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RESEARCH ON ACTIVE VIBRATION CONTROL TECHNOLOGIES FOR A COMPOSITE SHELL

Chen Yong Tao BaoQi Liu Guo Wan JianGuo Jin Jiang
The Key Lab of Smart Materials and Structures,
Nanjing University of Aeronautics and Astronautics, Nanjing, P.R.of CHINA 210016

Abstract. In this article, a strain actuation mechanical model is given to analyze the matching relationship between actuators, composite structures and bonding layers. A finite element model is derived to make vibration analysis of a composite shell with surface bonding piezoelectric ceramics using decoupled mechanic-electrical method. An adaptive control system is set up to suppress the transverse vibration response and noise radiation of a glass fiber/epoxy cylindrical shell. Results show that transverse vibration modes of the shell can be effectively suppressed, and the radiation noise is also decreased.

Introduction

With the development of high performance aircrafts, composite structures have been widely used. The aircraft structures tend to have more flexibility and poor passive damping. Composite shell is one of the most typical structure. Engineers should consider its dynamic properties, response and fatigue performances to ensure that it would not be damaged by excessive vibration and acoustic fatigue in service. In structural design, $\sqrt{2}\varpi$ principle is generally adopted to avoid resonance of the structure. However, thin wall composite shells possess many vibration modes in narrow frequency band, it is very difficult to achieve satisfactory results, and passive damping can only provide limited improvement because of heavy mass added and processing difficulties. Thus, active vibration control system for flexible structure is becoming more and more important.

Nowadays, there are many research activities which focus on applying distributed sensors and actuators to composite structures to control the unfavorable vibration and the radiated noise from structures. Piezoelectric ceramic possesses piezoelectric and piezoelectric converse effect, it is characterized with broad frequency band and easy to control , and can be used as either sensors or induced strain actuators, and has been widely used in adaptive structures. Jinhao Qiu[1] etc. conducted experimental research on the vibration and noise control of a Magnetic Response Image equipment with PVDF. C.R Fuller[2] achieved 12 dB reduction of noise level from aircraft panel by the control of structure vibration with piezoelectric actuators, and get to the conclusion that it is more effective than passive damping.

The objective of this paper is to attach piezoelectric ceramic patches to the wall of a glass/epoxy composite cylindrical shell. By adaptively control the deformation of the distributed induced strain actuators, we can increase the electric damping so as to suppress the transverse vibration of the shell and its acoustic radiation.

The composite shell for experiment

The composite shell for vibration control using distributed piezoelectric actuators is shown in figure 1. The radius of the shell is $\phi 1000$ mm, the height is 530mm, the thickness of the wall is 5mm. The shell is made of glass/epoxy with ply orientation [-45/45]s, $E_1 = E_2 = 14.8GPa$, $\mu_{12} = 0.37$, $\rho = 1.8 \times 10^3 kg/m^3$. Six piezoelectric actuators (PZT5H, $40 \times 9 \times 1mm$) are symmetrically bonded to the outer surface of the shell, $d_{31} = d_{32} = 1.7 \times 10^{-10} m/V$, $E_p = 83.3GPa$ $\rho_p = 7.5 \times 10^3 kg/m^3$.

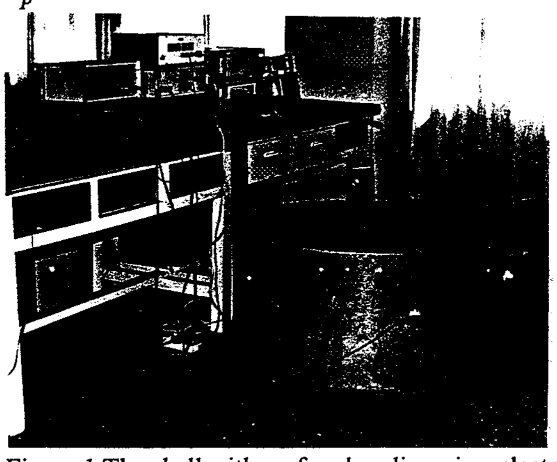

Figure 1 The shell with surface bonding piezoelectric actuators

The composite shell's transverse stiffness is relatively low, and under the excitation of external interfering forces, it is prone to cause serious vibration problems and noise radiation. In the experiment, we will suppress its vibration and acoustic radiation by the control of surface bonding piezoelectric ceramics.

