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Aeroelastic Tailoring of Composite Box Beams

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Abstract

Aeroelastic instabilities are an important factor in the design of modern high-speed, flexible aircraft. The current trend is toward the creative use of composites to delay these instabilities. To obtain an optimum design, an accurate as well as efficient model is required. As a first step towards this goal, aeroelastic analysis is carried out for a swept composite box beam, using a linear structural model and a linear 2-D unsteady aerodynamic theory. Structurally, the wing is modeled as a thin-walled composite box beam of rectangular cross section. Theodorsen's theory is used to get the 2-D unsteady aerodynamic forces, which are integrated over the span. The flutter solution is obtained using the V-g method and divergence speeds are calculated by using a very low reduced frequency in the flutter analysis. The variation of critical speeds with composite ply layup is plotted for various sweep angles. These trends are compared with those available in literature.

Nomenclature

$\bar{b}$ = semi-chord w.r.t beam axis
C, C_{ij} = 4 × 4 cross sectional stiffness matrix
g = artificial damping parameter
h = airfoil plunge motion
$[K]$ = stiffness matrix
$\bar{L}$ = aerodynamic lift
$\bar{M}$ = aerodynamic pitching moment
$[M]$ = mass matrix
$q, \bar{q}, \tilde{q}$ = generalized coordinates
Q = applied load (aerodynamic)
$\tilde{Q}$ = generalized aerodynami loads
T = kinetic energy of the beam
u, u_i = displacements of the beam
U = strain energy of the beam
x_i = cartesian coordinates
α = airfoil pitch motion
γ = strains in the beam
κ = curvatures of the beam
ω = frequency of oscillation
$[\Phi]$ = displacement modeshape matrix
$[\Phi^*]$ = strain modeshape matrix
θ, θ_i = rotations of the beam
$[\Upsilon]$ = free-vibration modeshape matrix

Supersripts

$(\ )'$ = derivative w.r.t beam axis
$(\dot{\ })$ = time derivative

Introduction

Aeroelastic tailoring is defined in Ref. 1 as

"Aeroelastic tailoring is the embodiment of directional stiffness into an aircraft structural design to control aeroelastic deformation, static or dynamic, in such a fashion as to affect the aerodynamic and structural performance of that aircraft in a beneficial way."

Aeroelastic tailoring is not a new concept; a similar design concept was used as early as 1949 by Munk[2] to design "propellers containing diagonally disposed fibrous material." The grain (fibers) of wood were oriented in the blade so as to twist it elastically and favorably as the thrust changes.

In recent years there have been lot of studies in aeroelastic tailoring with the advent of composites. Advanced composite materials combine vastly superior specific stiffness and strength characteristics and can be designed (tailored) to meet specified directional stiffness requirement. Thus, tailoring with composites is now a

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natural part of the design process. The best exapmle of the creative use of composites in design stage is the X-29 Research Aircraft with forward-swept wing. Until recently such an aircraft design was unconceivable due to the very low divergence speed of forward-swept wings.

The reintroduction of swept-forward wings is due to N. J. Krone[3] who showed that divergence of swept-forward wing could be avoided with no weight penalty using tailored composites. This brought about a renewed interest in aeroelastic tailoring, and consequently, this configuration has been the focus of much of the research. Among the more notable are a series of studies by Weisshaar[4,5,6] dealing with static aeroelastic problems including spanwise lift redistribution, lift effectiveness and aileron effectiveness. The papers also include a discussion on the various techniques used in literature to reduce a laminated plate to an equivalent beam model and introduce a bending-twist coupling parameter K to be used in tailoring. Sherrer *et al.*[7] experimentally demonstrated the principle of aeroelastic tailoring through low-speed wind tunnel tests on a variable sweep cantilever wing model.

One of the earliest parametric studies was done by Housner and Stein[8] they presented a computer program for flutter analysis, which calculated the variable stiffness properties by using a laminated, balanced ply, filamentary composite plate theory. The parametric studies included the effect of filament orientation upon the flutter speed for wings with various sweep, mass ratios and skin thickness. Recently, Hollowell and Dugundji[9] did an analytical and experimental investigation of flutter and divergence behavior of unswept, rectangular wings made of graphite/epoxy, cantilevered plates with various amounts of bending-twist coupling. Lottati[10] did an analytical investigation to determine flutter and divergence speeds of a cantilevered, composite, forward-swept rectangular wing, again by varing the bending-twist coupling. Green[11] concentrates on the aeroelstic problems of a transport aircraft with high aspect ratio aft swept wings.

Most of the work cited above use a very simplified and unrealistic structural model, like, a plate-beam model or a box beam composed of two rigidly attached plates. These models proove the concept of aeroelastic tailoring but are useless for design of real wings. In literature one does find design studies using more detailed formulations, like numerical solution of 3-D unsteady Euler/Navier-Stokes equation[12] but are computationally two expensive for preliminary design.

A large aspect-ratio wing can be modeled accurately as a thin walled beam. Librescu and Song[13] analyzed the divergence instability of a swept-forward, composite wing modeled as a thin-walled, anisotropic, composite beam. The model incorporates a number of non-classical effects, including, anisotropy of the material, transverse shear deformation and warping effects. The present work uses a similar structural model, though warping effects are not considered. The cross-sectional stiffness coefficients are derived from an variationally and asymptotically consistent theory for anisotropic thin-walled beams, developed by Berdichevsky *et al.*[14] The theory gives closed form expressions for the beam stiffness coefficients. The stiffness coefficients, static response and dynamic response[15] are in agreement with finite element predictions, other closed-form solutions and test data.

