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## HELICOPTER REDUNDANT STATION KEEPING FLIGHT CONTROL LAW

A. P. Oliva

*Instituto de Aeronáutica e Espaço (IAE) - Divisão de Sistemas Espaciais*

*CNPq Researcher*

*12228-904, São José dos Campos, SP - Brasil*

*e-mail: oliva@ase2.iae.cta.br*

### ABSTRACT

A proposal about how to implement a helicopter flight control system designed by linear quadratic method without the use of as many sensors as states required by the control law and maintaining the same flight performance of the basic system as also introducing analytical redundancy in the system is presented. The analytical redundancy allows to have a half of the sensors required to use the designed flight control law. The idea is applied to the helicopter dynamics without the rotor dynamics effects and also to the helicopter dynamics with coupling effects due to the rotor dynamics. Comparison between the optimal control law and the proposed simplification is analysed with respect to flying qualities and also with respect to stability characteristics.

### 1. INTRODUCTION

This work makes use of a helicopter flight control law designed in McLean<sup>1</sup> to propose a way to implement this control law with less sensors than the required by the original control law, as also suggests the introduction of some degree of analytical redundancy by the method proposed by Rynaski<sup>2</sup>. The control law used as example is designed to a station-keeping system, that is, a flight control system that gives the helicopter a way to maintain its position fixed in space, for a long period of time. The example used in this work is for a Sikorski S-61 helicopter also taken from McLean<sup>1</sup>. The control law has been designed by the linear quadratic method (LQR), and has been designed to the helicopter without the inclusion of the rotor dynamics, as also to the helicopter with the inclusion of the effects of rotor dynamics coupling. For both cases an alternative control law is designed with the use of a robust observer, suggested by Doyle<sup>3</sup>. The observer is designed by two methods, the first one for the case of helicopter without rotor dynamics, suggested by Chen<sup>4</sup> and the second one, for the helicopter with rotor dynamics included outlined in Friedland<sup>5</sup>.

### 2. BASIC FLIGHT CONTROL LAW

The helicopter dynamics plus rotor dynamics is described by the mathematical model given by,

$$\dot{x} = A x + B \eta \quad (1)$$

where the state vector is,

$$x^T = [\theta_R \ \phi_R \ p_R \ q_R \ \theta_F \ \phi_F \ p_F \ q_F \ u \ v] \quad (2)$$

and the control vector is given by,

$$\eta^T = [\delta_A \ \delta_B] \quad (3)$$

For this model the control law has been designed in McLean<sup>1</sup> based on the LQR method, and so resulting in a control of the form,

$$\eta = r - Kx \quad (4)$$

So, as it is possible to notice, to implement this kind of control law it will be necessary 10 sensors, since it uses 10 states. In order to get a simplified control law it is possible to use a simplified model of the helicopter dynamics, as described in McLean<sup>1</sup>, given by,

$$\dot{x}_1 = A_1 x_1 + B_1 \eta_1 \quad (5)$$

$$x_1^T = [\theta_F \ \phi_F \ p_F \ q_F \ u \ v] \quad (6)$$

$$\eta_1^T = [-\phi_R \ \theta_R] \quad (7)$$

$$\eta_1 = r - K_1 x_1 \quad (8)$$

and it is possible to notice that to implement this control law it will be necessary the use of only six sensors, since it uses six states.