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Mechanical model of strain actuation

A important factor of strain actuate structures is the stress transferred from the piezoelectric actuators to the structure. Many scholars have been paying attention to it. One of the studies involving the mechanical model of the piezoelectric actuators bonding on a beam was made by Crawley and de Luis[3]. This model was latter revised by Baz, Burk, Clark and Jerome [4, 5,6]. Dimitriadis and Wang had further expanded these models into a two-dimensional plate and multi-ply piezoelectric coupled structure[7,8]. However, these investigations were based on a lot of assumptions and did not include some important factors influencing the actuating process.

In this part, a model based on shearing-lag theory is adopted to analyze the force of the piezoelectric actuator on the shell. Using this model, we can further discuss the factors which are important to the mechanical-electric system.

Since the actuators are relatively small comparing with the radius of the shell, we can simplify the shell into a laminate. A typical arrangement for piezoelectric ceramic actuators bonding on the laminate is shown in Fig. 2, The arrows E and P indicate the direction of the electric field applied and the polarizing direction of the actuators, respectively. If the electric field is applied according to the direction of the arrows, the laminate will deform in extension. If the electric field is applied to one element in the direction of the arrow, and reversed to the other, the laminate will be deformed in bending.

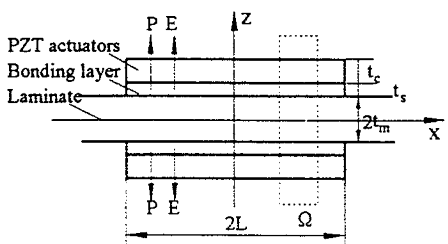


Figure 2 Arrangement of actuators bonding on the laminate

In order to establish the mechanical model, a differential area Ω outlined by the dashed area in Fig 2 will be considered. Assuming that all materials are linear-elastic, under the assumption of pure extension strain, the equilibrium equations in the x direction for the area Ω can be drawn

$$\frac{d\sigma_x^c}{dx} - \frac{\tau(x)}{t_c} = 0 \tag{1}$$

$$\frac{d\sigma_x^m}{dx} + \frac{\tau(x)}{t_m} = 0 \tag{2}$$

$$\frac{d\sigma_x^s}{dx} + \frac{G^s}{t_s} \cdot \frac{u_x^c - u_x^m}{t_s} - \frac{\tau(x)}{t_s} = 0 \tag{3}$$

The stress-strain relationship for the piezoelectric actuators can be expressed as

$$\sigma_x^c = Y^c\left(\varepsilon_x^c - d_{31}E_3\right) \tag{4}$$

The stress-strain relationships for the bonding layer and the laminate are

$$\sigma_x^m = Y^m \varepsilon_x^m \tag{5}$$

$$\sigma_x^s = Y^s \varepsilon_x^s = Y^s \varepsilon_x^m \tag{6}$$

The corresponding strain-displacement relationships are

$$\varepsilon_x^c = \frac{du_x^c}{dx} \tag{7}$$

$$\varepsilon_x^m = \frac{du_x^m}{dx} \tag{8}$$

In above expressions, the superscripts c, m and s represents piezoelectric ceramic, laminate and bonding layer respectively.

Integrating equations (1) to (8), we can get a pair of coupled second order differential equations :

$$\begin{cases} Y^c t_c \dfrac{d^2u_x^c}{dx^2} - Y^s t_s \dfrac{d^2u_x^m}{dx^2} - \dfrac{G^s}{t_s}\left(u_x^c - u_x^m\right) = 0 \\ Y^m t_m \dfrac{d^2u_x^m}{dx^2} + Y^s t_s \dfrac{d^2u_x^m}{dx^2} + \dfrac{G^s}{t_s}\left(u_x^c - u_x^m\right) = 0 \end{cases} \tag{9}$$

