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PREDICTION OF FATIGUE CRACK GROWTH FROM BOLT HOLES IN A TITANIUM DISC

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Abstract

The growth of a corner crack emanating from tie bolt holes in a representative gas turbine compressor disc is analysed using a crack closure model. Experimental observations showed that a corner crack first initiated at the hole edge and then propagated to become a through crack, whose growth lead ultimately to the failure of the disc. It is found that when the influence of the biaxial stress is ignored, viz, the problem is treated as a corner crack growing from a hole under uniaxial loading, much shorter fatigue life is predicted. To account for the biaxial loading effect, it is assumed that the ratio of the stress intensity factors under biaxial loading and uniaxial loading is the same for a corner crack and a through crack, for a given crack length. This allows the influence of the biaxial loading to be readily determined using the through crack solution, which can be obtained using a two-dimension weight function method. Predictions using the improved stress intensity factor appear to correlate reasonably well with the experimental results obtained from spin rig tests.

Introduction

Prediction of fatigue crack propagation in failure critical gas turbine engine components subjected to cyclic loading conditions is an important tool in the life management philosophies[1]. One of the present research objectives of AMRL is to acquire and apply advanced lifing methodologies in order to accurately predict fatigue crack growth in engine compressor and turbine discs, which are often the life limited components.

To predict fatigue crack propagation rates accurately, crack closure based modelling techniques have been extensively studied and developed. During fatigue crack propagation, crack surfaces remain fully open at high tension stresses (during loading), while the crack surfaces come into premature contact or remain closed at low stress levels (during unloading) due to residual plasticity, crack surface roughness and oxidation [2].

The aim of this paper is examine fatigue crack propagation emanating from tie bolt holes in a representative gas turbine compressor disc, with particular emphasis on the effect of biaxial stress inherent in a rotating disc. To make a direct comparison with crack growth data measured in the spin rig test, the nominal stresses on the cracked disc (a corner crack at a hole and then a through thickness crack between two holes), were determined first using three dimensional finite element analysis. A solution was obtained by assuming that the ratio of the stress intensity factors under biaxial loading and uniaxial loading is the same for a corner crack and a through crack, for a given crack length. The crack propagation rate predicted by the modified crack model is found to correlate better with the experimental data measured in the spin rig test than that predicted by ignoring the biaxial load effect.

Spin Rig Test Results

The low cycle fatigue test of a representative disc was conducted in the spin rig test facility developed at AMRL[3]. The spin rig was driven by an air turbine operating over a cyclic constant rotational speed (8,000 — 37,000 — 8,000 rpm). The experiment was carried out at ambient temperature, in air at low vacuum. Crack length was measured by rubber replication. Crack lengths versus speed cycles during spin rig testing, starting from bolt hole A in Figure 1, are given in Table 1.

Table 1 Crack growth data from spin rig test

Crack growth mode	Cycles	Crack length (mm)
Corner	8533	0.39
ditto	8758	0.54
ditto	8963	0.88
ditto	9178	1.36

ditto	9400	3.00
Through thickness	9400	3.00
ditto	9401	3.10
ditto	9403	3.30
ditto	9405	3.55
ditto	9407	3.95
ditto	9409	4.00
ditto	9414	4.40
ditto	9419	4.70
ditto	9424	5.55
ditto	9434	9.70
ditto	9439	10.40
ditto	9449	18.90
ditto	9459	32.00
ditto	9469	Failure

Figure 1 shows the failed disc in spin rig test after 9469 cycles. During the test, a significant portion of the fatigue life was found to be spent in growing the crack from the hole A towards the hole B, as indicated by an arrow shown in Figure 1.

A sketch of the failure surface from hole A to hole B is shown in Figure 2. Fractographic analysis revealed that after a certain number of cycles a fatigue crack "initiated" firstly at one corner edge of hole A. After about 9400 cycles, the corner crack penetrated through the surface of the disc and became a through-thickness crack. Subsequently, the through-thickness crack propagated rapidly from hole A towards hole B, and finally the disc failed at 9469 cycles.

