ICAS-98-6,6,3

A98-31686

AIRCRAFT OPERATIONAL MANAGEMENT BASED ON STATE-ESTIMATION

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RESUME

The technical state of a system changes randomly during its operation. It can be determined by so called internal system parameters. Therefore the momentary technical state of the system can be described mathematically as a point of the multidimensional state space defined by its internal parameters. The main goals of the technical diagnostics are the determination of momentary location of the examined system and to forecast direction and velocity of its movement in the abovementioned state space. Knowing momentary technical state and its changing, you can decide the most optimal operational strategy and forecast the needed service work. This paper will show the mathematical diagnostic modeling and its possibility of use of state-estimation based on mathematical modeling for the operational management by case of pneumatic brake system of the helicopter MI - 8 HIP.

1. INTRODUCTION

During the operation, system of aircraft wears out by stochastic effects ⁽⁸⁾. Therefore its technical state goes through continuous and cumulative changes. During the usage, technical state generally changes in a negative sense, while during the maintenance or repair, it changes positively. Technical state of the system is determined by its internal parameters (for example stiffness of spring, resistance). Therefore momentary technical state can be characterized as a point of the multidimensional state space defined by internal parameters.

Main goals of the technical diagnostics are

determination of momentary location of the investigated system and to forecast direction and velocity of its movement in this state space ⁽⁶⁾.

In the practice, these internal parameters cannot be determined directly because of technical and economic problems. Their values and changing velocities can be determined using a state-estimation method based on mathematical modeling. Knowing momentary technical state and its changing velocity, the optimal operational strategy and needed service work can be decided.

2. STATE-ESTIMATION BASED ON MATHEMATICAL MODELING

A model gives the most concise characterization of the investigated phenomenon ⁽⁹⁾. The mathematical model is the mathematical equation or system of equations which describes the internal principles of process occurring on the system, and its solution ⁽⁴⁾.

The setting up of a mathematical model should start from splitting up of the investigated system into its functional units. These units should be examined and interdependencies between their input and output parameters should be established mathematically. A mathematical model can be written by the WHITE BOX method (by analytical equations on the basis of scientific knowledge) or the BLACK BOX method (by analyzing of output parameters responded to known input ones) (1).

The equations described above form the model of the system, which can be written in general case by the following vector-equation:

$$f(y) = g(x) \tag{1}$$

For setting up of the linear diagnostic model of the investigated system, the mathematical model should be linearized. For linearization you can use the LOGARITHMIC LINEARIZATION or the DIRECT DIFFERENTIATION methods, TAYLOR or LIE-MAGNUS series expansions of the general (non-linear) mathematical system of equations. System of equation got in one of the above-mentioned ways describes interdependencies between

$$\delta \eta = \frac{d\eta}{\eta_{nom}} \tag{2}$$

relative changes of internal and external parameters. This linear model can be written in the following matrix formula:

$$A\delta y = B\delta \underline{x} \tag{3}$$

In the basis of the equation (3) and knowledge of values of measurable external parameters and coefficient matrices, the values of internal parameters, that is technical state of the system, should be estimated. This task can be solved using the

$$\underline{\underline{D}} = \underline{\underline{A}}^{-1} \underline{\underline{B}} \tag{4}$$

"classical diagnostic matrix".

Knowing the nominal values of internal parameters, that is the matrix

$$\underline{X} = \begin{bmatrix} x_{1nom} & K & K & 0 \\ 0 & x_{2nom} & M \\ M & O & M \\ 0 & K & K & x_{pnom} \end{bmatrix} ,$$
(5)

and their momentary measured parameters (\underline{x}) , vector of their relative changes is:

$$\delta \underline{x} = \underline{\underline{X}}^{-1} \underline{x} - \underline{e}_p \tag{6}$$

Then the equation (4) should be modified to

$$\delta \underline{y} = \underline{\underline{D}} \underline{X}^{-1} \underline{x} - \underline{\underline{D}} \underline{e}_{p} \tag{7}$$