### 3. PROPOSED CONTROL LAW FOR THE HELICOPTER WITHOUT ROTOR DYNAMICS

In order to reduce the number of sensors used by the control law and to introduce some redundancy it is proposed a control law that uses a robust observer as described by Rynasky<sup>2</sup>. With this new control law it will be possible to use only four sensors, for the states :  $\theta_F$ ,  $\phi_F$ ,  $p_F$  and  $q_F$ . In this way it is not necessary to use sensors for the states  $u$  and  $v$ . This new implementation it will also allow the system to support a triple sensor failure and still working close to the performance given by the base control law, due to the redundancy included. To design this control law it is supposed that the system is described by,

$$\dot{x}_1 = A_1 x_1 + B_1 \eta_1 \quad (9)$$

$$y = C_1 x_1 \quad (10)$$

$$\eta_1 = r - K_1 x_1 \quad (11)$$

The observer in this case is given by,

$$\dot{z} = Fz + Gy + H\eta_1 \quad (12)$$

and the estimated states by,

$$\hat{x}_1 = \begin{bmatrix} C_1 \\ T \end{bmatrix}^{-1} \begin{bmatrix} y \\ z \end{bmatrix} = M y + N z \quad (13)$$

So the control law to be implemented will be given by,

$$\eta_1 = r - K_1 \hat{x}_1 \quad (14)$$

and the augmented system, that is, helicopter, control law and observer is described by,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_1 + B_1 K_1 M C_1 & B_1 K_1 N \\ G C_1 + H K_1 M C_1 & F + H K_1 N \end{bmatrix} \begin{bmatrix} x_1 \\ z \end{bmatrix} + \begin{bmatrix} B_1 \\ H \end{bmatrix} r \quad (15)$$

The observer is designed following the method described in Chen<sup>4</sup> and the idea of Doyle<sup>5</sup>. The poles of the  $F$  matrix are chosen among the transmission zeros of the open loop transfer function  $y/u_1$ . The  $F$  matrix is then chosen as a diagonal matrix in order to make the design as simple as possible. After this the controllability of the pair  $(F, G)$  is checked, and if it is controllable the design go ahead, if not, it is necessary to choose another  $F$  matrix. Then, it is the necessary to solve the Lyapunov equation,

$$T A_1 - F T = G C_1 \quad (16)$$

in order to get  $T$ , and so finally  $H$  is obtained by,

$$H = T B_1 \quad (17)$$

This procedure has been done for the cases,

$$y = \theta_F ; y = \phi_F ; y = p_F \text{ and } y = q_F$$

and so four alternative control laws have been obtained. At section 5 the parameters for the case  $y_1 = \theta_F$  are given as an example to illustrate the procedure.

### 4. PROPOSED CONTROL LAW FOR THE HELICOPTER WITH ROTOR DYNAMICS

Again in this case a control law that uses a robust observer is proposed in order to reduce the number of necessary sensors, in this case it is necessary 10 sensors to implement the control law designed in McLean<sup>1</sup>. Here, the proposed control law it will use only four states, the same used in section (3). So it will be necessary only four sensors, for the following states :  $\theta_F$ ,  $\phi_F$ ,  $p_F$  and  $q_F$ . This implementation is going to introduce also some redundancy in the system. In this case the system dynamics is given by,

$$\dot{x} = A x + B \eta \quad (18)$$

$$y = C x \quad (19)$$

and equation (18) is splitted into ,

$$\dot{x}_1 = A_{11} x_1 + A_{12} x_2 + B_{11} \eta \quad (20)$$

$$\dot{x}_2 = A_{21} x_1 + A_{22} x_2 + B_{21} \eta \quad (21)$$

with  $x_1$  the available state, that is, the sensed state and  $x_2$  the remaining states. The observer dynamics is given by,

$$\dot{z} = F z + G y + H \eta \quad (22)$$

and the estimated states are given by,

$$\hat{x}_2 = L y + z \quad (23)$$

with the control law given by,

$$\eta = r - K_{11} \hat{x}_1 - K_{12} \hat{x}_2 \quad (24)$$

and finally the augmented system is given by,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_{11} - B_{11}K_{11} - B_{11}K_{12}L & A_{12} & -B_{11}K_{12} \\ A_{21} - B_{21}K_{11} - B_{21}K_{12}L & A_{22} & -B_{21}K_{12} \\ G - HK_{11} - HK_{12}L & 0 & F - HK_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ z \end{bmatrix} + \begin{bmatrix} B_{11} \\ B_{21} \\ H \end{bmatrix} r \quad (25)$$

The observer in this case has been designed by the method given in Friedland<sup>5</sup> and with the approach of Doyle<sup>3</sup>.

