AERODYNAMICAL AND DYNAMICAL INVESTIGATION OF HELICOPTER ROTORS

Tamás Gausz
Department of Aircraft and Ships
Technical University of Budapest

Keywords: *helicopter, rotor, aerodynamics, dynamics, elasticity*

Abstract

The present article deals with rotor dynamics and aerodynamics. The combined blade-element and momentum theory and the ONERA semi empirical model were applied in the aerodynamical calculations. The unsteady and compressibility effects can be calculated by using the ONERA semi empirical aerodynamic model. The equation of the flapping motion was solved by applying the induced velocity field. So the generalised equilibrium path of the rotor blades was approximated by an asymptotic solution.

The equations of the rotor blade motion were solved in linear and non-linear case too. Also the equation of the feathering was solved in order to find the required control moment of the rotor blades.

1 Introduction

In this article was assumed that the helicopter flies horizontally with constant speed. The aim is approximate determination the effects on the rotor blade of the unsteadiness and compressibility, blade bending deflections and mechanical non-linearities.

In order to determine the forces acting on the helicopter rotor we should investigate the rigid motion and the elastic deflections of the rotor blades, the flow over the rotor disk and the aerodynamic forces on the rotor blades together.

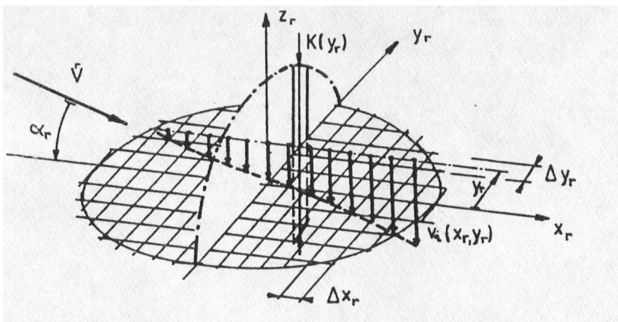
The aerodynamical basic of the calculation is the combination of the blade-element and the momentum theory. Using them with the semi-empirical ONERA model the induced velocity distribution and the unsteady-compressibility effects can be determined.

The motion of the rotor blades is calculated on the ground of the classical differential equation of the flapping motion and also of the full non-linear differential equation too. The generalised equilibrium state of the rotor blade can be calculated by successive approximation.

This calculation method was applied to the main rotor of the Hughes MD 500 E helicopter. This helicopter type flies in Hungary and some investigations to determine the rotor blade loads were done [6].

2 The Momentum Theory

The classical momentum theory was developed by Glauert. In this a distinct stream tube with a circular cross section was defined - the radius of the circle is equal to the rotor radius.



Airflow over the rotor disk
Fig. 1.

In this article the author applies a better estimation: the cross section of the stream tube is an ellipse which can be determined on the ground of the advanced vortex theory of the wings [8]. The cross section - shown in figure 1. - for the Hughes MD 500 can be characterised by $K(y_r)$ function:

$$K(y_r) = 3.3053\left[1-\left(\frac{y_r}{R}\right)^2\right]^{1/12} \qquad (1)$$

where:

R - the rotor radius.

The helicopter rotor acts on the airflow continuously, namely the induced velocity continuously changes - in general increases - from the "leading edge" along the rotor surface. The induced velocity (v_i) at a given y_r co-ordinate - according to the figure 1. - can be calculated:

$$v_i(x_r, y_r) = \int_{x_{le}}^{x_r} \frac{p(x_r, y_r)}{2\rho\, V_r K(y_r)}\, dx_r\,; \qquad (2)$$

where:

$p(x_r, y_r)$ - is the load distribution over the rotor disk;

V_r - is the flying velocity.