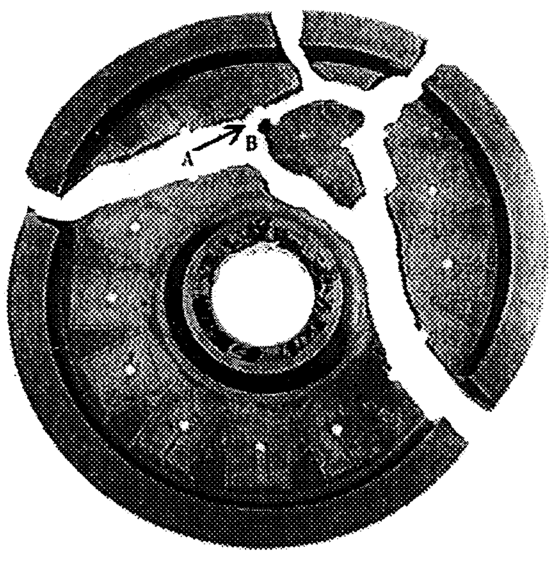

Figure 1 Failed disc in spin rig test

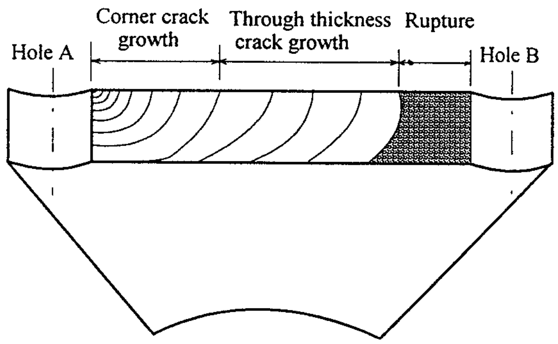

Figure 2 Fracture surface between the hole A and the hole B in failed disc

Disc Geometry and Loading Analysis

To determine the stresses present in the disc during the spinning test, a three-dimensional finite element analysis of the disc was carried out; the finite element mesh is shown in Figure 3. All dimensions are given in millimetres. The thickness in the region of the bolt holes is 3 mm, and the thickness at the hub and the rim is 15 mm. The mesh consists of 1248 20-node three-dimensional quadratic brick elements. The cyclic speed range used for the disc was 8,000 — 37,000 — 8,000 rpm. The state of stresses in the disc obtained from finite element calculation is found to be biaxial, that is, the structure in the bolt hole region is subjected to both radial and tangential (or hoop) stresses. In fact, the stress state is very close to equi-biaxial.

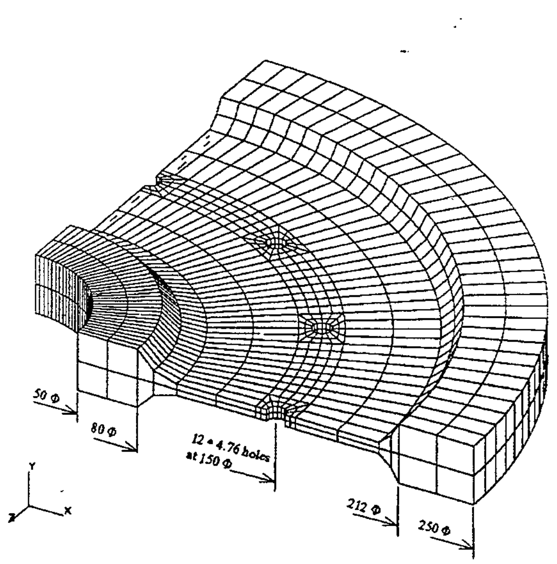

Figure 3 Disc geometry and 3D finite element mesh

The disc material used in the spin rig test is duplex annealed Ti-8Al-1Mo-1V titanium alloy[4]. Its ultimate tensile stress σ_u is 1049 MPa, and yield stress (0.2% offset) σ_y is 922 MPa. The elastic modulus E is 126 GPa. Poisson's ratio ν is 0.33 and the density ρ is 4373 kg/m^3.

From finite element calculation, the stress state in the bolt hole region is approximately uniform (with the variation over a distance equal to the diameter of the hole being negligible) It is thus not unreasonable to assume that the radial and tangential stresses are uniformly distributed along the boundary, as depicted in Figure 4. This will considerably simplify subsequent crack growth analysis. Here the symbol λ denotes the ratio of the stress along the x axis, which is the prospective crack path , to the stress along the y axis. The state corresponding to λ=0 represents uniaxial loading, λ=1 equi-biaxial, and λ=−1 shear loading.