From the coupled equations (9), the relationship of the displacement in the x axis direction between the actuators and the laminate can be derived:

$$u_x^c = -\frac{Y^m t_m}{Y^c t_c} u_x^m + C_1 x + C_2 \tag{10}$$

Where c_1, c_2 are constants and can be determined by following boundary conditions:

$$u_x^c = u_x^m = 0 \quad \text{at} \quad x = 0 \tag{11}$$

$$\varepsilon_x^c = \frac{du_x^c}{dx} = d_{31}E_3,\ \varepsilon_x^m = \frac{du_x^m}{dx} = 0 \text{ at } x = L \tag{12}$$

Substituting the boundary conditions into Eq. (10), the constants are found to be

$$\begin{cases} C_1 = d_{31}E_3 \\ C_2 = 0 \end{cases}$$

Substituting Eq. (10) into Eq. (9) , the displacement of the laminate in the x axis direction can be derived:

$$u_x^m = C_3 e^{\sqrt{K_1}x} + C_4 e^{-\sqrt{K_1}x} + \frac{K_2}{K_1}x \tag{13}$$

Where

$$K_1 = \frac{Y^c t_c + Y^m t_m}{Y^m t_m + Y^s t_s} \cdot \frac{G^s}{Y^c t_c t_s} \tag{14}$$

$$K_2 = \frac{G^s}{t_s\left(Y^m t_m + Y^s t_s\right)} d_{31} E_3 \tag{15}$$

Since $u_x^m(-x) = -u_x^m(x)$, $C_3 = C_4$ can be derived. Hence, the above equation can be rewritten as

$$u_x^m = Ash\left(\sqrt{K_1}x\right) + \frac{K_2}{K_1}x \tag{16}$$

Integrating Eq.(10) and Eq.(11), the displacement of the PZT actuators in the x axis direction can be obtained

$$u_x^c = -A\frac{Y^m t_m}{Y^c t_c} sh\left(\sqrt{K_1}x\right) + \left(d_{31}E_3 - \frac{K_2}{K_1}\frac{Y^m t_m}{Y^c t_c}\right)x \tag{17}$$

Where A is a constant and can be determined by the following boundary condition

$$\varepsilon_x^c = \frac{du_x^c}{dx} = d_{31}E_3, \varepsilon_x^m = \frac{du_x^m}{dx} = 0 \text{ at } x = L \tag{18}$$

Substituting the boundary condition into Eq. (13), A is then derived

$$A = \frac{-K_2}{K_1^{\frac{3}{2}} ch\left(\sqrt{K_1}L\right)} \tag{19}$$

After differentiating with respect to Eq. (17)and (18), the strain distribution of the laminate and the actuators can be obtained

$$\varepsilon_x^m = A\sqrt{K_1}\, ch\left(\sqrt{K_1}x\right) + \frac{Y^c t_c}{Y^m t_m + Y^c t_c} d_{31}E_3 \tag{20}$$

$$\varepsilon_x^c = -A\sqrt{K_1}\frac{Y^m t_m}{Y^c t_c} ch\left(\sqrt{K_1}x\right) + \frac{Y^c t_c}{Y^m t_m + Y^c t_c} d_{31}E_3 \tag{21}$$

By analyzing Eq(20), we can find that the dimension, and mechanical characteristics of the bonding layer, piezoelectric actuators and the laminate are important to the strain actuation process. The effect of some important factors on the performance of the piezoelectric actuators the laminate can be summarized as follows:

(1) The actuating force is transferred to the laminate by shearing force on the interface. The stress of the actuators and the laminate mainly concentrate nearby the boundary area.

(2) The strain of the laminate increases as the decrease of the thickness of the bonding layer. However, when the thickness of the bonding layer decreases to a certain degree, the strain of the laminate does not increase further more.

(3) As long as the elastic modulus of the adhesive is beyond a certain value, the actuators can actuate the laminate effectively. The increase in the elastic modulus of the adhesive results no improvement whatever the elastic moduli of the laminate is.

(4) The smaller the elastic modulus and the thickness of the laminate are, the easier the PZT actuates the laminate. Hence, if the condition permits, the laminate with smaller thickness and elastic modulus should be used.

