ICAS-98-R,5,7

A98-31650

THE METHOD FOR COMPUTING THE FAILURE PROBABILITY OF AIRCRAFT STRUCTURES

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Abstract Based on Moses' Incremental Load Method(ILM), Ditlevsen's narrow bounds' method , the developed criterion to select the critical elements and the integration method to compute the two-order joint failure probability of the authors of this paper, this paper proposes a new method of computing structural system failure probability. This method is very simple and easy for engineers to solve the complex large-scale structures because only the mean values of basic random variables are concerned to from the structural fault tree. The validity of the proposed method is demonstrated through 5 aircraft structure examples.*

Introduction

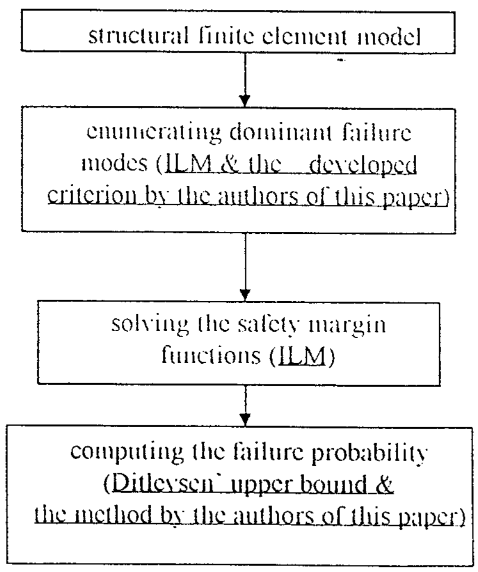
Estimation of structural system reliability is complicated exercise in probabilistic system. One of the currently adopted approaches is the reliability calculation based on the dominant failure modes. The methods to generate structural system dominant failure modes have been studied in references[1,2,3,4] . A procedure for frame structures called "Branching and Bounding Method" has been proposed by Murotsu . and it can be considered to be the special cases of the Truncated Enumeration Method presented by Melchers and Tang . An apparently quite different technique, also known as the "Incremental load Method"(ILM) proposed by Moses et al., is very simple and convenient for engineers to enumerate the structural system failure modes for large-scale complex structures, because only mean values of the random variables are concerned. In [5], Feng has developed the criterion in ILM to select the critical elements under every level incremental load and it can reduce the possibility with which some structural dominant failure modes are missed. In this paper, some defects of Feng's criterion are checked and many important improvements are also made. The first one is to revise the formula of selecting the critical elements to reduce the possibility of missing the structural dominant failure modes; The second one is to present two deleting criterion to avoid some identical failure modes being enumerated again and again and too many "non-dominant" failure modes being involved in the fault tree; The third one is to derive a formula to obtain the personally-invariant critical constants c, determined by the experience of engineers in Feng's criterion.

The ILM is also employed to solve the safety margin functions of structural dominant failure modes , it is very effective to the structures of plastic materials.

Ditlevsen' narrow bounds' method [6] is employed to compute the estimator of structural system failure probability. When the correlation coefficient between the two failure modes is great than 0.6, its computational error is a little greater because of the greater error of the two-order joint failure probability. In order to improve the precision of the result, an integration method presented in [7] is adopted to compute the two-order joint failure probability.

Theory and Method

Fig.1 is the flow chart of the method of this paper



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* The Authors Gratefully Acknowledges the Support of K.C. Wong Education Foundation, Hong Kong

21st ICAS Congress
13-18 September 1998
Melbourne, Australia

The developed Criterion for selecting Critical Elements

Based on Moses' incremental load method[4] and Y.S.Feng's criterion [5] , the developed criterion [8] can be used to select the critical elements under every incremental load.

The basic ideal of this method is that on the position where the real external load is placed ,the structure is loaded with the incremental loads step by step until the structure fails, and in every step, with the developed criterion, several elements are chosen as critical elements. Finally, the fault tree of the structure can be formed by this process.

The developed criterion for selecting critical elements is expressed as following.

