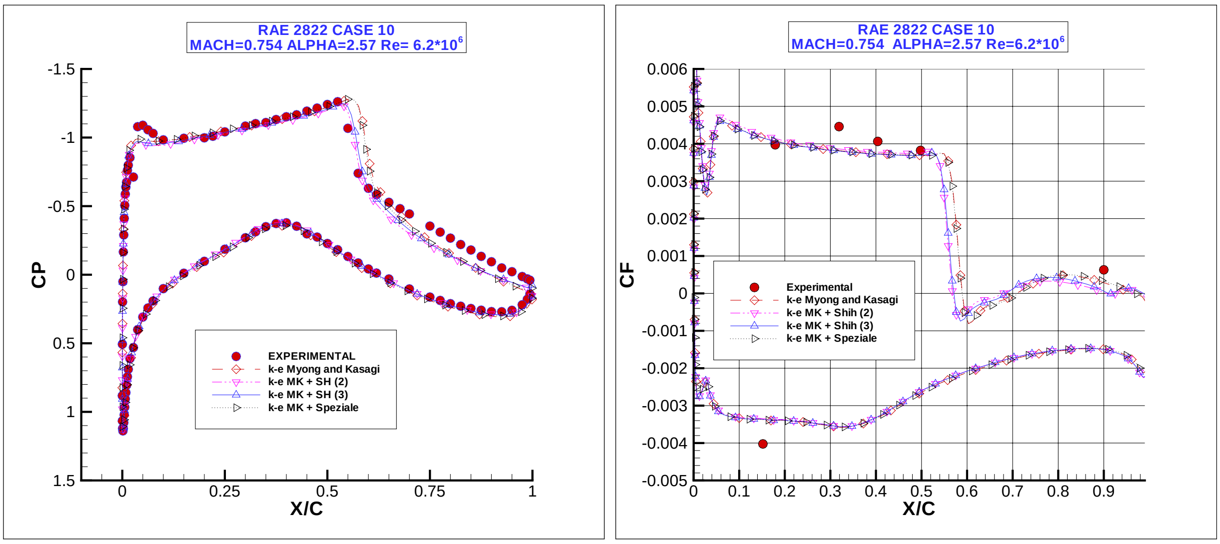

Fig. 1 RAE 2822 Case 10

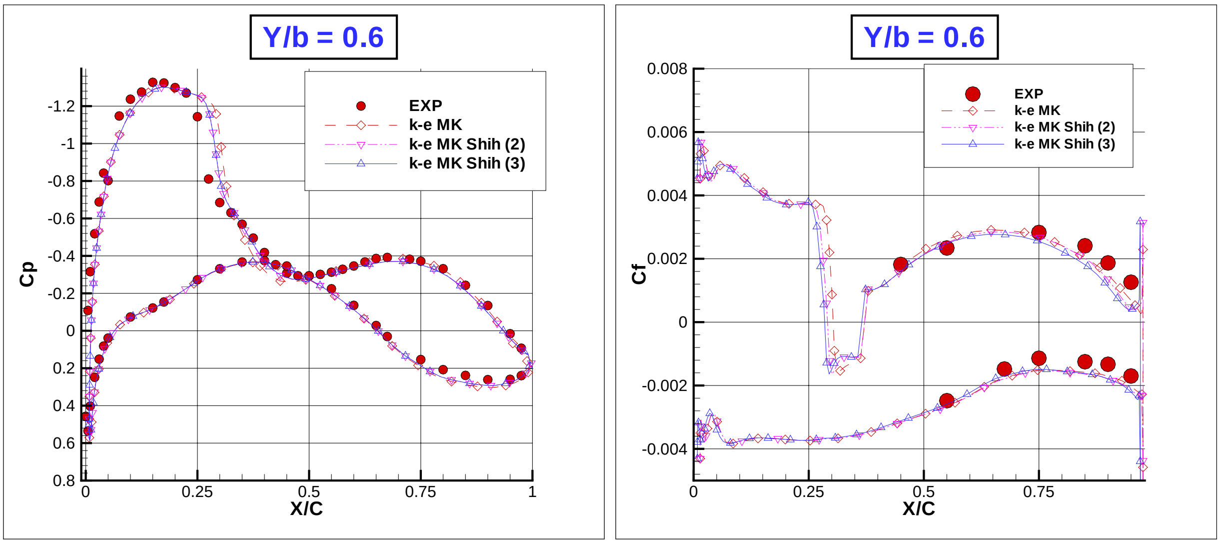

Fig. 2 Wing RAE M2155 - Mach=0.806 α=2.5 Re=$4.1*10^6$

tensor, and ensure the realizability of the model.

$$A_3 = \frac{\sqrt{1-\frac{9}{2}C_\mu^2\left(\frac{\kappa S^*}{\varepsilon}\right)^2}}{0.5+\frac{3}{2}\frac{\kappa^2}{\varepsilon^2}\Omega^* S^*} \qquad (5)$$

$$A_5 = \frac{1.6\mu_t}{\frac{\rho\kappa^4}{\varepsilon^3}\frac{7(S^*)^2+(\Omega^*)^2}{4}}$$

with

$$S^* = \sqrt{S^*_{ij}S^*_{ij}} \quad , \quad S^*_{ij} = S_{ij} - \frac{1}{3}S_{kk}\delta_{ij} \qquad (6)$$

and

$$\Omega^* = \sqrt{\Omega_{ij}\Omega_{ij}} \quad , \quad \Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \qquad (7)$$

It is worth to note that C_μ is not a constant but a function of κ, ε, and of the main strain and rotation tensor :

$$C_\mu = \frac{1}{4+A_s U^* \frac{\kappa}{\varepsilon}} \qquad (8)$$

where

$$A_s = \sqrt{6}\cos\phi \quad , \quad \phi = \frac{1}{3}\arccos(\sqrt{6}W^*) \tag{9}$$

with

$$W^* = \frac{S^*_{ij}S^*_{jk}S^*_{ki}}{(S^*)^3} \tag{10}$$

and

$$U^* = \sqrt{(S^*)^2 + (\Omega^*)^2} \tag{11}$$

2.1 Implementation issues

The turbulent stresses impact the diffusive term of the RANS momentum equations and the production term of the turbulent kinetic energy of the κ-ε equations. Both the terms need to be modified to take into account the non linear part of the stress-strain relationship 3.

The momentum Navier-Stokes equations can be written as :

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} + \frac{\partial p}{\partial x_i} = \frac{\partial T_{ij}}{\partial x_j} \tag{12}$$