3 Blade Element Theory

The blade element theory was used with combination of the momentum theory. The first one is very suitable for applying of the ONERA semi-empirical model. To apply the blade element theory we should know the rotor blade velocities:

$$U_t = \Omega[x_l \cos(\beta) + e] + V_r \cos(\alpha)\sin(\psi);$$

and

$$U_p = V_p + x_l \Omega \beta' + v_i + v_{elastic}\,;$$

where:

$$V_p = V_r \cos(\alpha)\cos(\psi)\sin(\beta) + \sin(\alpha)\cos(\beta);$$

and

α - is the angle of attack of the rotor;

β - is the flapping angle;

ψ - is the azimuth angle.

The values of steady lift- and drag-coefficients for NACA 0015 profile are given in [10]. In linear case according to [1] and [3] the unsteady part of the lift coefficient can be calculated:

$$\dot{c}_L = \lambda \dot{c} + \lambda s \dot{\Theta} + \sigma\left(\dot{\Theta} + \dot{c}\right) + s\,\ddot{\Theta}\,; \qquad (3)$$

where:

$\dot{c}$ - is the translation velocity of the section;

$\dot{\Theta}$ - is the rotational speed of the section.

The coefficients of the ONERA model are valid for the NACA 0012 profile, we can assume, that in this meaning the difference between the 0012 and the 0015 profiles is small.

After the estimation of the velocities the force coefficients can be calculated. From the equation (3) the time derivative of the lift coefficient can be determined. In the numerical calculations the total value of the lift coefficient will be taken as the sum of the steady value and its changing due to unsteadiness.

4 The Flapping and Feathering Motion of the Rotor Blades

The motion of the rotor blade was investigated in a coordinate system that was fixed to the rigid rotor blade. This system has three possible rotations (the lagging motion was neglected):

- together with the main shaft (Ω);
- along the flapping hinge (β);
- along the length axis of the blade (ϑ).

The equations of the flapping motion was derived by using with the above mentioned transformations. The full (non-linear) flapping equation:

$$a_0\beta'' - a_1\beta' + a_2 = \frac{M_a}{\Theta_y\Omega^2} + \varepsilon \sin(\beta)\cos(\vartheta); \tag{4}$$

where:

$a_0 = \cos(\vartheta);$

$a_1 = 2\sin(\beta)\sin(\vartheta);$

$a_2 = \sin(\beta)\cos(\beta)\cos(\vartheta);$

$\varepsilon = \dfrac{m\, x_{cg} e}{\Theta_y\Omega^2};$

m - the mass of the rotor blade;
x_{cg} - coordinate of the blade centre of gravity;
e - flapping hinge offset;
M_a - the aerodynamical moment;
M_g - the moment of the blade weight.

After some simplifications we get the classical differential equation of the rotor blade flapping:

$$\beta'' + (1+\varepsilon)\beta = \frac{M_a}{\Theta_y\Omega^2}.$$

The differential equation of the feathering motion was derived to calculate the control moment of the rotor blades:

$$M_x = \Theta_x\Omega^2\left(q_0 + \beta'(q_1 - q_2\beta') + \frac{d^2\vartheta}{d\psi^2}\right); \tag{5}$$

where:

$q_0 = \cos^2(\beta)\sin(\vartheta)\cos(\vartheta);$

$q_1 = \cos(\beta)\cos^2(\vartheta) - \cos(\beta)\left(1+\sin^2(\vartheta)\right);$

$q_2 = \sin(\vartheta)\cos(\vartheta);$

In calculation above we assumed that the aerodynamical moment is zero.

5 The Bending Deformation and Equations

The flapwise bending deformation is an important form of the rotor blade motions. This type of motion was investigated by using the normal modes [2]. There was used the first normal mode (equivalent with the rigid blade) and additionally the second and third normal modes ($S_i(x);\ \ i = 1,2,3$).