Using the nominal values matrix of external parameters and the k-dimensional summary vector \underline{e}_k , the measured values vector of external parameters can be determined by equation

$$\underline{y} = \underline{\underline{Y}} \delta \underline{y} + \underline{\underline{Y}} \underline{e}_k = \underline{\underline{Y}} \underline{D} \underline{X}^{-1} \underline{x} - \underline{\underline{Y}} \underline{D} \underline{e}_p + \underline{\underline{Y}} \underline{e}_k$$
 (8)

If "measured diagnostic matrix"

$$\underline{S} = \underline{Y}\underline{D}\underline{X}^{-1} \tag{9}$$

and auxiliary vector

$$\underline{u} = y + \underline{Y}\underline{D}\underline{e}_p - \underline{Y}\underline{e}_k \tag{10}$$

have been introduced, the equation (8) can be simplified

$$\underline{u} = \underline{Sx} \tag{11}$$

On the basis of momentary measured values of external parameters (that is vector \underline{y}), the measured values of internal ones should be determined by above equation. The vector \underline{x} that satisfies this equation should be estimated by using any search of optimum method in case of scalar-vector function:

$$f(\underline{x}) = (\underline{u} - \underline{S}\underline{x})^2 \tag{12}$$

To estimate vector \underline{x} you can use the GRADIENT, the RANDOM or the GAUSS-SEIDLER methods.

It is important to mention that the above described state-identification procedure cannot give unambiguous results because it uses any estimation of values method.

3. WEARING-OUT PROCESS OF SYSTEMS

During the operation, a technical system wears out by stochastic effects. Therefore its technical state goes through continuous, cumulative stochastic changes. In case of the usage the technical state generally changes in a negative sense, while during the maintenance or repair, it changes positively ⁽⁵⁾.

For demonstration, let general parameter n

characterize the technical state of the investigated system (see fig. 1). If the value of parameter η meets the η_b brake value, the system will break-down. Let τ be the parameter which characterizes the performance of the system. For example, this parameter can be the effective calendar time, effective operating hours (in case of the airframe), number of landings (in case of landing gear systems), or number of starts (in case of gas-turbine engines) from installation or the last overhaul.

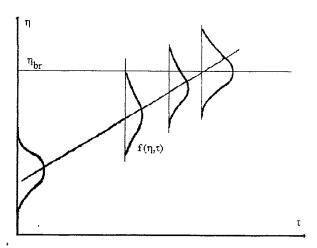


FIGURE 1 - The Wearing-out Process

In this case the wearing-out process of the system, that is the $\eta(\tau)$ stochastic function can be characterized by:

$$\eta(\tau)$$
 expected value function of the parameter η ; density function of parameter η .

Then the probability of good working state of the system:

$$P_{gw}(\tau) = P(\eta_{br} > \eta(\tau)) = \int_{-\infty}^{\eta_{br}} f(\eta, \tau) d\tau \quad . \tag{13}$$

The process of changing of parameter η can be describe by:

changing velocity of the parameter
$$\eta$$
; $\varphi\begin{pmatrix} 0 \\ \eta, \tau \end{pmatrix}$ density function of the changing velocity.

Then the "failure changing velocity" of parameter η is:

$$\eta_{br}(\tau, \Delta \tau) = \frac{\eta_{br} - \eta(\tau)}{\Delta \tau} \quad \text{if} \quad \eta_{br} > \eta(\tau) \tag{14}$$

and the probability of good working state of the system in the interval $(\eta, \eta + \Delta \eta)$:

$$P_{gw}(\tau, \Delta \tau) = P\begin{pmatrix} 0 \\ \eta_{br}(\tau) > \eta(\tau) \end{pmatrix} = \int_{-\infty}^{0} \varphi(\eta, \tau) d\eta$$
(15)

supposing that the system is ready to service at the start of the investigated performance interval.