with

$$T_{ij} = t_{ij} + \tau_{ij} \tag{13}$$

where t_{ij} is the laminar stress tensor :

$$t_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right) \tag{14}$$

and τ_{ij} is made up of a linear part τ^l_{ij} (first row of the expression 3) and of a non linear part τ^{nl}_{ij}.
By using the finite volumes approach the equation 12 becomes :

$$\frac{\partial}{\partial t}\int_\Omega \rho u_i dV + \int_{\partial\Omega} \rho u_i u_j n_j dS + \int_{\partial\Omega} p n_i dS = \int_{\partial\Omega}(t_{ij} + \tau^l_{ij})n_j dS + \int_\Omega \frac{\partial \tau^{nl}_{ij}}{\partial x_j} dV \tag{15}$$

In the above equation the Gauss theorem has been applied only to the terms containing the termodynamic pressure and the laminar and linear turbulent stress tensor, while the non linear turbulent stress tensor has been treated as a source term.

The κ-ε equations do not require any particular treatment; the production of the turbulent kinetic energy can be simply modified as follows :

$$P_\kappa = \tau_{ij}\frac{\partial u_i}{\partial x_j} = \tau^l_{ij}\frac{\partial u_i}{\partial x_j} + \tau^{nl}_{ij}\frac{\partial u_i}{\partial x_j} \tag{16}$$

3 Numerical results

In order to assess the behaviour of the model in case of a 3-D industrial application, the Myong and Kasagi κ-ε model together with the linear, and 2^{nd} order constitutive relation 3 has been applied to predict the flow at $Mach = 0.80$, $\alpha = 2.2$, and $Re/m = 10.6^6$ around the AS28 wing-body configuration.