The element i that satisfies the inequality (1a) is selected as a critical element under the 1st -level incremental load.

$$c_1 \le \overline{s_{i1}} \le 1 \qquad (1a)$$

$$\overline{S_{i1}} = {S_{icr}}/{S_{i1}} \qquad (1b)$$

$$S_{i1} = {\mu_{R_i^e}}/{a_{i1}} \qquad (1c)$$

$$s_{icr} = \min_i (s_{i1}) \qquad (1d)$$

where, $\mu_{R_i^e}$ is the mean value of the ith element's random strength R_i^e ; a_{i1} is the ith elemental internal force or equivalent stress under the 1st-level unit incremental load, it can be solved by the FEM (Finite Element Method); c_1 is a constant determined by the formula as follow,

$$c_1 = \frac{1-\sqrt{1-(1-A^2\overline{v}_s^{\,2})^2}}{1-A^2\overline{v}_s^{\,2}} \qquad (1e)$$

where, $A = -\Phi^{-1}(0.5Q)$, Q is a very small positive number , $\Phi(\bullet)$ represents the probability integration function of a standard normal random. If Q=1, then $A = -\Phi^{-1}(0.5)$ =0.5 represents the reliability index of the most critical element under 1st incremental external load ; $\overline{v}_s = \frac{1}{2}(v_r + v_s)$, v_r and v_s are the coefficient of variance of the structural elemental strength and external load, respectively.

The element i that satisfies the inequalities (2a) and (2b) is selected as a critical element under the pth-level ($p \ge 2$) incremental load, respectively.

$$c_p \le \overline{s_{ip}} \le 1 \qquad (2a)$$

$$s_{ip} \le 0 \qquad (2b)$$

$$\overline{s_{ip}} = \frac{\sum_{j=1}^{p-1} S_{jcr} + s_{pcr}}{\sum_{j=1}^{p-1} S_{jcr} + s_{ip}} \qquad (2c)$$

$$s_{pcr} = \mathrm{Min}_i (s_{ip}) \qquad (2d)$$

$$s_{ip} = {\mu_{R_i}^{(p-1)}}/{a_{ip}} \qquad (2e)$$

$$\mu_{R_i}^{(p-1)} = \mu_{R_i^e} - a_{i1}s_{1cr} - a_{i2}s_{2cr}\cdots - \cdots - a_{i(p-1)cr}s_{(p-1)cr} \qquad (2f)$$

where, a_{ij} is the ith elemental internal force or equivalent stress under the jth-level unit external incremental load. c_p also is a constant determined by the formula (1e). In these steps(p=2,3,⋯), the structures are some residual structures out of the structure-interested because p-1 elements to be supposed to have failed in succession .

With the help of the criteria (1a) 、(2a) and (2b), an initial structural fault tree can be formed and it can be simplified by removing the non-dominant failure modes.

Approach for the Safety Margin Function

Suppose 1,2,3,⋯m is one of the structural dominant failure modes, then, the following equations are obtained.

$$\left.\begin{array}{l} R_1^e = a_{11}s_{1cr} \\ R_2^e = a_{21}s_{1cr} + a_{22}s_{2cr} \\ \cdot \\ \cdot \\ R_m^e = \sum_{j=1}^{m-1} a_{mj}s_{jcr} + a_{mm}s_m \end{array}\right\} \qquad (3a)$$

Thus, the safety margin function is expressed as.

$$M = \sum_{j=1}^{m-1} s_{jcr} + s_m - S = b_0 + \sum_{j=1}^{m} b_j R_j^e - S \qquad (3b)$$

where, S represents the structural external load. s_{jcr}

and S_m can be obtained by formula (3a). It is obvious that M is the linear function with basic random variables.

Technique for computing failure probability

Ditlevsen's upper bound is expressed as.

$$P_{fs}^{u} = \sum_{i=1}^{m} P_i - \sum_{i=2}^{m} \max_{j<i}(P_{ij}) \qquad (4a)$$

where, P_i indicates the failure probability of failure mode i, and it can be obtained by following formulae.

$$P_i = \Phi(-\beta\ ) \qquad (4b)$$

$$\beta = \frac{\mu_M}{\sigma_M} \qquad (4c)$$

μ_M and σ_M are mean value and variance of the safety margin M, and they can be determined by formula (3b).