The 2-D unsteady aerodynamic theory in the frequency domain (Theodorsen's theory) is used for getting the aerodynamic loads. The aerodynamic lift and moment expressions for a swept wing are available in the aeroelasticity text by Bisplinghoff *et al.*[16] The V-g method is used for flutter analysis, divergence speed is obtained by tending the reduced frequency to zero.

Structural Model

A comprehensive beam modeling framework has been developed over the past decade which breaks up the complete 3-D elasticity problem into two less complicated problems. Firstly, the 2-D cross-section is modeled using a asymptotically correct cross-sectional analysis tool, followed by the 1-D beam analysis using the cross-sectional stiffnesses calculated earlier. It is based on 3-D elasticity, thus, asymptotically correct 3-D strain/stress can be recovered at any point within the structure.

Cross-sectional Analysis

A coordinate system $\hat{x} = \{\hat{x}_1\ \hat{x}_2\ \hat{x}_3\}$ is defined along the undeformed wing with $\hat{x}_1$ along the elastic axis. The displacements, rotations, strains and curvatures are denoted by u, θ, γ and κ respectively

$$u = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad \theta = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} \quad \gamma = \begin{Bmatrix} \gamma_{11} \\ 2\gamma_{12} \\ 2\gamma_{13} \end{Bmatrix} \quad \kappa = \begin{Bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix} \tag{1}$$

where, the strains and curvatures are related to the displacements and rotations as

$$\gamma = \begin{Bmatrix} \gamma_{11} \\ 2\gamma_{12} \\ 2\gamma_{13} \end{Bmatrix} = \begin{Bmatrix} u_1' \\ 2\gamma_{12} \\ 2\gamma_{13} \end{Bmatrix} \quad \text{and} \quad \kappa = \begin{Bmatrix} \theta_1' \\ \theta_2' \\ \theta_3' \end{Bmatrix} \tag{2}$$

now, for small rotations,

$$\theta_2' = -u_3'' \quad \text{and} \quad \theta_3' = u_2'' \tag{3}$$

Thus,

$$\kappa = \begin{Bmatrix} \theta_1' \\ -u_3'' \\ u_2'' \end{Bmatrix} \tag{4}$$

Berdichevsky *et al.*[14] used asymptotic analysis on two-dimensional shell theory to get the strain energy per unit length (Φ_2), for a beam, in terms of four strain parameters, u_1', θ_1', u_3'' and u_2''. Transverse shear strain is not included as a strain parameter in the energy expressions, but is consistently removed from the formulation in terms of other parameters.

$$\begin{aligned} \Phi_2 = \; & \tfrac{1}{2}\left[C_{11}(u_1')^2 + C_{22}(\theta_1')^2 + C_{33}(u_3'')^2 + C_{44}(u_2'')^2\right] \\ & + C_{12}u_1'\theta_1' + C_{13}u_1'u_3'' + C_{14}u_1'u_2'' \\ & + C_{23}\theta_1'u_3'' + C_{24}\theta_1'u_2'' + C_{34}u_3''u_2''. \end{aligned} \tag{5}$$

The constitutive relationships can be written in terms of stress resultants and kinematic variables by differentiating the above equation with respect to the associated kinematic variable. Thus we get an asymptotically correct constitutive law (cross-sectional stiffness) as,

$$\begin{Bmatrix} F \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{12} & C_{22} & C_{23} & C_{24} \\ C_{13} & C_{23} & C_{33} & C_{34} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{bmatrix} \begin{Bmatrix} u_1' \\ \theta_1' \\ u_3'' \\ u_2'' \end{Bmatrix} \tag{6}$$

where F is the force along the wing and M_1, M_2, M_3 are the torsional and two bending moments. $[C]_{4\times4}$ is the cross-sectional stiffness matrix. The analytical expressions for the C_{ij}'s can be calculated easily for a box beam.[15]

Beam Energy Formulation

Langrange equations of motion can be written in terms of the kinetic energy(T), the strain energy(U) and the external load(Q) as,

$$\frac{\partial}{\partial t}\left(\frac{\partial(T-U)}{\partial \dot{q}_i}\right) - \frac{\partial(T-U)}{\partial q_i} = Q \tag{7}$$

Kinetic energy is given by

$$T = \frac{1}{2}\int_0^l \begin{Bmatrix} \dot{u} \\ \dot{\theta} \end{Bmatrix}^T \begin{bmatrix} m & m\bar{\xi} \\ m\bar{\xi} & i \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{\theta} \end{Bmatrix} dx_1 \tag{8}$$

where m is the mass per unit lenth, $\bar{\xi}$ is a matrix of center of mass offset from the elastic axis and i is the matrix of polar moment of inertia. For our problem, the rotational kinetic energy due to beam bending in neglected, thus Eq. (8) becomes

$$T = \frac{1}{2}\int_0^l \begin{Bmatrix} \dot{u}_1 \\ \dot{\theta}_1 \\ \dot{u}_3 \\ \dot{u}_2 \end{Bmatrix}^T \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & i & m\xi_2 & m\xi_3 \\ 0 & m\xi_2 & m & 0 \\ 0 & m\xi_3 & 0 & m \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{\theta}_1 \\ \dot{u}_3 \\ \dot{u}_2 \end{Bmatrix} dx_1 \tag{9}$$