The procedure is the following :

First the desired poles of the observer are chosen among the open loop transmission zeros of the open loop transfer functions  $y / \eta$ . So the observer poles are given by vector  $P$ . After this the  $L$  matrix of the observer gains are obtained by pole placement design, with the help of the control toolbox of MATLAB<sup>6</sup>, that is,

$$L = \text{place} (A_{22}^T, A_{12}^T, P) \quad (26)$$

and finally the observer parameters are obtained by,

$$F = A_{22} - L^T A_{12} \quad (27)$$

$$H = B_{21} - L^T B_{11} \quad (28)$$

$$G = A_{21} - L^T A_{11} + FL^T \quad (29)$$

## 5 EXAMPLE OF BOTH DESIGNS

At this point it is useful to give an example for the designs described in sections 3 and 4.

### 5.1 DESIGN EXAMPLE FOR SECTION 3

In this case the matrices  $A_1$ ,  $B_1$  and  $K_1$  used here can be found in McLean<sup>1</sup>. The  $F$  matrix of the observer was chosen as a diagonal matrix of order ( 5x5 ) with its elements taken from the transmission zeros of the open

loop transfer function  $y / \eta_1$ . So from the transmission zeros of the open loop transfer functions :

$\theta_F / \eta_1$ ,  $\phi_F / \eta_1$ ,  $p_F / \eta_1$ ,  $q_F / \eta_1$ ,  $u / \eta_1$  and  $v / \eta_1$  the following open loop transmission zeros have been taken as the poles for the observer ,

$$F = \text{diagonal} ( -26.3, -0.50, -0.006, -0.003, -25 )$$

and  $G$  has been taken as ,

$$G^T = [ 1 \ 1 \ 1 \ 1 \ 1 ]$$

in this way the observer has been designed.

### 5.2 DESIGN EXAMPLE FOR SECTION 4

Here again the matrices  $A$ ,  $B$  and  $K$  have been taken from McLean<sup>1</sup> and used as an example. The  $P$  vector for the observer has its poles chosen among the transmission zeros of the open loop transfer functions  $y / \eta$  as in the previous case, so the observer poles are given by,

$$P^T = [ -13.73 \ -12.12 \ -14.2 \ -1.09 \ -0.70 \ -0.41 \\ -17.82 \ -22.13 \ -26.4 ]$$

Then by pole placement the gain vector of the observer ( $L$ ) has been obtained as ,

$$L = [ -152 \ 179.6 \ -4571.6 \ -8459.2 \ -86.6 \ 50.7 \\ 238.1 \ 2664.5 \ -6918.3 ]$$

After that it is possible to obtain the matrices  $F$ ,  $H$  and  $G$  with the equations (27), (28) and (29) respectively.

## 6 RESULTS OBTAINED

Both designs have been assessed against the basic control law in order to verify if they maintain the same flying qualities and stability characteristics given by the basic control law.

### 6.1 HELICOPTER WITHOUT ROTOR DYNAMICS

In figure ( 3 ) there is a time history comparison of  $\theta_F$  for a step in  $\phi_R$  for the basic control law against the observer based control law designed with  $y = \theta_F$ . It is quite clear that the same flying qualities are maintained by the alternative control law. So the tracking capability is well maintained.

In figure ( 4 ) there is a time history comparison of  $\theta_F$  for an initial disturbance in  $\theta_F$  for the basic control law against the observer based control law designed with  $y = \theta_F$ . It is again quite clear that also the regulator characteristics of the basic control law are well maintained by the observer based control law.

In figure ( 5 ) it is plotted the frequency response of the basic control law against the frequency response of the observer based control law designed with  $y = \theta_F$  obtained from the transfer function  $\theta_F / \phi_R$ .