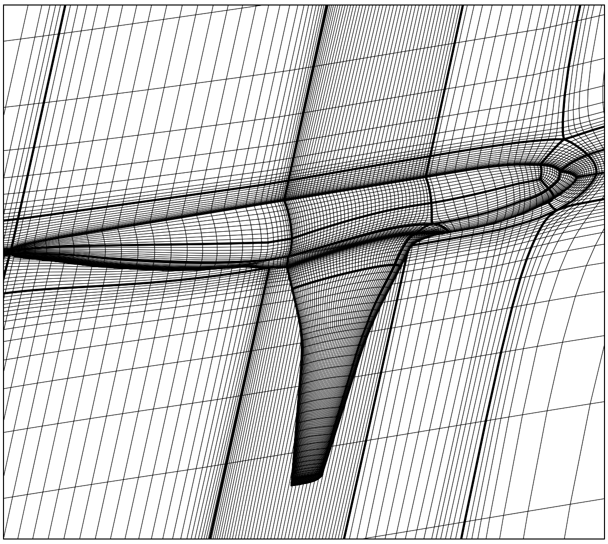

Fig. 3 AS28WB - Grid -

The grid (fig. 3) has been provided, in the framework of the project AVATC, by BAE and has about 5 million cells.

The numerical results, in terms of pressure and friction coefficients at several stations along the wing span, and velocity profiles are shown in the figures 4 - 9.