Strain energy is given by

$$U = \frac{1}{2}\int_0^l \begin{Bmatrix} \gamma \\ \kappa \end{Bmatrix}^T [C] \begin{Bmatrix} \gamma \\ \kappa \end{Bmatrix} dx_1 \tag{10}$$

so that

$$U = \frac{1}{2}\int_0^l \begin{Bmatrix} u_1' \\ \theta_1' \\ u_3'' \\ u_2'' \end{Bmatrix}^T \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{12} & C_{22} & C_{23} & C_{24} \\ C_{13} & C_{23} & C_{33} & C_{34} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{bmatrix} \begin{Bmatrix} u_1' \\ \theta_1' \\ u_3'' \\ u_2'' \end{Bmatrix} dx_1 \tag{11}$$

Generalized coordinates are introduced to make it a finite dimensional system (Rayleigh - Ritz type formulation). We represent the deflections u and θ with finite number of modeshapes.

$$\{\bar{u}\} = \begin{Bmatrix} u_1 \\ \theta_1 \\ u_3 \\ u_2 \end{Bmatrix} = [\Phi]\{q\} \tag{12}$$

where

$$[\Phi] = \begin{bmatrix} \{\phi_{u_1}\}_{1\times n} & 0 & 0 & 0 \\ 0 & \{\phi_{\theta_1}\}_{1\times o} & 0 & 0 \\ 0 & 0 & \{\phi_{u_3}\}_{1\times p} & 0 \\ 0 & 0 & 0 & \{\phi_{u_2}\}_{1\times q} \end{bmatrix} \tag{13}$$

Here, ϕ_{u_1}'s, ϕ_{θ_1}'s, ϕ_{u_3}'s, ϕ_{u_2} are the extentional, torsional, vertical bending, and inplane bending modeshapes, respectively and n,o,p,q are the corresponding number of each kind of modes. The normal modes for axial and torsional vibration of a uniform beam are known to be sinusidal. The analytical expressions for the normal bending modes of a uniform beam derived by Chang and Craig[17] are used. The analytical expressions of the modeshapes can be differentiated to get the corresponding modeshapes for u_1', θ_1', u_3'' and u_2''. Thus we can get,

$$\{\bar{\gamma}\} = \begin{Bmatrix} u_1' \\ \theta_1' \\ u_3'' \\ u_2'' \end{Bmatrix} = [\Phi^*]\{q\} \tag{14}$$

where

$$[\Phi^*] = \begin{bmatrix} \{\phi'_{u_1}\}_{1\times n} & 0 & 0 & 0 \\ 0 & \{\phi'_{\theta_1}\}_{1\times o} & 0 & 0 \\ 0 & 0 & \{\phi''_{u_3}\}_{1\times p} & 0 \\ 0 & 0 & 0 & \{\phi''_{u_2}\}_{1\times q} \end{bmatrix} \tag{15}$$

Substituting the expressions for $[\Phi]$ and $[\Phi^*]$ in the kinetic and strain energy expressions, the Lagrange's equations can be expressed as

$$[M]\{\ddot{q}\} + [K]\{q\} = \{Q\} \tag{16}$$

where

$$[M] = \int_0^l [\Phi]^T \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & i & m\xi_2 & m\xi_3 \\ 0 & m\xi_2 & m & 0 \\ 0 & m\xi_3 & 0 & m \end{bmatrix} [\Phi] dx_1 \tag{17}$$

$$[K] = \int_0^l [\Phi^*]^T \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{12} & C_{22} & C_{23} & C_{24} \\ C_{13} & C_{23} & C_{33} & C_{34} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{bmatrix} [\Phi^*] dx_1 \tag{18}$$

Calculation of $\{Q\}$ will be discussed later. Analytical expressions for the above integrals were also derived for the case of uniform cross-sectional beams.

Aerodynamic Model

To model the 3-D unsteady aerodynamics of a wing in a accurate way, for arbitrary motion, is a complex problem in itself, requiring a lot of computational effort. The goal of this excersise is to obtain an efficient aeroelastic tool, to be used for aeroelastic tailoring during preliminary design. Thus a simple 2-D aerodynamics in used which would give useful trends to the designer. The 2-D unsteady aerodynamic model is integrated over the span to get the total unsteady lift and moments.