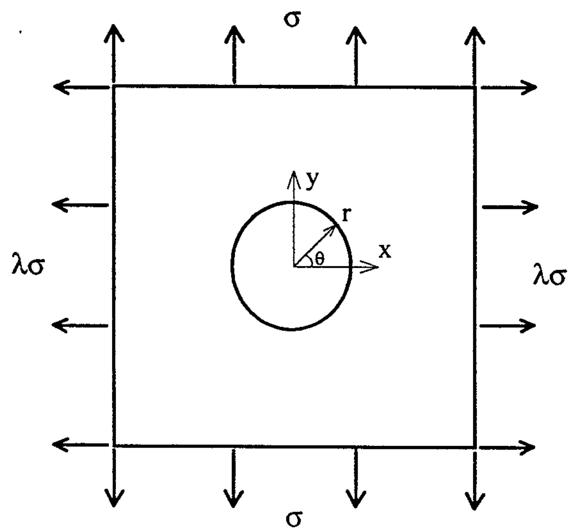

Figure 4 Stress state in the region of bolt hole in a rotation disc

At the peak rotation speed of 37,000 rpm, the radial stress σ is 650 MPa, and the biaxiality ratio λ is 0.92. At the minimum rotation speed of 8,000 rpm, the radial stress σ is equal to 30 MPa with the same λ. This implies that the stress ratio of the cyclic load is about 0.05.

Modelling of Biaxial Stress

Referring to Figure 4, the hoop stress along the prospective crack path is given by

$$\sigma_{\theta\theta}(x) = \frac{(\lambda+1)\sigma}{2}(1+\frac{r^2}{x^2}) - \frac{(\lambda-1)\sigma}{2}(1+\frac{3r^4}{x^4}) \quad (1)$$

where r represents the radius of the hole, as shown in Figure 4. Clearly the stress concentration factor K_t at x=r is,

$$K_t(\theta=0) = 3-\lambda \quad (2)$$

In the present case, since $\lambda \approx 1$, the stress concentration factor is approximately 2. Consequently, for the maximum applied stress of 650 MPa, the maximum hoop stress under *elastic* conditions around the hole is 1300 MPa, exceeding the yield stress of the material. Therefore, upon the first half cycle, local plastic yielding would occur around the hole edge. The total strain can be estimated by using the Neuber's rule, assuming the material is elastic-perfectly plastic,

$$\varepsilon_{total} = \frac{(K_t\sigma)^2}{\sigma_{ys}E} = 0.0145 \quad (3)$$

which is about twice the yield strain $\varepsilon_0 = \sigma_{ys}/E = 0.0073$. However, because the far-field applied stress is cycling with a stress ratio of R=0.05, the cyclic stress amplitude is less than the material's yield stress, ie., $K_t\Delta\sigma/2 < \sigma_{ys}$, and the first few load cycles generate compressive residual stresses such that there is no reverse yielding or cyclic plasticity around the hole. This implies that linear elastic fracture mechanics should still be applicable to quantifying the growth of short cracks emanating from the hole.

As there are no closed-form solutions available for the stress intensity factors of a corner crack at the root of a circular hole under biaxial stress, a detailed numerical solution, such as three-dimensional finite element analysis, is called for to determine the stress intensity factors along the crack front. For simplicity, in the present work, we will develop an approximate solution by combining the known solutions for a corner crack at a circular hole under uniaxial loading[5] and a through-thickness crack emanating from a circular hole under biaxial stresses. The ratio of the stress intensity factors under uniaxial stress and biaxial stresses is assumed to be the same for both a corner crack and a through-thickness crack.

In the case of a through-thickness crack, the stress intensity factor can be readily determined using a weight function method [6],

$$K = \int_0^a \sigma_{\vartheta\vartheta}(t+r)G(t,a)dt \qquad (4)$$

where the function $G(t,a)$ represents the stress intensity factor of crack, with length a, caused by a pair of concentrated forces acting at position t (measured from the edge of the hole). The solution for $G(t,a)$ is available[6]. A non-dimensional stress intensity factor f_λ can be defined as,

$$K_\lambda = f_\lambda K_{\lambda=0} \qquad (5)$$

where $K_{\lambda=0}$ denotes the stress intensity factor of a through-thickness crack emanating from a circular hole subjected to uniaxial stress. With equation (4), the results of the non-dimensional stress intensity factor f_λ are shown in Figure 5. It is evident from the figure that f_λ is a function of the biaxiality ratio and crack length to hole radius ratio. By assuming that this non-dimensional stress intensity factor is also applicable to a corner crack, we can modify the solution by Newman [5] for a corner crack emanating from a circular hole under uniaxial loading to account for the biaxial stress effects. It is seen from Figure 5 that for small crack length, the biaxiality ratio has a significant influence on the stress intensity factors. As crack gets longer, the biaxiality effect gradually diminishes, as the lateral stress (*T*-stress) does not contribute to the elastic stress intensity factors. It should be noted that, in the long crack case, the *T*-stress will affect both the crack opening stress level and the crack-tip opening displacement, hence the crack growth driving force. This will be the subject of a separate article.