4. OPERATIONAL MANAGEMENT METHOD

For exact and manageable comparison of different technical states and management of operational process, the so called leader parameter should be introduced. The leader parameter is the most important one for operation and maintenance of the system. This should be one of parameters estimated above or a parameter which can be determined directly from internal ones. For example, the leader parameter can be thrust or useful power in case of engines.

Depending on the momentary values of the leader parameter and its velocity, the needed service work can be decided. For decision, permissible value and velocity of the leader parameter should be determined on the basis of its breakdown value and permissible probability of risk ⁽⁷⁾.

Knowing the breakdown value η_{br} of the parameter η and performance interval between checks $\Delta \tau$, the permissible value η_p and permissible changing velocity to ready for working should be determined. Supposing that:

- the change of the parameter η on interval $\Delta \tau$ (see figure 2) is a linear one;
- density function of the changing velocity is independent on working time of the system.

4.1. Determination of Permissible Velocity

In this case, if value of the parameter η reaches the permissible value η_p at the *i*-th checking

and it changes with

$$\stackrel{\circ}{\eta} > \frac{\Delta \eta}{\Delta \tau}$$

velocity, the parameter η is going to reach breakdown value η_{br} before next (i+l-th) check, in the other words the operated system will break-down.

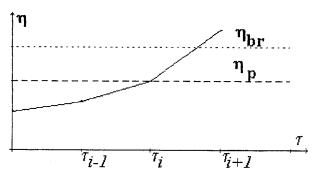


FIGURE 2 - Determination of the Permissible Parameter Values

Therefore, permissible velocity of parameter η to ready for working is:

$$\eta_p^{\circ} = \frac{\Delta \eta}{\Delta \tau}$$
(16)

The probability of breakdown is:

$$P_{br}(\Delta \tau, \Delta \eta) = P\left(\stackrel{\circ}{\eta} > \stackrel{\circ}{\eta}_{br}\right) = 1 - P\left(\stackrel{\circ}{\eta} \le \stackrel{\circ}{\eta}_{br}\right) =$$

$$=1-\int_{-\infty}^{\eta_{br}} \varphi(\eta) d\eta$$
 (17)

Knowing permissible probability of risk Q (permissible probability of breakdown), it is substituted into equation (17), equation

$$Q = P_{br}(\Delta \tau, \Delta \eta) = 1 - \int_{-\infty}^{\eta_p} \varphi(\eta) d\eta$$
 (18)

is got.

4.2. Determination of Permissible Value

If the density function of velocity η cannot be determined by statistical method, usage of one of

known density functions is suitable. For example (2):

UNIFORM distribution:

$$\varphi(\eta) = \frac{1}{\eta_{\text{max}} - \eta_{\text{min}}} = \frac{1}{\Delta \eta} \quad \text{(if } \eta_{\text{max}} > \eta > \eta_{\text{min}})$$

$$(19)$$

Then

$$Q = 1 - \int_{-\infty}^{\stackrel{\circ}{\eta_p}} \frac{1}{\circ} d \stackrel{\circ}{\eta} = 1 - \frac{\stackrel{\circ}{\eta_p}}{\circ} = 1 - \frac{\Delta \eta}{\Delta \tau \Delta \eta} , \qquad (20)$$

that is

$$\Delta \eta = (1 - Q)\Delta \tau \Delta \stackrel{\circ}{\eta} \quad . \tag{21}$$

EXPONENTIAL distribution:

$$\varphi(\eta) = \lambda e^{-\lambda \eta}$$
 (if $\eta > 0$) . (22)

Then

$$Q = 1 - e^{-\lambda \stackrel{\circ}{\eta}_p} \qquad , \tag{23}$$

and

(17)
$$\Delta \eta = -\frac{\ln(1-Q)}{\lambda} \Delta \tau \tag{24}$$

NORMAL (GAUSS) distribution:

$$\varphi(\mathring{\eta}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\mathring{\eta}-m)^2}{2\sigma^2}} \tag{25}$$

In this case simply solution cannot be gotlike to above ones which is deduced easily in algebraical way. Therefore, on the basis of its variance and expected value, transforming the normal distribution to the standard normal one, the permissible velocity of parameter η and parameter interval $\Delta \eta$ can be determined.