P_{ij} is the two-order joint failure probability of the failure modes i and j, in terms of [7], it can be computed by an integration method.

$$P \approx \sum_{i=1}^{p}\sum_{j=1}^{p} f(M_{1i}, M_{2j})\Delta S_{ij} \qquad (5a)$$

$$\Delta S_{ij} = (n/p)^2 \sigma_{M_1}\sigma_{M_2} \qquad (5b)$$

$$M_{1i} = (n/2p)(1-2i)\sigma_M \qquad (5c)$$

$$M_{1j} = (n/2p)(1-2j)\sigma_M \qquad (5d)$$

$$f(M_i, M_j) = \frac{1}{2\pi\sigma_{M_i}\sigma_{M_j}\sqrt{1-\rho^2}} \exp\{-\frac{1}{2(1-\rho^2)}$$

$$\left[\frac{(M_i-\mu_{M_i})^2}{\sigma_{M_i}^2} - 2\rho\frac{(M_i-\mu_{M_i})(M_j-\mu_{M_j})}{\sigma_{M_i}\sigma_{M_j}} + \frac{(M_j-\mu_{M_j})^2}{\sigma_{M_j}^2}\right]\} \qquad (5e)$$

where, ρ is the correlation coefficient between M_i and M_j, and it can be obtained through formula (3b). n and p are the constants to determine the computational precision, in this paper, author suggests

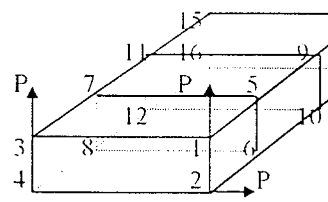
$$n \geq (3 \sim 4) + \max\left(\frac{\mu_{M_i}}{\sigma_{M_i}}\right), \ \text{p=16}.$$

Examples

In all examples of this paper, the coefficient of variation of external load and elemental strength are 0.12, and 0.1, respectively, the correlation coefficient among elemental strengths is 0.5.

Example 1 A 3-box structure is shown in Fig.2, the mean value of external load P is 230kg.

The fault tree obtained by the developed criterion is shown in Fig.3.



the notes of the element 30 are 5,1,2,6; the notes of the element 32 are 7,3,4,8; the notes of the element 33 are 6,8,4,2

Fig.2 The 3-box-structure of Example 1

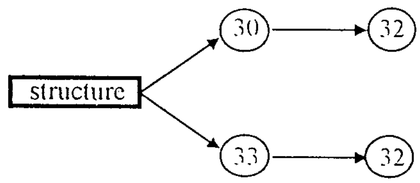


Fig.3 The fault tree of example 1

In this fault tree, the safety margin functions obtained by the formulae (3a) and (3b) are.

$$M_1 = 2.0\,R_{30}^{e} + 2.0\,R_{32}^{e} - P$$
$$M_2 = 8.0\,R_{32}^{e} - 8.0\,R_{33}^{e} - P$$

By the method of this paper, the Ditlevsen's upper bound of the structural system failure probability is 5.6122E-4

Example 2 A 9-box structure is shown in Fig.4, the mean value of the external load is 150kg, the allowable maximum displacement of any one of the structural notes is 0.1m.

The fault tree obtained is shown in Fig.5. The safety margin functions are.

$$M_1 = 4.0\,R_{78}^{e} + 4.0\,R_{68}^{e} - 4.0\,R_{77}^{e} - P$$
$$M_2 = 0.2\,R_{78}^{e} + 3.2\,R_{77}^{e} - P$$

The Ditlevsen's upper bound is 0.1493 E - 3.

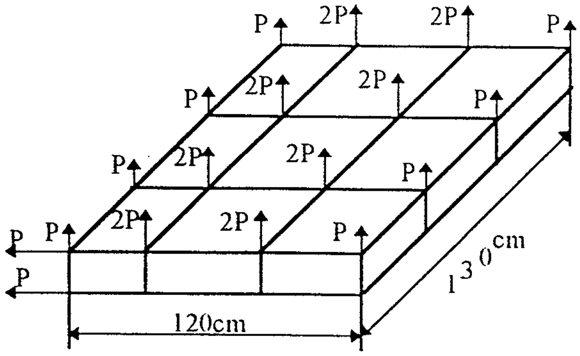


Fig.4 The 9-Box structure of Example 2

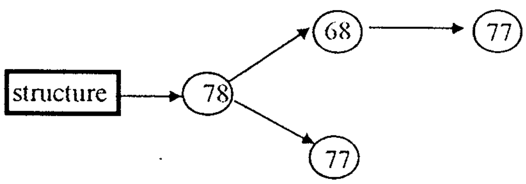


Fig.5 The fault tree of Example 2

Example 3 A circle stiffened frame structure is shown in Fig.6, the allowable maximum displacement of any one of the structural notes is 0.05m. The external load and computational results are shown in Table.1