Two-Dimensional Unsteady Aerodynamics

Expressions for the unsteady aerodynamic lift (L) and moment (M) on an airfoil undergoing simple harmonic motions have been derived by Theodorsen. The motion is restricted to a combination of plunge (h) and pitch (α) motions. The lift and moment expressions are derived in terms of the amplitudes of motion and a non-dimensional parameter called reduced frequency $k = \omega b/V$. ω is the frequency of oscillation, b is the semi-chord and V is the freestream velocity. For a swept wing the sweep angle (Λ) also effects the aerodynamics. The expressions for the lift and pitching moment about the elastic axis (axis of pitching) can be found in the aeroelasticity text by Bisplinghoff *et al.*[16]

$$\bar{L} = -\pi\rho\omega^2\bar{b}^3 \left\{ \frac{h}{\bar{b}} L_{hh} + \sigma L_{hh'} + \bar{\alpha} L_{h\alpha} + \bar{b}\tau L_{h\alpha'} \right\} \tag{19}$$

$$\bar{M} = \pi\rho\omega^2\bar{b}^4 \left\{ \frac{h}{\bar{b}} M_{\alpha h} + \sigma M_{\alpha h'} + \bar{\alpha} M_{\alpha\alpha} + \bar{b}\tau M_{\alpha\alpha'} \right\} \tag{20}$$

where,

$$\sigma = \frac{\partial h}{\partial \bar{y}} \quad \text{and} \quad \tau = \frac{\partial \alpha}{\partial \bar{y}} \tag{21}$$

and ρ is the air density. The quantities with a bar ($\bar{\ }$) are measured with respect to the swept wing coordinate system. Thus, $\bar{y}$ is the spanwise distance along the swept wing (elastic axis), $\bar{b}$ is the semichord perpendicular to the elastic axis and $\bar{\alpha}$ is the pitch angle measured about the elastic axis. The expressions for L_{hh}, $L_{hh'}$, $L_{h\alpha}$, $L_{h\alpha'}$, $M_{\alpha h}$, $M_{\alpha h'}$, $M_{\alpha\alpha}$ and $M_{\alpha\alpha'}$ can be written in terms of the reduced frequency (k), the sweep angle (Λ) and a the distance between the midchord and the elastic axis.

The total lift and moment can be written in terms of the kinematic variables by recognizing that,

$$h = -u_3 \qquad \bar{\alpha} = \theta_1 \tag{22}$$

Development of Generalized Aerodynamic Forces

Lagrange's equations were developed from kinetic and strain energy formulations for a uniform, cantilevered, thin-walled, closed-section beam and are given by

$$[M]\{\ddot{q}\} + [K]\{q\} = \{Q\} \tag{23}$$

where, $[M]$ is the $n \times n$ generalized mass matrix, $[K]$ is the $n \times n$ generalized stiffness matrix, $\{Q\}$ is the $n \times 1$ generalized force vector, and $\{q\}$ are the generalized coordinates. The generalized aerodynamic forces are developed from the principle of virtual work. The virtual work done on a two-dimensional airfoil section is,

$$\overline{\delta W} = \bar{L} \cdot \delta u_3 + \bar{M} \cdot \delta\theta_1 \tag{24}$$

where, $\bar{L}$, $\bar{M}$ are the section lift and moment, while δu_3, $\delta\theta_1$ are the virtual displacements. The virtual work can be written in a compact form as

$$\overline{\delta W} = \{P\}^T \{\delta\bar{u}\} \tag{25}$$

where P and δu are 4×1 column matrices given by

$$P = \begin{Bmatrix} 0 \\ \bar{M} \\ \bar{L} \\ 0 \end{Bmatrix} \quad \text{and} \quad \delta\bar{u} = \begin{Bmatrix} \delta u_1 \\ \delta\theta_1 \\ \delta u_3 \\ \delta u_2 \end{Bmatrix} \tag{26}$$

The virtual work can be expressed in terms of virtual generalized displacements by expanding the column of virtual displacements using the modal matrix (Φ), thus we get

$$\overline{\delta W} = \{P\}^T \left[\frac{\partial\bar{u}}{\partial q}\right] \{\delta q\} = \{P\}^T [\Phi] \{\delta q\} \tag{27}$$

The coefficient of δq is the transpose of the generalized aerodynamic forces on the section. Integrating over the wing span, we obtain the generalized aerodyanmic forces as

$$\{Q\} = \int_0^L [\Phi]^T \{P\}\, dx \tag{28}$$

The column matrix P can be represented in terms of the displacements and their spanwise derivatives i.e pitch $\bar{\alpha} = \theta_1$, plunge $h = -u_3$, and the spanwise derivatives $\sigma = \frac{\partial h}{\partial \bar{y}}$ and $\tau = \frac{\partial \alpha}{\partial \bar{y}}$, using Theodorsen's expressions for lift and pitching moment,

$$\begin{aligned}\{P\} &= \begin{Bmatrix} 0 \\ \bar{M} \\ \bar{L} \\ 0 \end{Bmatrix} \\ &= \omega^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \pi\rho\bar{b}^4 M_{\alpha\alpha} & -\pi\rho\bar{b}^3 M_{\alpha h} & 0 \\ 0 & -\pi\rho\bar{b}^3 L_{h\alpha} & \pi\rho\bar{b}^2 L_{hh} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \{\bar{u}\} \\ &+ \omega^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \pi\rho\bar{b}^4 M_{\alpha\alpha'} & -\pi\rho\bar{b}^3 M_{\alpha h'} & 0 \\ 0 & -\pi\rho\bar{b}^3 L_{h\alpha'} & \pi\rho\bar{b}^2 L_{hh'} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \{\bar{u}'\}\end{aligned} \tag{29}$$