To predict the crack growth rate, the crack opening stresses (or closure stresses) at the crack tip under either uniaxial or biaxial cyclic loading have to be calculated to correlate crack growth data properly with the effective stress intensity factor range. The effective stress intensity factor range is the difference between the maximum stress intensity factor and the minimum stress intensity factor at which the crack is just fully open during loading. It is now well known that on the basis of the effective stress intensity factor range, fatigue crack growth rates corresponding to different stress ratios can be collapsed onto a single curve. There are many factors contributing to the closure of fatigue cracks, with the plasticity induced closure being shown to be the most dominant. In this regard, the strip-yield model based method appears to be the most robust and reliable. One such example is an algorithm called FASTRAN developed by Newman[5]. However, it handles only the case of uniaxial loading.

An effort has been made to improve FASTRAN to take into account the effects of biaxial loading. To this end, the non-dimensional stress intensity factor obtained previously for a through-thickness crack emanating from a circular hole is applied to the analogous problem of a corner crack. This is achieved by modifying the stress intensity factors corresponding to a corner crack. The fatigue crack opening stress is assumed to be constant along the crack front, and cracks are considered to grow in two directions: at the maximum depth point and at the point of the intersection of the crack with the free surface.

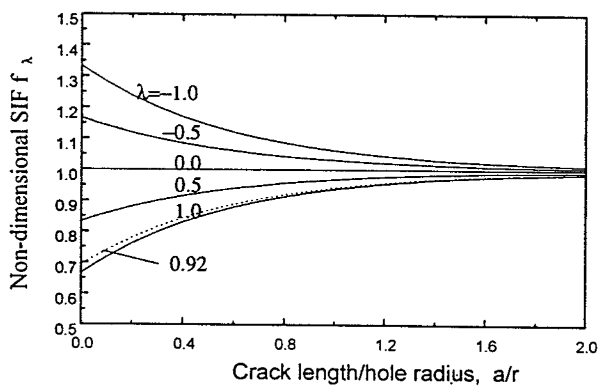

Figure 5 Non-dimensional stress intensity factor a through-thickness crack emanating from a circular hole under biaxial loading.

Ti-8Al-1Mo-1V Crack Growth Properties

The compressor disc being studied is made of a duplex annealed Ti-8Al-1Mo-1V titanium alloy, which has been tested by Hudson[7] under constant amplitude load with various stress ratios of 0, 0.25, 0.43, 0.67, and 0.85; the mean stress was kept constant at 173 MPa. The specimen was 610 mm long and 203 mm wide. The thickness was 1.27 mm. The test results are shown in Figure 6, where crack growth rates are plotted against the stress intensity factor range ΔK. Based on the crack growth data shown in

Figure 6, the baseline da/dN versus ΔK_{eff} relations can be obtained using plasticity-induced crack closure modelling technique. Under constant amplitude uniaxial loading, the only unknown for the calculation of the baseline is the constraint factor, α. In the present work, we take α=1 to obtain the baseline data for through-thickness crack growth, and α=2.5 for the corner crack growth.

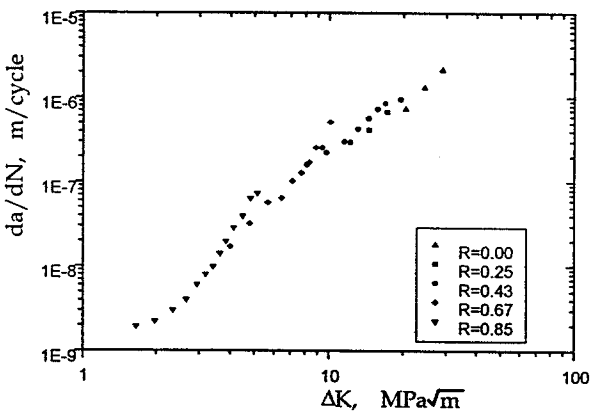

Figure 6 Crack growth data of Ti 8-1-1 at 26.7 °C

The correlated baseline relation da/dN against ΔK_{eff} for α=2.5 is shown in Figure 7. The baseline equations correlated for both plane stress and plane strain are given by the following Paris-Erdogan relationship,

$$\frac{da}{dN} = C(\Delta K_{eff})^m \tag{6}$$

where $C = 2.6\times10^{-10}$, $m = 3.14$ for α=1 and $C = 3.16\times10^{-10}$, $m = 2.86$ for α=2.5.