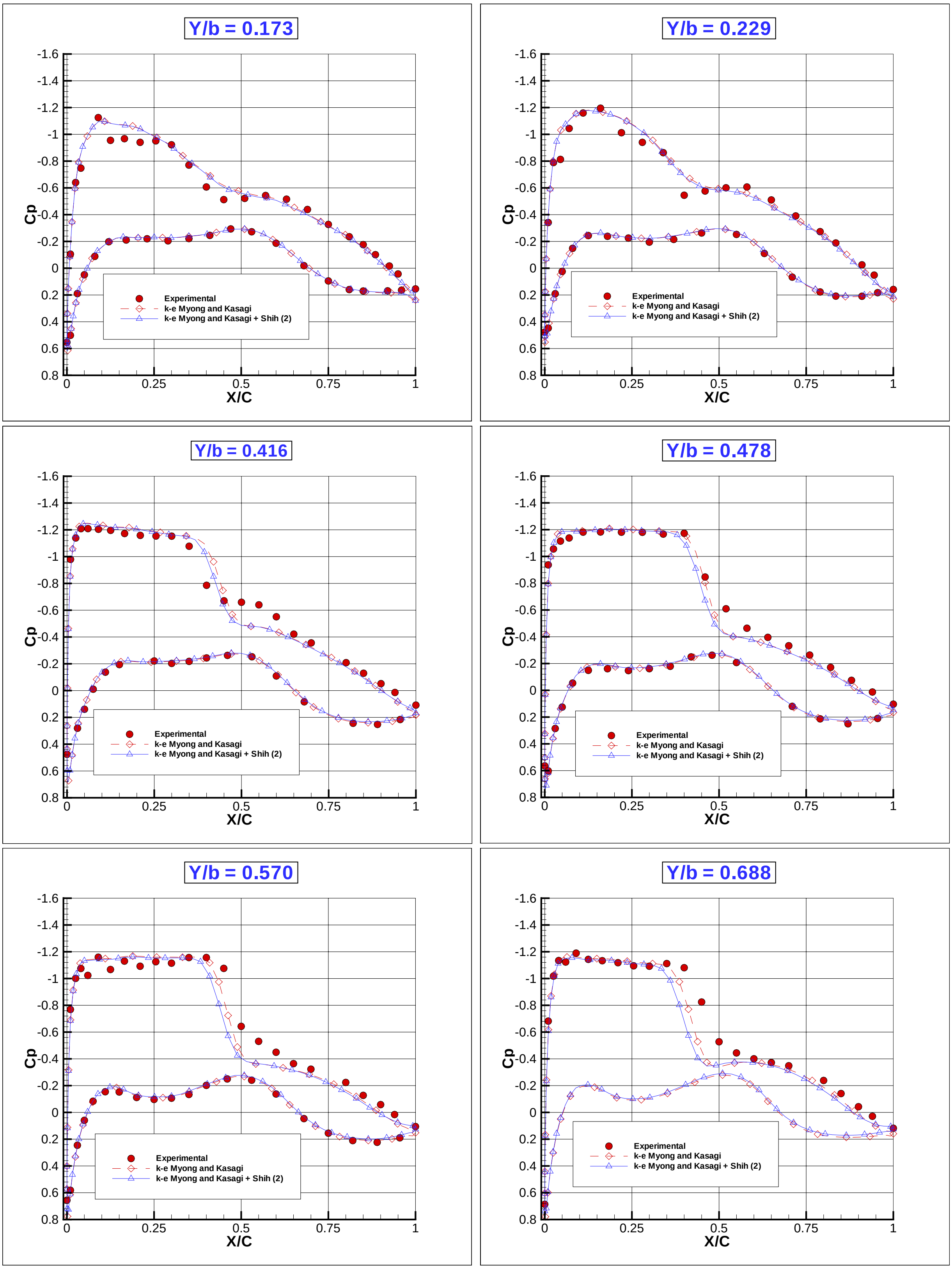

Fig. 4 AS28WB - Pressure Coefficients -

The agreement between the computed and the experimental pressure coefficients, keeping into