Now, the column matrices $\bar{u}$ and $\bar{u}'$ could be expanded interms of the generalized coordinates as

$$\{\bar{u}\} = [\Phi]\{q\} \quad \text{and} \quad \{\bar{u}'\} = [\Phi']\{q\} \tag{30}$$

where, $[\Phi']$ contains the first spanwise derivatives of the modeshapes. Thus we get the generalized aerodynamic forces as

$$\begin{aligned}\{Q\} &= \omega^2 \int_0^L \left([\Phi]^T [A] [\Phi] + [\Phi]^T [D] [\Phi']\right) dx \{q\} \\ &= \omega^2 [\bar{Q}] \{q\}\end{aligned} \tag{31}$$

where,

$$[A] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \pi\rho\bar{b}^4 M_{\alpha\alpha} & -\pi\rho\bar{b}^3 M_{\alpha h} & 0 \\ 0 & -\pi\rho\bar{b}^3 L_{h\alpha} & \pi\rho\bar{b}^2 L_{hh} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$[D] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \pi\rho\bar{b}^4 M_{\alpha\alpha'} & -\pi\rho\bar{b}^3 M_{\alpha h'} & 0 \\ 0 & -\pi\rho\bar{b}^3 L_{h\alpha'} & \pi\rho\bar{b}^2 L_{hh'} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{32}$$

Aeroelastic Analysis

Analysis of the flutter problem is greatly simplified when the mass matrix $[M]$ and the stiffness matrix $[K]$ in Eq. (23) are diagonal matrix. The mass and stiffness matrices in our analysis, however, contains off-diagonal terms that account for inertial/stiffness couplings in the beam. Inertial coupling arise when the center of mass does not coincide with the reference line, while stiffness coupling are due to the anisotropic nature of the composite material. To solve this problem, the equations of motion in Eq. (23) is recast in terms of a diagonalized mass and stiffness matrix. To diagonalize the system, the free vibration problem of the wing is solved first.

Free vibration analysis

For free vibration analysis, $\{Q\} \equiv 0$, thus Eq. (16) becomes

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \tag{33}$$

Now assuminq simple harmonic motion, i.e. $\{q\} = \{\bar{q}\}e^{i\omega t}$, we get an eigenvalue problem, whose eigenvalues give the frequencies and eigenvectors the modeshapes of vibration.

$$[K]\{\bar{q}\} = \omega^2[M]\{\bar{q}\} \tag{34}$$

The eigenvectors $\bar{q}_i$ and eigenvalues ω_i^2 of this problem are obtained and used to simplify the equations of motion in the flutter problem. This is done by defining a new set of genaralized coordinates in terms of the coupled, free vibration modes.

$$\{q\} = [\Upsilon]\{\bar{q}\} \tag{35}$$

where,

$$[\Upsilon] = [\bar{q}_1 \quad \bar{q}_2 \quad \ldots \quad \bar{q}_n] \tag{36}$$

Pre-multiplying the Langrange's equations, Eq. (23, by $[\Upsilon]^T$ and using the above relation for $\{q\}$, we get,

$$\left[\Upsilon^T\right][M][\Upsilon]\{\ddot{\bar{q}}\}+\left[\Upsilon^T\right][K][\Upsilon]\{\bar{q}\} = \omega^2\left[\Upsilon^T\right]\left[\bar{Q}\right][\Upsilon]\{\bar{q}\} \quad (37)$$

i.e. $$[M]^D\{\ddot{\bar{q}}\}+\left[K^D\right]\{\bar{q}\}-\omega^2\left[\tilde{Q}\right]\{\bar{q}\}=0 \quad (38)$$

The matrix $\left[M^D\right] = [\Upsilon]^T[M][\Upsilon]$ is the diagonalized mass matrix, and the matrix $\left[K^D\right] = [\Upsilon]^T[K][\Upsilon]$ is the diagonalized stiffness matrix. Matrix $\left[\tilde{Q}\right] = [\Upsilon]^T\left[\bar{Q}\right]\Upsilon$ is the new generalized aerodynamic force matrix.

Structural Damping

The Eq. (38) gives the equation of motion of a wing in an airstream. But, the aerodynamic model chosen is valid only for simple harmonic motion. So to force the system to undergo harmonic oscillations, an artificial structural damping term is added. The structural damping is know to be a force proportional to the displacement but in-phase with velocity. For a system undergoing simple harmonic motion $x = x_0e^{i\omega t}$, the the velocity is $ix_0\omega e^{i\omega t}$, thus the structural damping force can be represented by $F_D = i \times const. = igK$, where K is a stiffness parameter and g is an ad hoc artificial damping coefficient. Adding artificial structural damping to our system, Eq. (38) becomes

$$\left[M^D\right]\{\ddot{\bar{q}}\}+ig\left[K^D\right]\{\bar{q}\}+\left[K^D\right]\{\bar{q}\}-\omega^2\left[\tilde{Q}\right]\{\bar{q}\}=0 \quad (39)$$

Flutter Solution

Eq. (39) gives the flutter equation to be solved. After the addition of the artificial damping term the system undergoes harmonic oscillations. Thus, $\bar{q} = \bar{q}e^{i\omega t}$. The flutter equation thus becomes,

$$(-\omega^2)\left[M^D\right]\{\bar{q}\}+ig\left[K^D\right]\{\bar{q}\}+\left[K^D\right]\{\bar{q}\}-\omega^2\left[\tilde{Q}\right]\{\bar{q}\}=0 \quad (40)$$