The bending stiffness was determined on the ground of measuring [6]. The second and third normal modes were calculated by the „method of assumed modes” [2]. For the calculation of the bending deflections the next two modal equations can be written:

$$\varphi_i'' + \lambda^2\varphi_i = \frac{Q_i}{\Omega^2 R^2 m_i}; \qquad i = 2, 3; \tag{6}$$

where:

φ_i - the i[th] generalised co-ordinate;
$\lambda_i\Omega$ - the i[th] natural frequency;

$$Q_i = \int_0^L p_b(x)\, S_i(x)\, dx;$$

and

p_b is the external load along the rotor blade.

The modal equations were integrated together with the other differential equations.

6 The Collective and Cyclic Control

The most important control of the helicopters is the collective and cyclic control of the blade pitch. This control is characterised by the following equation:

$$p = p_0 + p_1\cos(\psi) + p_2\sin(\psi); \tag{7}$$

where:

p_0 - is the collective control coefficient;
p_1 - is the cyclic control coefficient;
p_2 - is the cyclic control coefficient.

The feathering motion is connected with the flapping motion: the flapping motion causes feathering motion too by which the original flapping motion is damped. The relationship can be characterised by the equation:

$$\sin(\vartheta) = \frac{p}{f} + \frac{r_0 - e}{f} tg(\beta); \qquad (8)$$

or in the simplest linear form:

$$\vartheta = \frac{p}{f} + \frac{r_0 - e}{f} \beta; \qquad (8/a)$$

where:

r_0 and f – are geometrical parameters of the control system.

The equation (8/a) can be applied for linear calculation and equation (8) in the non-linear case.

7 Description of the Numerical Computation

The numerical calculation was done for the main rotor of Hughes MD 500E. For the realising of this procedure a general computer code was developed.

First the program reads the geometrical, aerodynamical and further data. The second step is the computation of the preliminary induced velocity distribution on the ground of the Glauert's approximation.

Then the program numerically integrates the differential equations of the flapping and bending motions during one revolution. In the calculation of the flapping motion and bending deflections there are included the unsteady-compressible lift coefficient and the equation of the flapping-feathering connection. This calculation uses the polar coordinate system.

The force distribution over the rotor surface is known - the corresponding (new) induced velocity distribution can be calculated in a Descartes coordinate system.

On the ground of these calculations - in order to investigate the equilibrium state of the helicopter - can be determined the horizontal, side and trust force of the main rotor.

After these steps the program goes back to the flapping calculation - while the rotor blade turns to the generalised equilibrium state. This can be realised practically after 10-15 revolutions.

In this point we have possibility to investigate the equilibrium of the whole helicopter. If the equilibrium state is not reached, then the parameters of the collective and pitch angle control of the rotor blades (p_0, p_1, p_2) can be changed.

At the end of this procedure we can get the asymptotic solution of the rotor blade motion and the induced velocity distribution.

8 Results of the Computation

In the present calculation run three variations of the general program:

- rigid rotor blade, steady lift and linear system (RSL system);
- elastic rotor blade, unsteady lift and linear system (EUL system);
- elastic rotor blade, unsteady lift and non-linear system (EUN system).

In all cases was applied the same set of control parameters, on this ground there was possible the comparison of the results of the different program variants.

8.1 Calculation of the Forces

The forces (T - Thrust, H - Horizontal, S - Side force) are summarised in the following table:

Table 1.

	T [N]	H [N]	S [N]
RSL	12018	301	-155
EUL	12092	360	-143
EUN	11957	363	-142

Changing in thrust is smaller than 1% - it is quit small. Changing in side force is bigger - approximately 8% - and the greatest is the change of the horizontal force, it is approximately 17%. This means, that the thrust is practically constant and the inclination of the rotor has change.

8.2 The Flapping Angle

The flapping angle characterizes only the rigid blade motion, but in the elastic cases the path of the rotor blade tip is much more interesting. The flapping angles are shown in figure 2.