(5) The strain and stress of the laminate is linearly related to the intensity of external electric field applied on the PZT and increases with the intensity of the electric field.

Analytical model of the piezoelectric actuating composite shell

Analysis of the composite shell is based on following assumptions

1. The shell is symmetrical about its center line, the transverse shearing is negligible.
2. The thickness of the shell is uniform.
3. The polar axis of the piezoelectric ceramic is aligned with the radical direction at each location.
4. All materials are physically and geometrically linear.
5. The paired actuators are perfectly bonded to the surface of the shell, boundary shear effect can be ignored.

Using above assumptions, finite element method (FEM) can be used to predict the dynamic response of the composite shell under the action of external forces and electric fields on the distributed piezoelectric actuators.

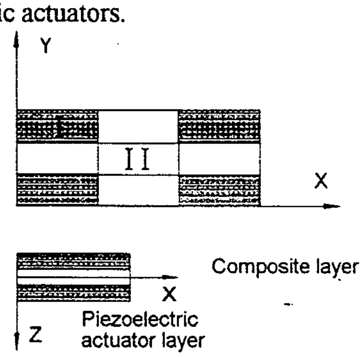


Figure 3 Two types of element in FEM model

In the analysis , two types of elements can be used.. Type I element includes the composite shell and the actuators, type II does not contain the actuator layer, as shown in figure 3.

The stress-strain relation in ply-layer is

$$[\sigma] = [Q]\left\{[\varepsilon] - [\varepsilon^e]\right\} \tag{22}$$

$[Q]$ is the module matrix of ply-layer, $[\varepsilon^e]$ is the strain caused by piezoelectric converse effect, in the composite ply-layer, $[\varepsilon^e] = 0$.

According to Kirchhoff hypothesis, the strain in the element can be written as

$$[\varepsilon] = [\varepsilon^0] + z[\kappa] \tag{23}$$

where $\left[\varepsilon^0\right]$ and $\left[\kappa\right]$ is the strain and curvature of mid-plane in the element.

The strain-stress relation of the laminate element can be derived as follows

$$\begin{Bmatrix} N \\ M \end{Bmatrix} + \begin{Bmatrix} N^e \\ M^e \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \tag{24}$$

where A, D, B is extension, bending and extension / bending coupling stiffness matrix respectively, they can be differently expressed depending upon type I or II element. $\left[N^e\right], \left[M^e\right]$ are equivalent induced in-plane forces and moment of piezoelectric ply-layer by external electric field.

$$\left[N^e\right] = \int_h [Q]\left[\varepsilon^e\right] dz \tag{25}$$

$$\left[M^e\right] = \int_h [Q]\left[\varepsilon^e\right] z dz \tag{26}$$

It is obvious that in type II element we have $\left[N^e\right] = 0, \left[M^e\right] = 0$.

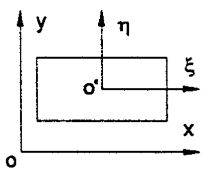


Figure 4 rectangular thin plate element

4 nodes, 20 degrees of freedom rectangular thin plate element can be used in the FEM model of the composite shell, as shown in figure 4. $xoy, \xi o'\eta$ is the global and partial coordinate respectively, $x = x_0 + a\xi, y = y_0 + b\eta$. The displacement in the element can be interpolated as

$$\begin{aligned} u &= \sum_{i=1}^{4} N_{pi} u_i \\ v &= \sum_{i=1}^{4} N_{pi} v_i \\ w &= \sum_{i=1}^{4} (N_{bi} w_i + N_{bxi}\theta_{xi} + N_{byi}\theta_{yi}) \end{aligned} \tag{27}$$

where $(u_i, v_i, w_i, \theta_{xi}, \theta_{yi})$ is the displacement of the ith node, $\theta_{xi} = -w_{,y}|_i, \theta_{yi} = -w_{,x}|_i$, $i = 1,2,3,4$. N_{pi} and $(N_{bi}, N_{bxi}, N_{byi})$ are the interpolation function of in-plane and bending strain, expressed as the following