Rearranging the terms, the above equation can be posed as a eigenvalue problem,

$$\left[\left[K^D\right]^{-1}\left(\left[M^D\right]+\left[\tilde{Q}\right]\right)\right]\{\bar{q}\} = \frac{1+ig}{\omega^2}\{\bar{q}\} \quad (41)$$

If the reduced frequency k, is specified then the generalized aerodynamic forces can be calculated, which in general will be complex quantities. The above complex eigenvalue problem can be solved to get the complex eigenvalues, $(\frac{1+ig}{\omega^2})$, and eigenvectors for a given reduced frequency. The eigenvalues give the frequency (ω) and damping coefficient (g) of the motion. Knowing the reduced frequency ($k = \frac{\omega b}{V}$), the velocity is found. Similar computations are done over a large range of reduced frequencies and the variation of ω and g is plotted. At the point where g of any mode becomes zero, the flutter is reached, cause at that point no structural damping is required to keep the system in harmonic oscillations. The corresponding modeshape is the flutter modeshape.

Numerical Results

A modular computer code has been developed which is a direct implementation of the aforementioned structural and aerodynamic theories. The code uses a linearized beam model (solution based on the Rayleigh-Ritz method) and 2-D Theodorsen's unsteady aerodynamic formulation (strip theory). First, a static aeroelastic tailoring example is considered and compared with available results. Next, flutter analysis of a test wing is done for validation, followed by some dynamic aeroelastic tailoring results.

Static Aeroelastic Tailoring

Given a geometry and material distribution for the wing cross section, the aeroelastician must have at hand consistent stiffness constants to be used in the beam analysis. Up to now, few asymptotically correct cross-sectional analysis formulations are available. For our case of thin-walled single-cell composite beams, the work of Ref. 14 is used. Even though not asymptotically correct, the work of Ref. 18 is also used in this paper as a simple way to get approximate transverse shear stiffness constants analytically.

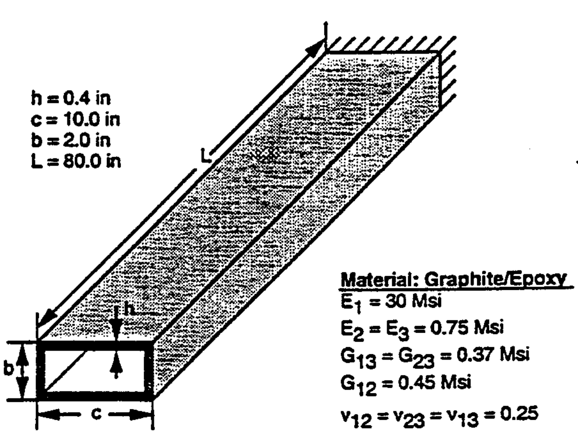

Fig. 1. Geometry of the box beam

As a first configuration test, the wing used in Ref. 13 is considered for the present numerical study.

The wing is prismatic, and the planform is shown in Fig. 1. The structural model is a box beam made of Graphite/Epoxy (properties described in Fig. 1) and the ply angle can be varied from $-90°$ to $90°$, depending on the configuration of the cross section (Fig. 2). The sweep angle Λ is allowed to vary.

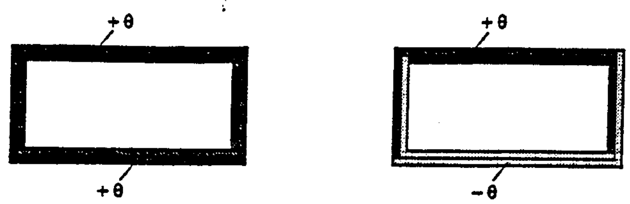

Fig. 2. CUS (left) and CAS (right) configurations (Ply angle θ measured w.r.t outward normal axis)

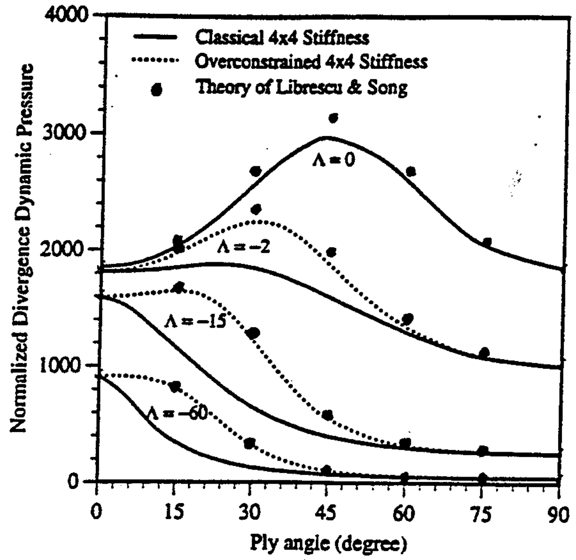

Fig. 3. Variation of divergence dynamic pressure with ply angle for a CUS configuration (normalized with respect to the divergence dynamic pressure for 0° ply angle)

Fig. 3 shows the variation of divergence dynamic pressure with ply angle for a circumferentially uniform stiffness (CUS) configuration, including different values of the sweep angle Λ. The CUS configuration produces extension-twist coupling and the fiber orientation in the cross section is represented in Fig. 2 (left). When the authors of Ref. 13 studied this configuration, they were interested in the effects of transverse shear in the divergence speed. The symbols showed in Fig. 3 are samples of their numerical results without the inclusion of transverse shear. As discussed in Ref. 19, there are basically two ways to get a 4×4 stiffness model from a 6×6 stiffness formulation for the anisotropic beam. The first is achieved by just neglecting the transverse shear effects all together from the stiffness matrix. This does not lead to a correct 4×4 matrix, over-estimating some of the stiffness constants (see dotted lines in Fig. 3). The second approach is the consistent one, done by minimization of the strain energy with respect to the transverse shear measures (solid lines in Fig. 3). By doing so, the important contribution of the coupling terms between transverse shear and the classical measures are correctly accounted for. This result can be directly achieved by using an asymptotically correct classical formulation, as done in Refs. 14.