## 6.2 HELICOPTER WITH ROTOR DYNAMICS

In figure ( 6 ) there is a  $\theta_F$  time history response comparison between the basic control law and the observer based control law for a step in  $\delta_A$ . It is again clear that the same flying qualities are maintained by the observer based control law.

In figure ( 7 ) it is showed the time history response for the basic control law and the observer based control law for a disturbance in  $\theta_F$  in order to assess the regulator characteristics, and again it is showed that the observer based control law maintains the same regulation characteristics of the basic control law.

In figure ( 8 ) there is a frequency response comparison between the basic control law and the observer based control law for the transfer function  $\theta_F / \delta_A$ , and it is possible to notice that the observer based control law maintains the same frequency response given by the basic control law, as also the stability characteristics.

## 7 CONCLUSIONS AND COMMENTS

From the performed work and the analysis of the results the following can be concluded :

In the case of the helicopter without rotor dynamics it is possible to use the observer based control law instead of the basic control law, since the observer based control law maintains the same flying qualities and stability characteristics given by the basic control law, except for the case of regulation performance with respect to the helicopter states forward speed ( u ) and lateral speed ( v ) .

It is possible to use just four sensors, in the case of the helicopter without rotor dynamics, that is , for  $\theta_F$ ,  $\phi_F$ ,  $p_F$  and  $q_F$ , eliminating the sensors for u and v, that is, for angle of attack and angle of sideslip.

It is possible to use four observer based control laws, named as,  
observer based CL1 ( control law n.1 ) designed for the case with  $y = \theta_F$   
observer based CL2 ( control law n.2 ) designed for the case with  $y = \phi_F$   
observer based CL3 ( control law n.3 ) designed for the case with  $y = p_F$   
observer based CL4 ( control law n.4 ) designed for the case with  $y = q_F$

and in this way introducing analytical redundancy into the system and so allowing to the system support a sensor failure and working with a performance much closer to the that given by the basic control law.

In the case of the helicopter with rotor dynamics it is also possible to use the observer based control laws instead of the basic control law, since they also maintains the same flying qualities and stability characteristics given by the basic control law, except for the case of regulation performance with respect to the helicopter states u and v .

Again in this case it is possible to use only four sensors, that is, for  $\theta_F$ ,  $\phi_F$ ,  $p_F$  and  $q_F$  and so it is not necessary to use sensors for the remaining six states (  $\theta_R$ ,  $\phi_R$ ,  $p_R$ ,  $q_R$ , u and v ) .

Here it is also possible to use four control laws, the same listed in the case of the helicopter without rotor dynamics, and so also introducing analytical redundancy into the system.

It can be noticed that the only disadvantage of the observer based control laws with respect to the basic control law is the regulation performance with respect to the helicopter states forward speed ( u ) and lateral speed ( v ) a feature that can be improved with a more careful design for the observers.

It is also noticed that the observer matrix F is the same for the four control laws ( CL1 ,CL2 ,CL3 and CL4 ), and also the G matrix of the observer. This is a fact that simplifies the implementation.

The observers used for the alternative control laws have the same characteristics of the Doyle-Stein observer, namely , maintains the same frequency response of the basic control law and has its poles at the transmission zeros of the open loop transfer function  $y / \eta$ . However they have not  $H = 0$  as expected in a Doyle-Stein observer