$$\begin{cases} N_{pi} = (1+\xi_0)(1+\eta_0)/4 \\ N_{bi} = (1+\xi_0)(1+\eta_0)(1+\xi_0+\eta_0-\xi^2-\eta^2)/8 \\ N_{bxi} = b\eta_i(1+\xi_0)(1+\eta_0)(1-\eta^2)/8 \\ N_{byi} = -a\xi_i(1+\xi_0)(1+\eta_0)(1-\xi^2)/8 \end{cases} \tag{28}$$

in Eq.(28), $\xi_0 = \xi_i\xi, \eta_0 = \eta_i\eta$.

According to classic laminate theory, the deformation in a thin plate element is

$$\begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} u_x \\ v_y \\ u_y + v_x \\ -w_{xx} \\ -w_{yy} \\ -2w_{xy} \end{bmatrix} = \begin{bmatrix} B_p & 0 \\ 0 & B_b \end{bmatrix} \begin{Bmatrix} \delta_p \\ \delta_b \end{Bmatrix} \tag{29}$$

$$\left\{\delta_p\right\} = \begin{bmatrix} u_1 \\ v_1 \\ \vdots \\ u_4 \\ v_4 \end{bmatrix}, \left\{\delta_b\right\} = \begin{bmatrix} w_1 \\ \theta_{x1} \\ \theta_{y1} \\ \vdots \\ w_4 \\ \theta_{x4} \\ \theta_{y4} \end{bmatrix} \tag{30}$$

$$\left[B_p\right] = \left[B_{p1} \quad B_{p2} \quad B_{p3} \quad B_{p4}\right] \tag{31}$$

$$\left[B_b\right] = \left[B_{b1} \quad B_{b2} \quad B_{b3} \quad B_{b4}\right] \tag{32}$$

$$\left[B_{pi}\right] = \begin{bmatrix} N_{pi,x} & 0 \\ 0 & N_{pi,y} \\ N_{pi,y} & N_{pi,x} \end{bmatrix} = \frac{1}{ab} \begin{bmatrix} bN_{py,\xi} & 0 \\ 0 & aN_{pi,\eta} \\ aN_{pi,\eta} & bN_{pi,\xi} \end{bmatrix} \quad (i = 1,2,3,4) \tag{33}$$

$$\left[B_{bi}\right] = -\begin{bmatrix} N_{bi,xx} & N_{bxi,xx} & N_{by,xx} \\ N_{bi,yy} & N_{bxi,yy} & N_{byi,yy} \\ N_{bi,xy} & N_{bxi,xy} & N_{byi,xy} \end{bmatrix} \quad (i = 1,2,3,4) \tag{34}$$

The stiffness equation of the laminate element (including both composite and piezoelectric ply-layers) can be expressed as

$$\{R\} + \left\{R^e\right\} = [K]\{\delta\} + [c]\{\dot{\delta}\} + [M]\{\ddot{\delta}\} \tag{35}$$

where

$\{\delta\} = \left[\delta_p^T \quad \delta_b^T\right]^T, \{R\} = \left[R_p^T \quad R_b^T\right]^T, \left\{R_p\right\}$, $\left\{R_b\right\}$ is the equivalent force of $\left\{\delta_p\right\}$ and

$\{\delta_b\}$, $\{R^e\}=\left[(R_p^e)^T \quad (R_b^e)^T\right]^T$, $\{R_p^e\}$, $\{R_P^e\}$ is the equivalent node force induced by piezoelectric actuators under external electric field. $[K]$ is the stiffness matrix of the element as follows

$$\{R^e\}=\iint\begin{bmatrix} B_p & 0 \\ 0 & B_b \end{bmatrix}^T \begin{Bmatrix} N^e \\ M^e \end{Bmatrix} dxdy \tag{36}$$

$$[K]=\iint\begin{bmatrix} B_p & 0 \\ 0 & B_b \end{bmatrix}^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} B_p & 0 \\ 0 & B_b \end{bmatrix} dxdy \tag{37}$$

Obviously that in type II element, $\{R^e\}$ .equals Zero

In this way , the coupled mechanical-electric system can be treated as elastic system under the action of equivalent external forces and the analysis process of the structure is greatly simplified. According to standard FEM schedule, we can combine each stiffness and mass matrix of the element in the shell into a global stiffness and mass matrix, and analyze the vibration mode to predict the dynamic response of the composite shell .