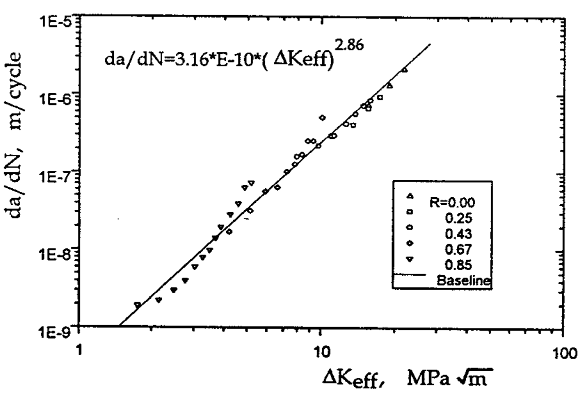

Figure 7 Baseline relation of da/dN vs. ΔK_{eff} (α=2.5)

Figure 7 shows that the crack growth data for different stress ratios appear to collapse within a narrow band, and the baseline curve fits the test data quite well for all R ratios.

Fatigue Life Prediction

Using the non-dimensional stress intensity factor, a modification was implemented in FASTRAN II source code to predict the fatigue crack growth behaviour under biaxial loading conditions. In this study the biaxiality ratio λ is 0.92, and the non-dimensional stress intensity factor shown in Figure 5 can be represented by the following 4th order polynomial in the region $a/r < 2$.

$$f_\lambda = 0.693 + 0.505\frac{a}{r} - 0.376\left(\frac{a}{r}\right)^2 + 0.143\left(\frac{a}{r}\right)^3 - 0.022\left(\frac{a}{r}\right)^4 \tag{7}$$

One difficulty frequently encountered in trying to predict the total fatigue life of a component is the choice of the initial crack size (a_i), which in principle should be determined based on non-destructive inspection and metallographic analysis. In reality, the choice of the initial crack size, especially when it is very small, will greatly affect the predicted fatigue life, as the crack growth rate would obviously approach zero as the initial crack size approaches zero. In the absence of an appropriate initial crack size, a sensitivity study is helpful in evaluating the effect of initial crack size on the structural durability. It has been reported that initial semi-circular surface cracks for Ti-6Al-4V titanium alloy were 5.0 and 25 μm.[8] In this study, the initial corner crack lengths at the bolt hole are chosen to be 1.0 and 5.0 μm as they may vary with the heat treatment history of the material. It is assumed that initial corner crack depth is equal to the length (circular corner crack). In the spin rig test, the smallest crack length that was obtained in the test was 0.39 mm, after approximately 8533 cycles. It is assumed that initial corner crack depth is equal to the length(circular corner crack).

Using the modified crack growth code, along with the experimentally derived da/dN vs ΔK_{eff} baselines in Eqn.(6), crack growth from hole A towards hole B in the disc as shown in Figure 2 can be predicted. For comparison, predictions based on the modified method and that method where the biaxial effect is ignored are presented in Figure 8. Including biaxial effects improves prediction accuracy, and assuming a very short initial crack leads to a better prediction. However compared with the spin rig test results, both predictions are relatively conservative, essentially because of the difficulty of predicting crack growth in the short crack regime.

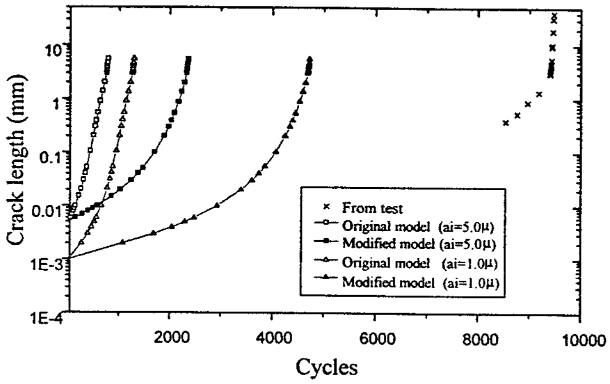

Figure 8 Predicted and tested crack length vs cycles

Summaries

Fatigue crack propagation emanating from tie bolt holes in a representative gas turbine compressor disc is predicted by accounting for the effects of the biaxial stress state present in a rotating disc. It is found that the approximate method presented in this article does provide a considerable improvement over the conventional method where the biaxial loading is simply ignored.

Nevertheless, the present work also highlights a number of issues that need to be addressed if a better prediction is to be achieved. First of all, a more appropriate method in determining the stress intensity factors along the crack front for a corner crack emanating from a circular hole under biaxial loading would be to utilise a three-dimensional weight function approach or Green function approach. The variation of the constraint factor along the crack front, especially during the transition from a corner crack to a through-thickness crack needs to be studied.

In the study, it has also been found that a large part of the fatigue life in a rotating disc is spent in initiating and growing small corner cracks. Further research is required to investigate the behaviours of short fatigue crack growth emanating from a hole in a rotating disc.

Acknowledgments

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