## 8 . FIGURES

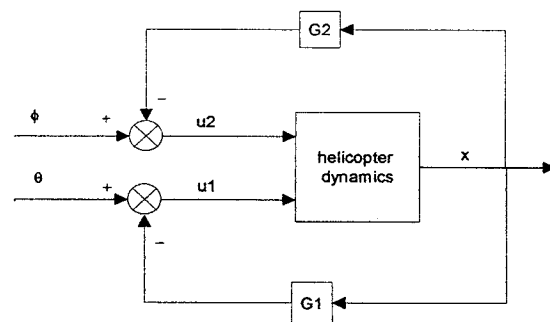


Figure 1 - Basic Control Law

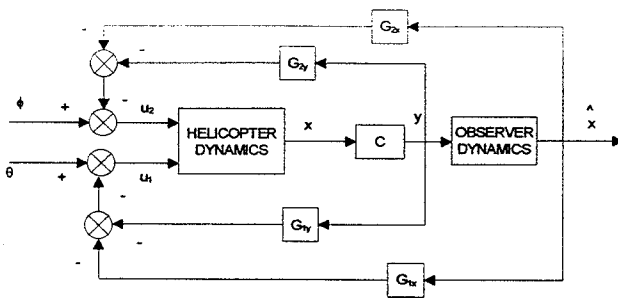


Figure 2 - Observer Based Control Law

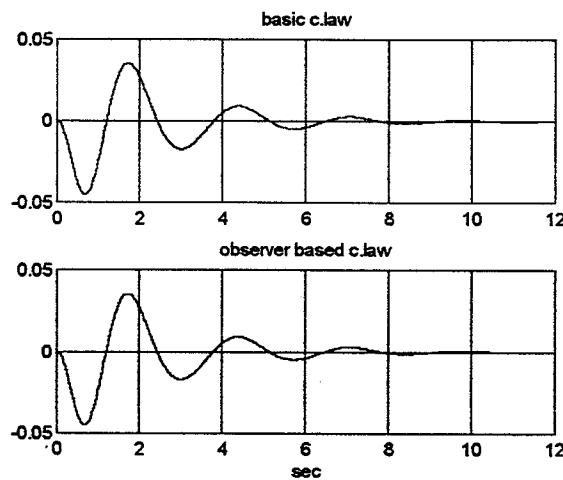


Figure 3 - time history comparison of  $\theta_F$  response for a step input in  $\phi_R$  basic control law ( above ) and observer based control law ( below ) with  $y = \theta_F$  helicopter without rotor dynamics included

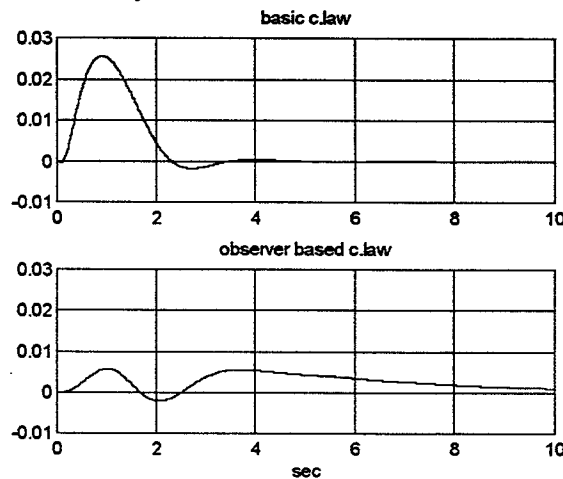


figure 4 - time history comparison of  $\theta_F$  response for an initial disturbance in forward speed ( u ) basic control law ( above ) and observer based control law ( below ) with  $y = \theta_F$  helicopter without rotor dynamics included

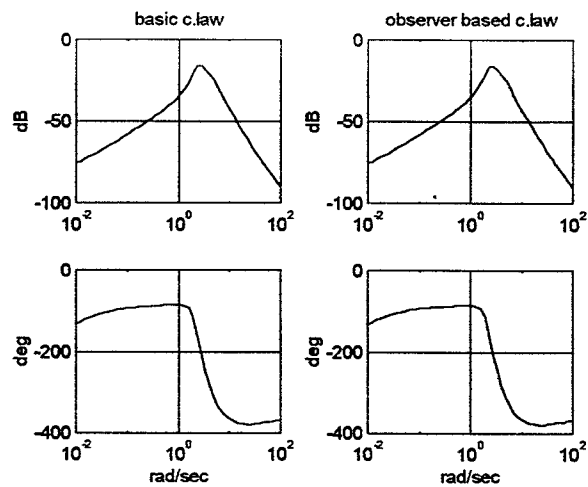


figure 5 - frequency response comparison of the transfer function  $\theta_F / \phi_R$  for the basic control law ( left ) and for the observer based control law , ( right ) with  $y = \theta_F$  helicopter without rotor dynamics