The permissible value of the parameter η to

ready for working:

$$\eta_p = \eta_{br} - \Delta \eta \qquad . \tag{26}$$

If momentary values η and η smaller that those determined by equation (26) and (16), the system will not break down till the next check with probability of least 1-Q.

5. USAGE OF THE METHOD

For demonstrating the possibility of use of above mentioned method, the setting up and usage of mathematical model of brake-system of the helicopter MI-8 will be shown.

The task of pneumatic system of helicopter MI-8 are the braking of main undercarriage wheels and to be compressed air-source in case of field-operation ⁽³⁾.

After splitting up of the system into functional units - using the above-mentioned WHITE BOX method - the interdependencies between their input and output parameters are written mathematically. The diagnostic model was set up by LOGARITHMIC LINEARIZATION of these equations.

The internal and external parameters of the examined system were determined by equations (27) and (28):

$$\underline{y} = \begin{bmatrix} p_H \\ p_{br} \\ p_{ab} \\ z_1 \\ \vdots \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} , \qquad (27)$$

$$\underline{x} = \begin{bmatrix}
 P_{c} \\
 F_{s1} \\
 F_{s2} \\
 F_{s3} \\
 F_{1} \\
 S_{2} \\
 F_{1} \\
 S_{2} \\
 F_{2} \\
 S_{3} \\
 F_{3} \\
 F_{3} \\
 F_{4} \\
 P_{t}$$
(28)

The coefficient matrices of the vectors are described by equations (29) and (30).

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 3,42e-1 \\ -1,08 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1,05 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1,05 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1,05 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1,05 & 0 & 0 & 0 & 1 & 0 \\ 3,42e-4 & 1,63e-2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} ,$$

$$(29)$$

The nominal value matrix of internal parameters:

The nominal value matrix of external parameters:

The "classical diagnostic matrix" of the pneumatic system of the helicopter MI-8:

```
-2,29e-2 \quad 9,87e-2 \quad -1,12e-4 \quad -1,42e-5 \quad 7,15e-21 \quad -1,42e-5 \quad 7,15e-21 \quad -1,42e-5 \quad 7,15e-21 \quad -1,42e-5 \quad 6,59e-19 \quad -3,5e-19 \quad
     1,28
     1.38
                                          -2,48e-2 4,85e-2 -2,01e-2 -1,53e-5 -6,33e-21 -1,53e-5 -6,33e-21 -1,53e-5 -6,33e-21 -1,53e-5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          211e-19 -3.79e-1
     145
                                                                                    1,7le-2 -2,1le-2 -1,29e-3 -1,79e-2 -1,6le-5 4,39e-21 -1,6le-5
                                                                                                                                                                                                                                                                                                                                                                                                   4,39e-21 -1,61e-5
                                                                                     1.71e-2 -2.11e-2 -1.61e-5 2.37e-19 -1.29e-3 -1.79e-2 -1.61e-5
    145
                                                                                                                                                                                                                                                                                                                                                                                                 4.39e-21 -1.6le-5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           5.32e-19 -3.98e-1
     145
                                          -2.6e-2
                                                                                     1.71e-2 -2.11e-2 -1.61e-5
                                                                                                                                                                                                                2,37e-19 -1,6le-5
                                                                                                                                                                                                                                                                                                          237e-19 -128e-3 -179e-2
                                                                                                                                                                                                                                                                                                                                                                                                                                       -16le-5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          5.32e-19 -3.98e-1
     145
                                                                                     1,7le-2 -2,1le-2 -1,6le-5 7,65e-19 -1,6le-5
                                                                                                                                                                                                                                                                                                        7.65e-19 -1.61e-5 7.65e-19 -1.28e-3 -1.79e-2 -3.98e-1
                                           4,12e-4 \quad 1,75e-4 \quad 3,28e-4 \quad 4,15e-5 \quad -1,33e-20 \quad 4,15e-5 \quad -1,33e-20 \quad 4,15e-5 \quad -1,33e-20 \quad 4,15e-5 \quad -7,54e-20
-2.29e-2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     103
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        (33)
```