Noise radiation model of the cylindrical shell

The composite shell will vibrate and radiate noises by the excitation of external interference forces. The noise level depends upon the characteristics of exciting forces and the vibration mode of the shell. The radiated noise power from a cylindrical shell can be written as

$$P = W\rho_0 c / 2\pi r \tag{38}$$

where w is the radiated noise power by per length of the shell, ρ_0 and c is the density of the air and sound speed in the air.

$$w=\frac{\pi\rho_0 f^2}{2c}\left(\frac{4\pi A}{\rho_0\omega}\right)^2 \tag{39}$$

In Eq.(39), the radiated noise power is the 2th power of vibration amplitude and 4th power of vibration frequency. By the control of transverse response of the shell, the noise radiation can be effectively suppressed. In the case of same vibration suppers ratio, the higher the frequency is , the better result can we expect.

<u>Experimental test setup</u>

In the experiment, filtered-X LMS theorem is applied as shown in figure 5. A piezoelectric sensor is bonded to the surface of the composite shell. d is uncontrolled output signal of charge amplifier from the structure, e is the controlled output signal. y is the output from adaptive controller, H_1 and H_2 is the weight of FIR filter function of external excitation and piezoelectric actuator respectively, W is the weight of the adaptive controller, X is the reference signal. In the experiment, reference signal is directly sampled from external excitation to ensure that it is relative to external excitation force.

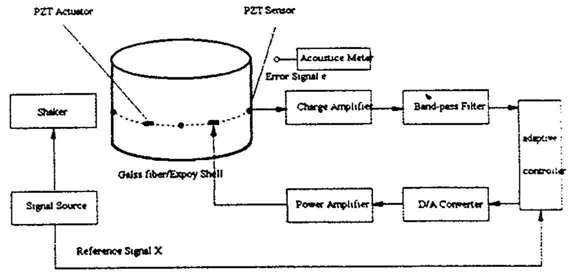


Figure 5 Vibration control system of the shell

<u>Experiment of adaptive control</u>

Adaptive vibration control experiment is made on the composite shell and the acoustic radiation is also measured with a precision acoustic meter*(ND2)*. The transverse mode of the shell is excited by a shaker. By adaptively control the signal on the actuators, the vibration and acoustic radiation of the shell can be successfully suppressed. Figure 7 shows the time and frequency domain comparison result of 380 Hz.

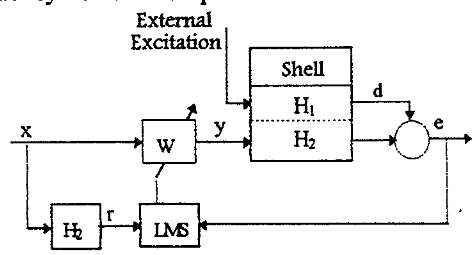
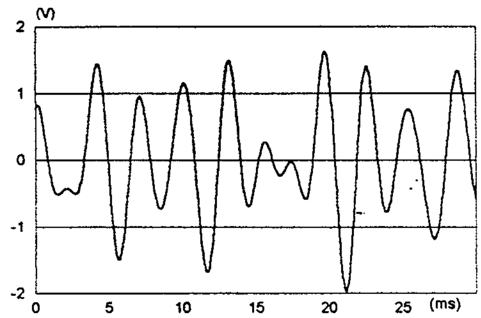
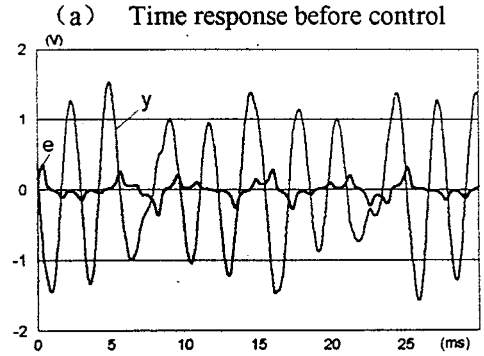