For this particular example, the 6×6 cross-sectional stiffness constants were obtained by using Ref. 18. Even though not described in Ref. 13, the results suggest that the authors used the first method of disregarding transverse shear effects. As one can see from Fig. 3, when the bending stiffness starts dominating the behavior of the wing (large sweep angles and $15° \leq \theta \leq 60°$), even the qualitative behavior changes from the two stiffness models. The missing effects that are totally associated with transverse shear[13] are in part caused by the reduction of the effective bending stiffness due to the bending-shear coupling (present in a CUS configuration). This just reinforces the fact that the aeroelastician has to have available a consistent stiffness model to be used in the analysis.

Flutter Tailoring

For lack of published flutter results for composite box beams, the code is validated by comparing the flutter speed of Goland's typical wing[20] Fig. 4 shows the V-g plot obtained for this case. The flutter and divergence point can be easily spotted. The present theory gives a flutter speed of 445 fps as compared to the exact flutter speed of 450 fps, and the flutter frequencies are, respectively, 70 rad/s and 70.7 rad/s (both with 1.0% relative error).

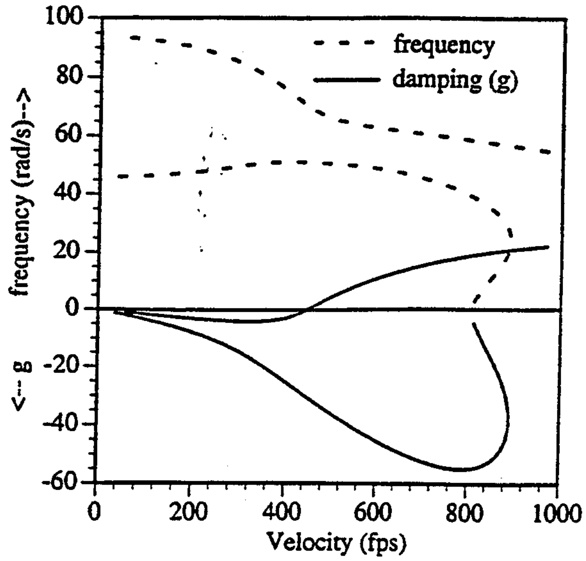

Fig. 4. V-g plot for Goland's wing

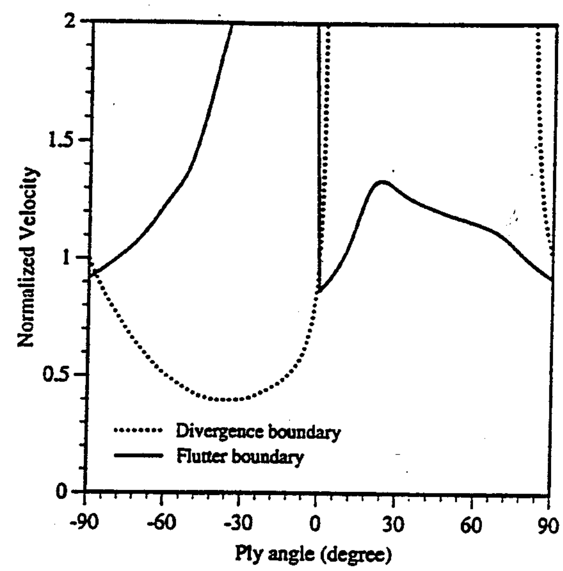

Fig. 5. Variation of flutter and divergence velocities with ply angle for a box beam wing (CAS configuration) – normalized with respect to the divergence velocity for 0° ply angle

Now for the composite box beam wing, consider the variation of the divergence and flutter speeds with ply angle for a circumferentially asymmetric stiffness (CAS) configuration. The CAS configuration produces vertical bending-twist coupling, the fiber orientation in the cross section is represented in Fig. 2 (right). As for divergence, positive ply angles produce a favorable bending-twist coupling, leading to a very high divergence speed, whereas a negative ply angle shows lower divergence speed (see Fig. 5). The flutter results are more interesting and thought provoking. Flutter involves dynamic interaction of various modes. Also, the normal modes of vibration of the composite beam change with ply angle, thus leading to a change in the flutter mode shape. In Fig. 5, only the lowest flutter speed is represented. The plot is not smooth due to the changes of the lowest flutter mode shape. Future work will include examining these flutter mode shapes and its variation with ply angle, which should provide a better understanding about the phenomenon.

Conclusions

Aeroelastic stability is an important factor in the design of modern flexible wing aircraft. The possibility of using material couplings in the structural tailoring process opens new frontiers to the design of a composite wing. As shown, both static and dynamic aeroelastic stability can be altered by those couplings. The present work discussed some basic analysis tools to be used in preliminary design of high-aspect ratio composite wings.