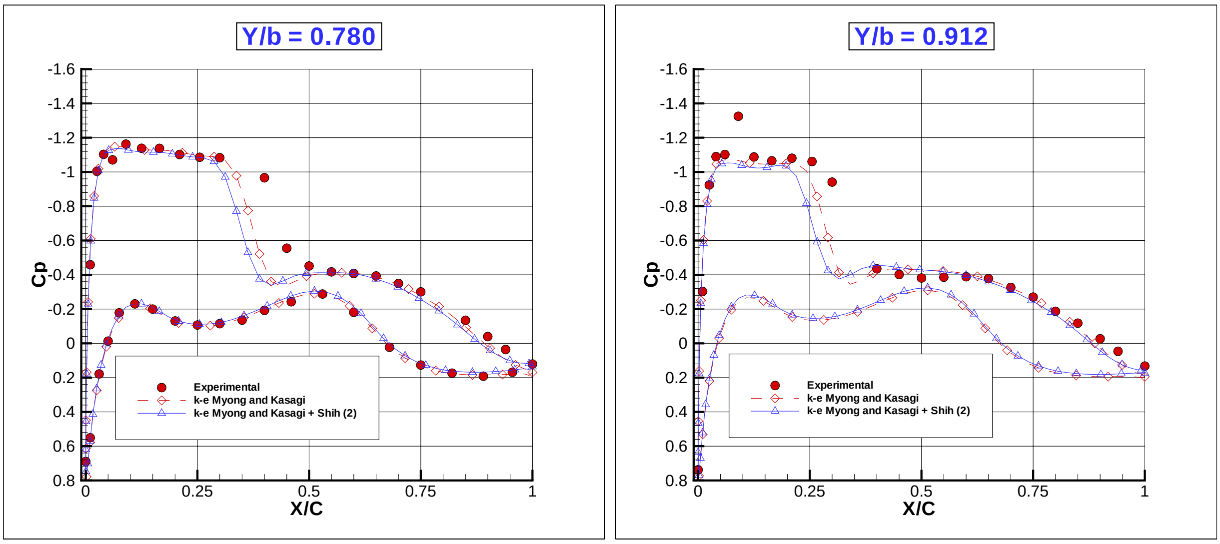

Fig. 5 AS28WB - Pressure Coefficients -

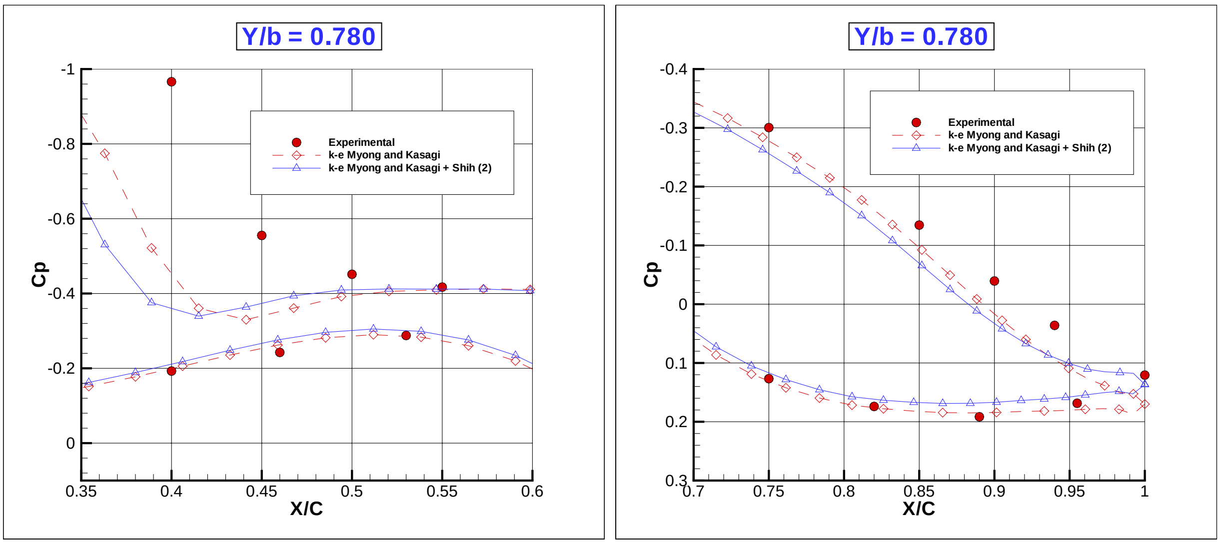

Fig. 6 AS28WB - Pressure Coefficients (zoom at the station $y/b = 0.780$) -

account that the lift coefficient is not matched, is good.
There are not experimental data available for the friction coefficients, but however the data compare well with the other numerical results achieved in the AVTAC project.

The non linear turbulent stresses show their effectiveness, improving the pressure recovery behind the shock and at the trailing edge, in the outer region of the wing whereas the shock-boudary layer interaction is stronger (fig. 6).
Since the fluid dynamic conditions are not severe, and the flow on the wing is everywhere attached the effect of the non linear model can be only seen in the details of the flow field (fig. 9).

4 Conclusions

Non linear eddy viscosity turbulence models have been developed by the authors in the framework of the CEC BRITE-EURAM project AVTAC, and so far validated by predicting the transonic flow around the RAE2822 airfoil and the wing RAE M2155 placed in a wind tunnel.