The "measured diagnostic matrix":

```
1,49e1
        -1,38e-4 1,07e-5 -5,37e-8 -6,34e-6 3,15e-21 -6,34e-6 3,15e-21 -6,34e-6 3,15e-21 -6,34e-6
                                                                                                           2,9le-19 -1,73el
 5,18e-1 -4,81e-6 1,70e-7 -3,11e-7 -2,21e-7 -9,01e-23 -2,21e-7 -9,01e-23 -2,21e-7 -9,01e-23 -2,21e-7
                                                                                                           3,00e-21 -6,04e-1
3,51e-1 -3,26e-6 3,85e-8 -2,10e-7 -1,20e-7 -1,64e-4 -1,49e-7 4,08e-23 -1,49e-7 4,08e-23 -1,49e-7
                                                                                                           4,88e-21 -4.09e-1
4,27e9
        -3,97e4
                  4,69e2
                           -2,56e3
                                     -1,82e3
                                               2,65e-11 -1,49e-7 -2,00e6
                                                                             -1,82e3
                                                                                                            5,95e-11 -4,98e9
                                                                                       4,9le-13 -1,82e3
4,27e9
        -3,97e4
                  4,69e2
                           -2,56e3
                                     -1,82e3
                                               265e-11 -1,82e3
                                                                   265e-11 -1,46e5
                                                                                      -2,0026
                                                                                                 -1,82e3
                                                                                                            5,95e-11 -4,98e9
4,27e9
         -3,97e4
                  469e2
                           -2.56e3
                                     -182e3
                                               8,55e-11 -1,82e3
                                                                   8,55e-11 -1,82e3
                                                                                       8,55e-11 -1,46e5
                                                                                                           -2,0026
                                                                                                                     -49869
-6,77e7
         6,28e2
                  L3lel
                            3,98e1
                                     4,70e3
                                              -1,49e-12 4,70e3
                                                                  -1,49e-12 4,70e3
                                                                                                           -843e-12
                                                                                      -1,49e-12 4,70e3
                                                                                                                      1,28e10
                                                                                                                          (34)
```

The decrease of the brake-effort and brake asymmetry were chosen as leader parameters. To determine the permissible value and velocity of this leader parameters,

$$Q = 0.025$$

permissible probability of risk was used.

The quantity of data is not sufficient for statistical estimation of their distribution. Therefore, for determination of the permissible value and velocity of the resultant brake-effort, the density of its changing velocity is supposed as an uniform one see equations (19); (20) and (21).

Working time of the helicopter chosen for investigation was 52 hours 06 minutes (649.10 - 701.16) from 19 June to 20 November. The following figures show the test results depending on operating hours and depending on calendar time.

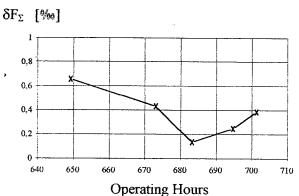


FIGURE 3 - Decrease of the Brake-Effort Depending on Operating Hours

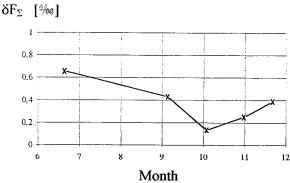


FIGURE 4 - Decrease of the Brake-Effort Depending on Calendar Time (of the Investigating Year)

$$\frac{d\delta F_{\Sigma}}{dt_{w}} \text{ [f hour}^{-5} \text{]}$$

Operating Hours
FIGURE 5 - Changing Velocity of the Brake-Effort
Depending on Operating Hours