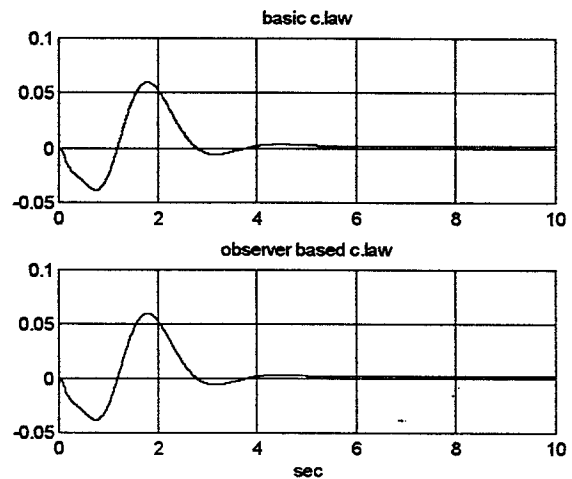


figure 6 - time history comparison of  $\theta_F$  response for a step in  $\delta_A$  basic control law ( above ) and observer based control law ( below ) with  $y = \theta_F$  helicopter with rotor dynamics included

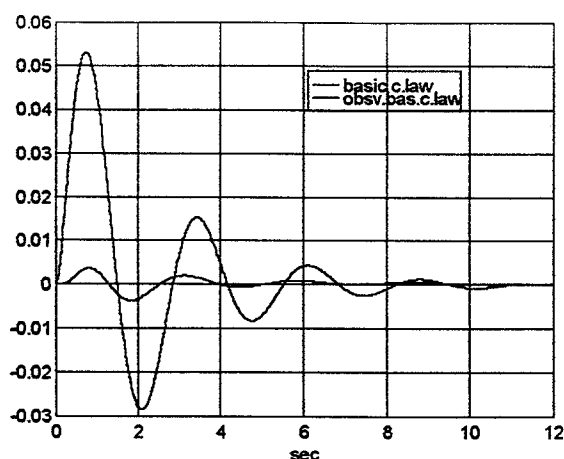


figure 7- time history comparison of  $\theta_F$  response for an initial disturbance in forward speed ( u ) basic control law and observer based control law with  $y = \theta_F$  helicopter with rotor dynamics included

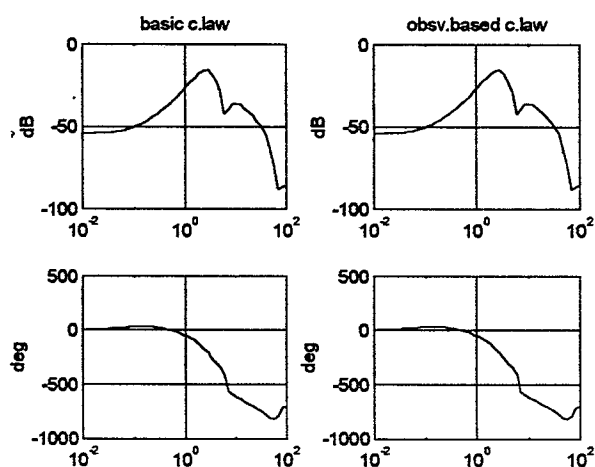


figure 8 - frequency response comparison of the transfer function  $\theta_F / \delta_A$  for the basic control law ( left ) and for the observer based control law ( right ) with  $y = \theta_F$  helicopter with rotor dynamics included

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