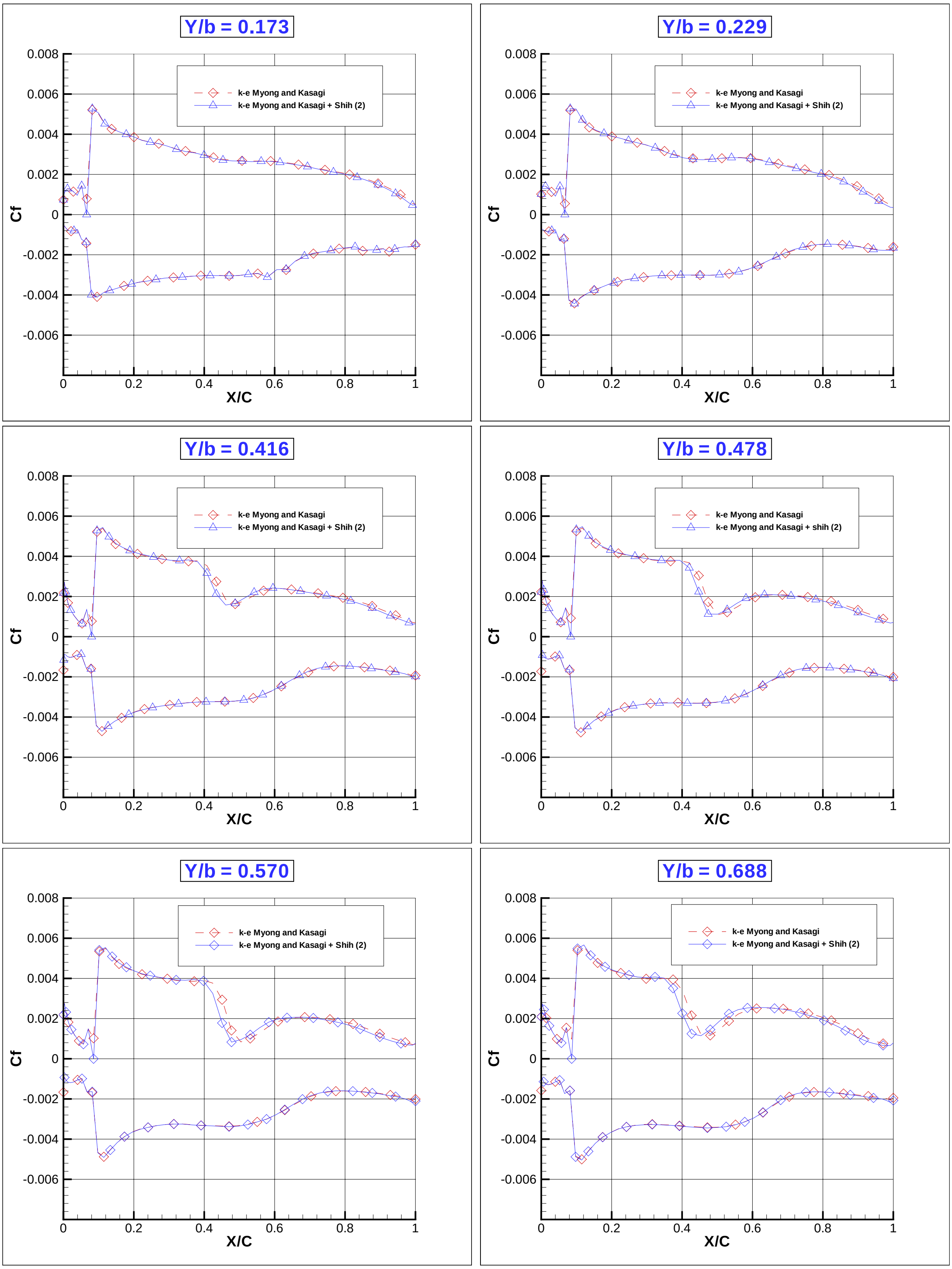

Fig. 7 AS28WB - Friction Coefficients -

The behaviour of these models has now been assessed for the flow around the AS28 wing-body