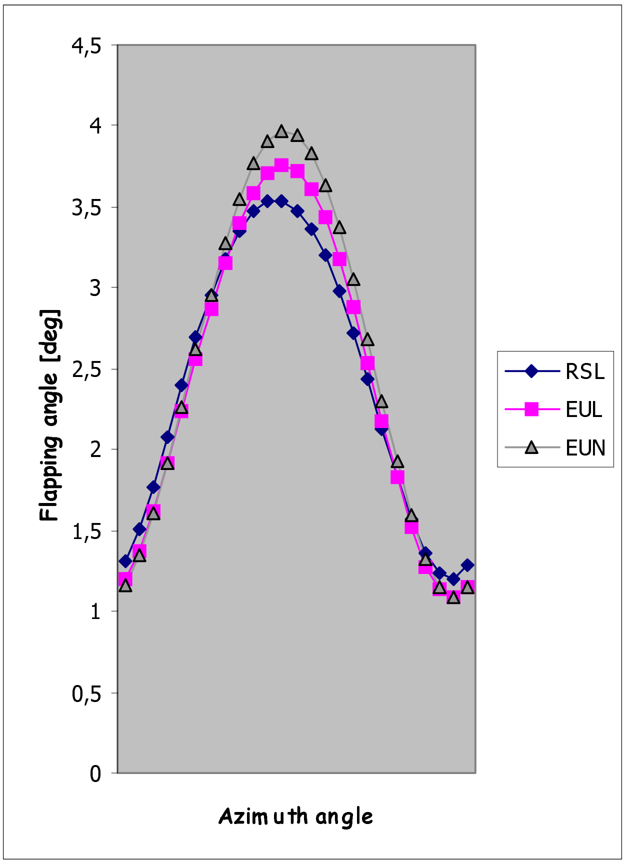


Figure 2.

As it shown in figure 2. the flapping angles are similar - the calculation gives the biggest results in the EUN case. It means, that in the non-linear case the coning angle has greater values but the longitudinal and the lateral flapping has only small changes.

8.3 The Tip Path

The tip path of the rotor blades is much more characteristic for the rotor blade motion because it include the elastic deflections. The three tip path of the rotor blades can be shown in figure 3.:

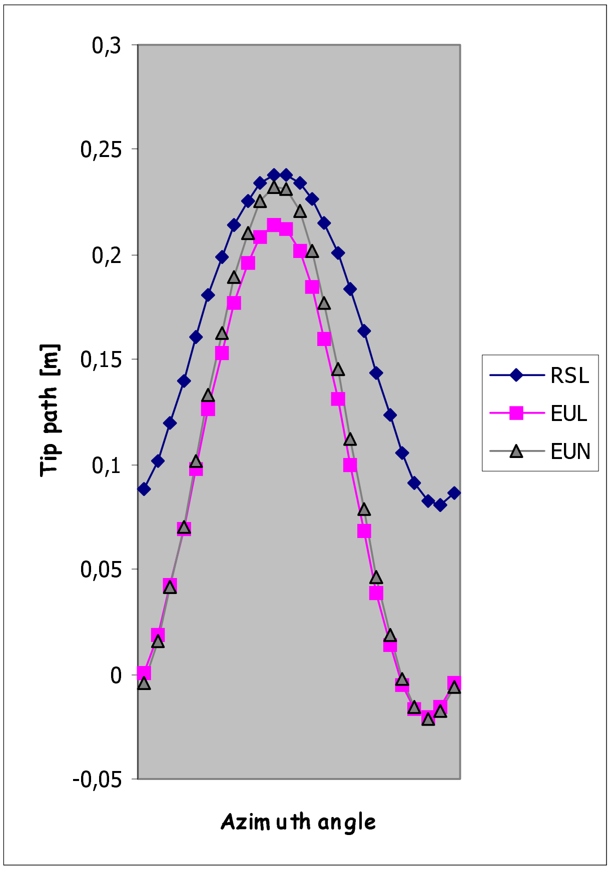


Figure 3.