Aeroelastic analysis has been implemented for linear models of anisotropic closed section beams using Theodersen's function. Goland's typical wing is used to verify the procedure. Box beam configurations with different levels of vertical bending-twist and extension-twist couplings are used to represent composite wings. Results indicate the possibility of changing aeroelastic characteristic of a wing by tailoring the composite structure. Unlike earlier parametric studies, this study used a box-beam configuration which is a closer approximation of a wing. Due to the complexity of the possible couplings present in the structure, special attention must be given to the cross-sectional stiffness constants used in the analysis and their effects in the aeroelastic response.

References

[1] Shirk, M. H., Hertz, T. J., and Weisshaar, T. A., "Aeroelastic Tailoring – Theory, Practice, and Promise," *J. Aircraft*, Vol. 23, No. 1, Jan. 1986, pp. 6–18.

[2] Munk, M. M., "Propeller Containing Diagonally Disposed Fibrous Material, U. S. Patent 2,484,308,1111, Oct. 1949.

[3] Krone, N. J., *Divergence Elimination with Advanced Composites*, Ph.D. Thesis, University of Maryland, College Park, MD, Dec. 1974.

[4] Weisshaar, T. A., "Divergence of Forward Swept Composite Wings," *J. Aircraft*, Vol. 17, No. 6, June 1980, pp. 442–448.

[5] Weisshaar, T. A., "Aeroelastic Tailoring of Forward Swept Composite Wings," *J. Aircraft*, Vol. 18, No. 8, Aug. 1981, pp. 669–676.

[6] Weisshaar, T. A., "Aeroelastic Tailoring – Creative Uses of Unusual Materials," In *Proceedings of the 28th Structures, Structural Dynamics, and Materials Conference*, Monterey, California, April 6–8, 1987, AIAA-87-0976-CP.

[7] Sherrer, V. C., Hertz, T. J., and Shirk, M. H., "Wind Tunnel Demonstration of Aeroelastic Tailoring Applied to Forward Swept Wings," *J. Aircraft*, Vol. 18, No. 11, Nov. 1981, pp. 976–983.

[8] Housner, J. M. and Manuel Stein, "Flutter Analysis of Swept–Wing Subsonic Aircraft with Parametric Studies of Composite Wings," Technical report, NASA–TN–D–7539, Sept. 1974.

[9] Hollowell, Steven J. and Dugundji, John, "Aeroelastic Flutter and Divergence of Stiffness Coupled, Graphite/Epoxy Cantilevered Plates," *J. Aircraft*, Vol. 21, No. 1, Jan. 1984, pp. 69–76.

[10] Lottati, I., "Flutter and Divergence Aeroelastic Characteristics for Composite Forward Swept Cantilevered Wing," *J. Aircraft*, Vol. 22, No. 11, Nov. 1985, pp. 1001–1007.

[11] Green, J. A., "Aeroelastic Tailoring of Aft-Swept High-Aspect-Ratio Composite Wings," *J. Aircraft*, Vol. 24, No. 11, 1987, pp. 812–819.

[12] Guruswamy, G. P. and Byun, C., "Fluid-Structural Interactions Using Navier-Stokes Flow Equations Coupled With Shell Finite Element Structures," In *Proceedings of the AIAA 24th Fluid Dynamics Conference*, Orlando, FL, July 6–9, 1993, pp. 1–10, AIAA-93-3087.

[13] Librescu, L. and Song, O., "On the Static Aeroelastic Tailoring of Composite Aircraft Swept Wings Modelled as Thin-Walled Beam Structures," *Composites Engineering*, Vol. 2, No. 5–7, 1992, pp. 497–512.

[14] Berdichevsky, V., Armanios, E., and Badir, A., "Theory of Anisotropic Thin–Walled Closed–Cross–Section Beams," *Composites Engineering*, Vol. 2, No. 5–7, 1992, pp. 411–432.

[15] Armanios, E. A. and Badir, A. M., "Free Vibration Analysis of Anisotropic Thin–Walled Closed–Section Beams," In *Proceedings of the 35th Structures, Structural Dynamics, and Materials Conference*, Hilton Head, South Carolina, April 1994, pp. 164–171, AIAA-94-1327-CP.

[16] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison – Wesley Publishing Co., Reading, MA, 1955.

[17] Chang, Tish-Chun and Craig, Roy R., "On Normal Modes of Uniform Beams," Technical Report EMRL 1068, Engineering Mechanics Research Laboratory The University of Texas at Austin, January 1969.

[18] Rehfield, L. W., Atilgan, A. R., and Hodges, D. H., "Nonclassical Behavior of Thin-Walled Composite Beams with Closed Cross Sections," *Journal of the American Helicopter Society*, Vol. 35, May 1990, pp. 42–50.

[19] Hodges, D. H., Atilgan, A. R., Cesnik, C. E. S., and Fulton, M. V., "On a Simplified Strain Energy Function for Geometrically Nonlinear Behaviour of Anisotropic Beams," *Composites Engineering*, Vol. 2, No. 5 – 7, 1992, pp. 513 – 526.

[20] Goland, M., "The Flutter of a Uniform Cantilever Wing," *Journal of Applied Mechanics*, Vol. 12, No. 4, December 1945, pp. A197 – A208.