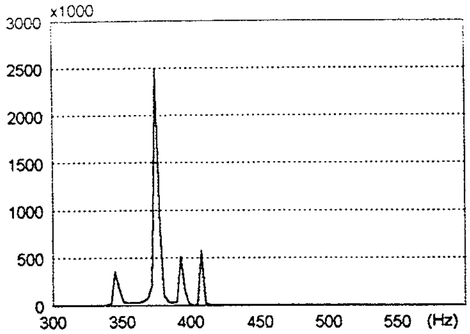
Figure 6 Filtered-X LMS controller



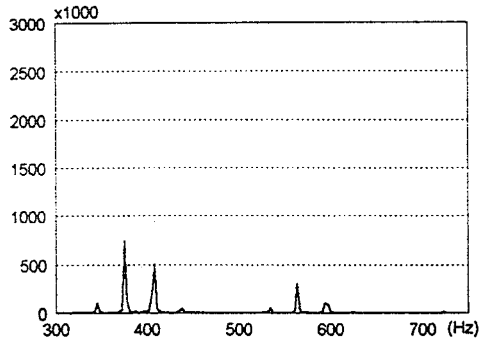
(a) Time response before control



(b) Time response after control



(c) Power spectrum before control



(d) Power spectrum after control

Figure 7 Time and frequency domain comparison results

In figure 7(c), the composite shell has several transverse mode in narrow band, and the spectrum of response possesses many frequency ingredients because of dynamic coupling. By adaptive control, the transverse vibration has been suppressed in all frequency domain, acoustic radiation is also decreased. However, the controlled response has more frequency ingredients than that before, the reason is that digitized controlling signal has higher order frequency ingredients and the higher vibration modes of the composite shell are excited.

Figure 8 shows the frequency response of the composite shell, table 1 is the acoustic radiation. We can get the conclusion that by adaptively control the electric field on piezoelectric actuators, the transverse response of the shell has been effectively suppressed and the noise radiation is also sharply deceased.

Table 1 Acoustic radiation of the shell

Exciting Frequency (Hz)	390	531	654	881
No control dB(A)	72	80	79.5	89.5
control dB(A)	60	71	68.5	86

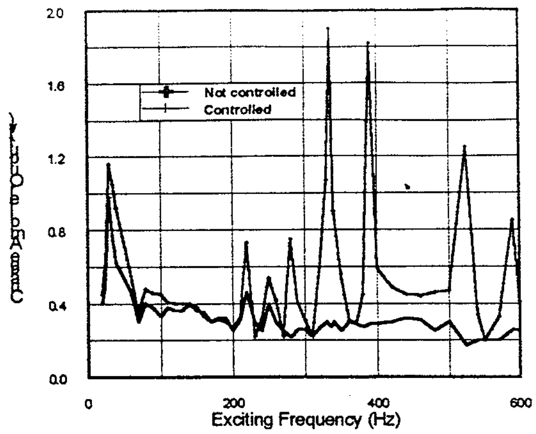


Figure 8 Frequency response of the composite shell

Conclusion

In this article, the strain actuation model of piezoelectric actuators on laminate is analyzed. The principle of vibration control using a surface bonding piezoelectric actuators of a composite shells is discussed. By adaptive control, the transverse vibration and acoustic radiation is successfully suppressed. The following conclusion can be obtained from the analysis and experiment results.

(1) In piezoelectric actuated structure, there is a matching relationship to increase the effect of strain actuation.

(2) The thin wall composite shell possesses many elastic modes in narrow band. By the adaptive control of surface bonding piezoelectric actuators, not only the main exciting frequency can be suppressed, the high order frequency ingredients are also decreased.

(3) By the control of structural vibration, the acoustic radiation is also decreased. This characteristic is very useful in the improvement of structural acoustic fatigue performance in aircrafts.

(4) Piezoelectric ceramic chip actuators can be easily used in composite structures, and served as a new method of structural vibration control.

Acknowledgment

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