$$\frac{d\delta F_{\Sigma}}{dt_{c}} [\text{day}^{-6}]$$

$$\downarrow 15$$

$$\downarrow 10$$

$$\downarrow 5$$

$$\downarrow 0$$

$$\downarrow 5$$

$$\downarrow 0$$

$$\downarrow 5$$

$$\downarrow 0$$

$$\downarrow -5$$

$$\downarrow -10$$

$$\downarrow -15$$

$$\downarrow 9$$

$$\downarrow 10$$

$$\downarrow 11$$

$$\downarrow 12$$

$$\downarrow 10$$

$$\downarrow 11$$

$$\downarrow 12$$

$$\downarrow 10$$

$$\downarrow 11$$

$$\downarrow 12$$

FIGURE 6 - Changing Velocity of the Brake-Effort Depending on Calendar Time (of the Investigating Year)

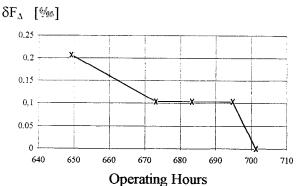


FIGURE 7 - Brake-Asymmetry Depending on Operating Hours

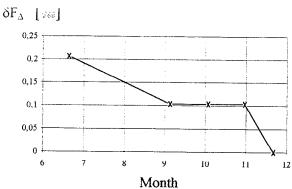
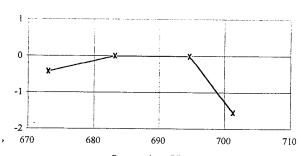


FIGURE 8 - Brake-Asymmetry Depending on Calendar Time (of the Investigating Year)

$$\frac{d\delta F_{\Delta}}{dt_{w}}$$
 [f.hour⁻⁵]



Operating Hours
FIGURE 9 - Changing Velocity of BrakeAsymmetry Depending on Operating Hours

$$\frac{d\delta F_{\Delta}}{dt_{c}} [\text{day}^{-6}]$$

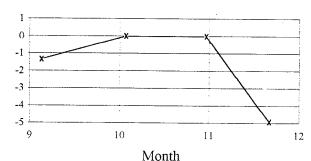


FIGURE 10 - Changing Velocity of Brake-Asymmetry Depending on Calendar Time (of Investigating Year)

5. SUMMARY

In this study the management method of aircraft system operation, as a Markov-process, based on state-estimation has been formulated in case of pneumatic system of helicopter MI-8. Technical data needed for usage of mathematical diagnostics and operational management method can be obtained by using gages of the helicopter and instrument of technical service team. Operation management method based on mathematical diagnostics is able to minimization of technical service work with adequate safety of operation. The possibility of use of shown method has been proven by examination of pneumatic brake-system a regular helicopter

6. NOMENCLATURE

6.1. Scalars

F	-	force;
m	-	mean value;
P	-	probability;
p	-	pressure;
Q	-	permissible probability of risk;
S	-	stiffness of spring;
Z	-	break-clearance;
η	-	general parameter;
σ	-	variance;
τ	-	general performance parameter

6.2. Vectors

<u>e</u>	-	summary vector;
\underline{u}	-	auxiliary vector;
<u>x</u>	-	vector of internal parameters;
¥	*	vector of external parameters;

6.3. Matrices

$\underline{\underline{A}}$	-	coefficient matrix of external
		parameters;
<u>B</u>	-	coefficient matrix of internal
		parameters;
$\underline{\underline{D}}$	-	diagnostic matrix;
<u>D</u> <u>S</u>	-	measured diagnostic matrix;
X	-	nominal value matrix of internal
		parameters;
Y	-	nominal value matrix of external
		narameters:

6.4. Others

ab - after break;

br - break / break-down;

c - control; qw - good work;

H - ambient;

max - maximum;

min - minimum; nom - nominal;

p - permissible;

s - spring;

t - tank;

 Δ - different:

 δ - relative different;

 Σ - resultant;

- expected;

° - velocity.

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