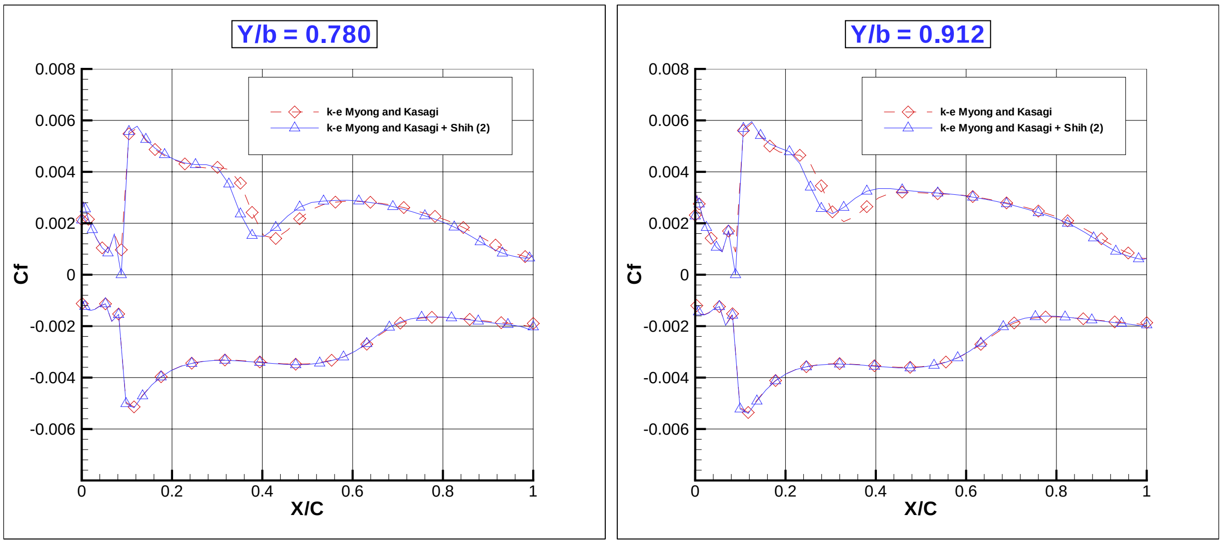

Fig. 8 AS28WB - Friction Coefficients -

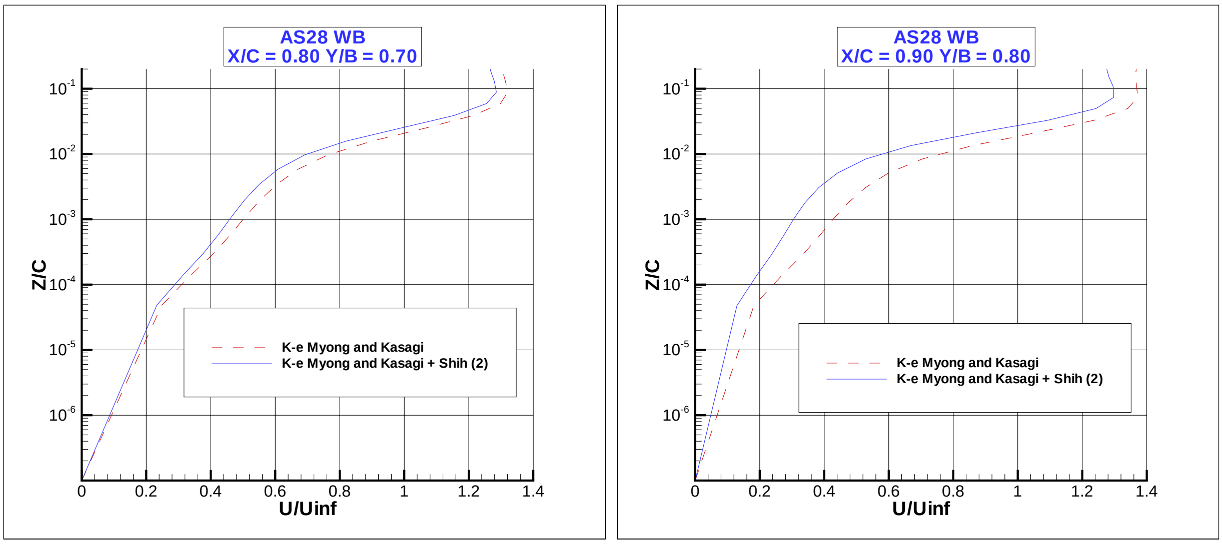

Fig. 9 AS28WB - Velocity profiles -

configuration.

The main features of the non linear models have been confirmed, and the pressure recovery behind the shock and at the trailing edge has been improved in the outer region of the wing whereas the shock-boundary layer interaction is stronger. However for engineeristic applications of external aerodynamics and for attached flows when the details of the flow field are not very important, the improvement due to the use of non linear models seems to be negligible.

The non linear eddy viscosity models will be tested by the authors in case of flows in more severe fluid dynamic conditions, such as high lift performances of airfoils and wings, and air intake internal flows.

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