The tip path of the elastic blades goes under the tip path of the rigid blade, however the flapping angle in the case of EUN was the biggest. It is so, because the centrifugal force increases strongly outward of the rotor blade and the rotor blade have commonly a downward elastic camber. This camber was demonstrated in [6] too.

The non-linearity has no considerable effect expect that close to the maximum there is noticeable difference.

8.4 The Control Moment

The control moments are shown in the figure 4:

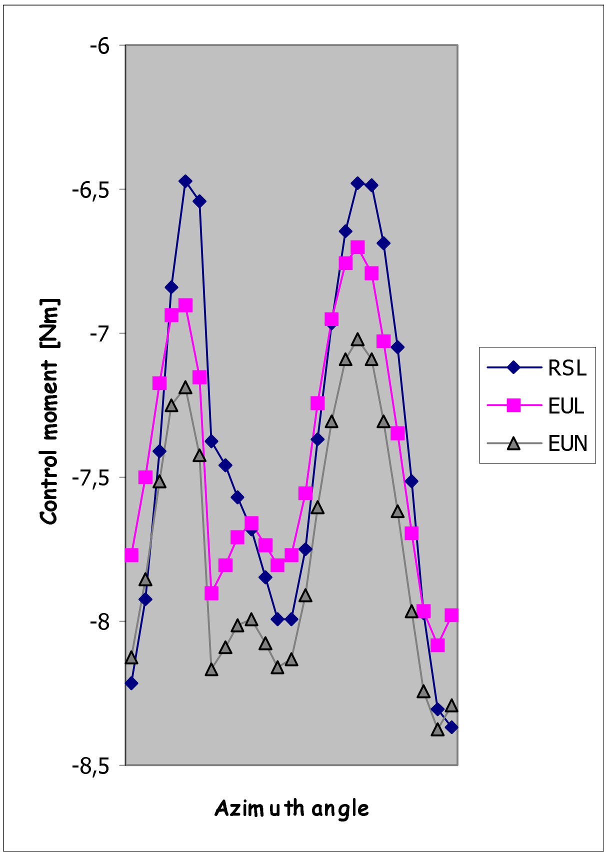


Figure 4.

The control moments are similar, the greatest values are given for EUN case. But the change of the control moments is the smallest also for EUN case. It means, that the elastic and unsteady calculations give a compensative effect.

9 Summary

This approximate calculations are quite complicated but gives acceptable, preliminary results. It is possible to extend this computations for general flight too.

The time effort of the calculations is not too big, it give possibility to create a real time rotor and helicopter simulator.

The computer program has a modular structure, the developing of the modules or the whole program is easy.

References

[1] Dat, R.: *Development of Basic Methods needed to Predict Helicopter Aeroelastic Behaviour* Vertica, Vol. 8. No. 2. pp. 209-228, 1984

[2] Bramwell, A. R. S.: *Helicopter Dynamics* Edward Arnold Ltd. London, 1976

[3] Bergh, H.-Wekken, A.J.P.: *Comparison between Measured and Calculated Stall-Flutter Behaviour of a One-Bladed Model Rotor* Vertica, Vol. 11. No. 3. pp. 447-456, 1987

[4] Stepniewsky, W.Z.: *Rotary-Wing Aerodynamics* Dover Publications, New York, 1979

[5] Etkin, B.: *Flugmechanik und Flugregelung* Berliner Union, Stuttgart, 1966

[6] Őry, H-Lindert, H.W.: *Calculation of Rotor Blade Air Loads from Measured Structural Response Data* Zeitschrift für Flugwissenschaften, Vol. 17. No. 4.

[7] Gausz, T.: *Helicopters (in Hungarian)* Postgraduate Institute of TU Budapest, 1982

[8] Jones, R. T.: *Wing Theory* Princeton University Press, 1990

[9] Gausz, T.: *Helicopter Rotor Aerodynamics and Dynamics* 5th MiniConf. on Vehicle System Dynamics, Budapest 1996.

[10] *SUMMARY OF AIRFOIL DATA* NACA Report No. 824