Table.1 The structural external load and computational results of Example 3

load cases	μ_{P_1}	μ_{P_1}	P_{fs}
A	*99925.9kg*	3397.5kg	0.6396E-6
A	5195.6± *758.1kg**	3398.1kg	1.1462E-10
B	*75275.3 kg*	*2709.9kg*	*0.4515E-9*
L_A	*97745.5kg*	*3518.8kg*	*0.3610E-6*

* *the external loads of structural nodes 5 ,15 and 6,14 are taken "+" and "-", respectively.*

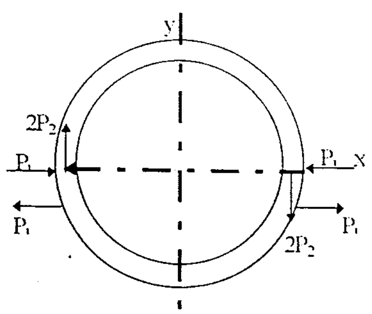


Fig.6 The Circle Stiffened Frame Structure of the Example 3

Example 4 A elliptical stiffened frame structure is shown in Fig.7, the allowable maximum displacement of any one of the structural notes is 0.05m. The external load and computational results are shown in Table.2

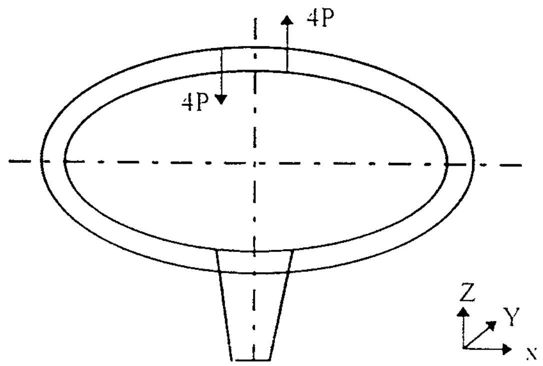


Fig.7 The elliptical stiffened frame structure of Example 4

Table.2 The Computational Results of Example 4

load cases	P_{fs}
A	*0.6268E-14*
ZH	*0.3638E-6*
4	*0.1751E-9*
C	*0.4747E-12*
F.*20*	*0.1832E-12*
F.15	*0.4225E-12*

Example 5 The finite element model of a wing structure is shown in Fig.8. Its fault tree obtained by the developed criterion is shown in Fig.9.

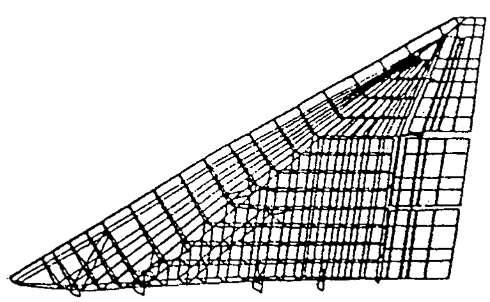


Fig.8 The finite element model of a wing structure

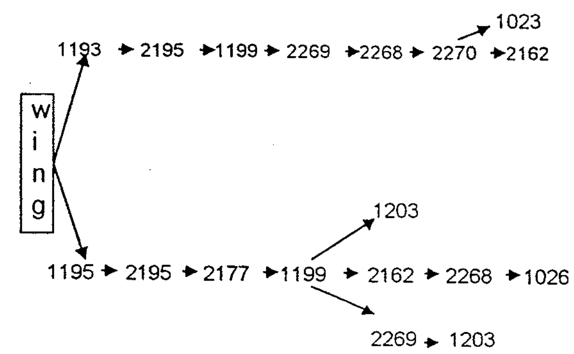


Fig.9 The fault tree of the wing Structure in Fig.8

By the method of this paper, $P_{fs}=0.0176\times E-5$.

Conclusion

Based on the Incremental Load Method and Ditlevsen's narrow method, it has been shown that a quite general, but economic procedure is developed for computing the structural system failure probability. This procedure is simple, convenient and effective technique to estimate the complex structures with many dominant failure modes and basic random variables.

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