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OPTIMAL MAIN HELICOPTER ROTOR PROJECTION MODEL OBTAINED BY VISCOUS EFFECTS AND UNSTEADY LIFT SIMULATIO

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Abstract

This paper presents a model of airfoil which allows obtaining a method for optimal main helicopter rotor projection by viscous effect and unsteady lift simulation through algorithm and set of program entireties, applicetely directed to ideological and main project of helicopter rotor. Numerical analysis considerations in this paper can be applied, on basis of a real rotors theoretical consideration, whit sufficient accuracy in analysis and constructive realizations of helicopter rotor in real conditions. The method for unsteady viscous flow simulations by inviscid techniques is developed.
The based aim of this paper is to determine helicopter rotor blade lift with most possible accuracy by using singularity method and to define such optimal conception model of aerodynamic rotor projection which is corresponding to rotor behavior in real condition and with sufficiently quality from aspect et engineer use.

Nomenclature

V_∞ = freestream velocity
Γ = vortex intensity
z = complex variables, $x + iy$
$\bar{z}$ = $x - iy$
x,y,z = Cartesian coordinates
f = potential function
$\bar{f}$ = complex conjugate potential function
ϕ = velocity potential
ψ = stream function
w = complex potential
V = complex velocity
Cp = pressure coefficient
n = number of vortices in wake
δ = wake thickness
β = stagnation point position
α = angle of attack
u_x', u_x'' = velocities at upper and lower side of the shear layer
a = radius
$z_{1/2}$ = stagnation point position
i_ω = unit vector in z direction
R_e = Reynolds number
ν = kinematic viscosity
μ = dynamic viscosity
ρ = density
θ = mapping angle
$d\zeta/dz$ = mapping derivative
δ = Dirac δ function
π = 3.14159...
L = characteristic length
t = time

Introduction

The basic aim of this paper is to determinate the lift of helicopter rotor blades by using singularity method, that is to develop an optimal model of aerodynamic rotor planning by simulating unsteady flow and viscous effects. The model is considered corresponding to rotor behavior in real conditions and with high quality from aspect of engineer use. The idea of unsteady lift modeling and viscous effects simulation by using singularity method is based on the need for avoidance very expensive experiments in the first stage of rotor planning by using contemporary aerodynamic analysis applied to available computer technique.
It is necessary to modulate vortex wake, unsteady 2D flow field characteristics and blade dynamic characteristics for determination of aerodynamic forces acting upon helicopter rotor blade.
In the first part airfoil is approximated by vortex and source panels, and boundary layer by layer of vortices changing their position during time. Vortex trail and trail of separation are modulated by free vortices. The separation is modulated by free vortices. The dependence of coefficient of lift on angle of attack is determined for known motion of an airfoil. The behavior of one separated vortex in an article time model is spread into a series of positions at a certain number at different vortices. The separated flow velocity profile is approximated by superposition of displacement thickness of these vortices coupled with the potential solution model. After every time step the position of free vortices is changed, for what it is required generation of new vortices that would all together satisfy the airfoil contour boundary conditions.
In the second part model is developed for 3D for complete flow field and is applied on calculation of flow over

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helicopter blade. Helicopter rotor blade is divided into certain number of segments. Each segment defines an airfoil which flow condition is similar to those described over airfoil. The flow over entire helicopter rotor blade can be defined by use of certain intersection characteristics. This can be the basis for unsteady force acting upon blade, rotor and helicopter study.

In addition to a complex problem of analytical modeling these phenomena there is a problem of modeling interactions between effects of these phenomena.

This approach to helicopter rotor planning allows experimental investigations in tunnels and in flight to be final examinations. In that way, a very useful interaction between numerical calculation and experimental results is achieved.

In addition to standard decompression and synthesis method for realization of this paper singularity method, panel method, vortex surface method and PIC (particle in cell) method will be used.

The finale result is obtaining of unsteady lift dependence of angle of attack. Model established in such a way is characteristic for the helicopter rotor blade airflow and it is based on the influence of the previous lifting surface's wake influence on the next coming blade.

According to such analysis and a chosen model, a computer program is developed, used for the helicopter blade airflow analysis.

Foundations of the Irrotational 2-D Flow

The planar potential flow of incompressible fluid can be treated in Cartesian coordinates x and y. If physical plane is mapped to the complex plane by $z = x + iy$ where $i = \sqrt{-1}$. The symmetrical point with respect to the x-axis is $\bar{z} = x - iy$. Two dimensional potential incompressible flow is completely defined by the speed potential and stream function and is presented by Cauchy-Reimann equations:

$$\frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} = 0 \quad \text{i} \quad \frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y} = 0 \qquad (1)$$

where u and v are velocities in x and y directions respectively.

The Laplace partial differential equations can also be introduced:

$$\nabla^2 \phi = 0 \quad \text{i} \quad \nabla^2 \psi = 0 \qquad (2)$$

Where ∇^2 is the Laplace's operator.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad (3)$$

The fulfillment of Cauchy-Reimann conditions enables combining of the velocity potential and stream function:

$$w(z) = \phi\,(x,y) + i\psi\,(x,y) \qquad (4)$$

of complex variable $z = x + iy$.

This complex function entirely defines the planar potential flow of incompressible fluid as a function of a complex coordinate. The complex analytical function $w\,(z)$, called the complex flow potential, always has a unique value for the first derivative. This derivative of the complex potential is equal to the complex velocity at that point, i.e.:

$$\frac{dw}{dz} = u - iv = \overline{V} \qquad (5)$$

where u is its real part (velocity component in x-direction) and v is the imaginary part (velocity component in y-direction).

Circulation and flow are equal to zero for any closed curve in complex plane. Complex potential has no singularities except at the stagnation point.

Simulation of the moving vortex

If we assume that the moving vortex of intensity Γ_0 is at the distance z_0, than the perturbance potential of such a flow is:

$$w(z) = V_\infty z e^{-i\alpha} + V_\infty \frac{a^2}{z} e^{+i\alpha} + i2a \cdot V_\infty \cdot sin(\alpha - \beta) \ln z + \frac{i\Gamma_0}{2\pi} \ln(z - z_0) - \frac{i\Gamma_0}{2\pi} \ln\left(\frac{a^2}{z} - \bar{z}_0\right) \qquad (6)$$

and it's complex velocity:

$$\overline{V} = V_\infty e^{-i\alpha} - V_\infty \frac{a^2}{z^2} e^{+i\alpha} + \frac{i}{z} 2a \cdot V_\infty \cdot sin(\alpha - \beta) + \frac{i\Gamma_0}{2\pi} \frac{1}{(z - z_0)} + \frac{i\Gamma_0 (a^2/z^2)}{2\pi\left(\frac{a^2}{z} - \bar{z}_0\right)} \qquad (7)$$

In this case displacement of the stagnation point appears, which is now at the distance:

$$z = a \cdot e^{i\beta'} \qquad (8)$$

According to the request that complex velocity at the stagnation point must be zero:

$$\frac{dw}{dz} = \overline{V} = 0 \qquad (9)$$

we can determine position z and according to that β' i.e.:

$$\overline{V}(ae^{i\beta}) = 0 \qquad (10)$$

The angle β' defines new position of stagnation point, while the difference of angles is:

$$\Delta\beta = \beta - \beta' \qquad (11)$$

In order to keep the stagnation point at the steady position, a vortex of intensity Γ_1 must be released.

Intensity of the vortices that are released can be determined according to the Kelvin's theorem:

$$d\Gamma/dt = 0$$

So every change in circulation must be compensated by vortex inside of cylinder/airfoil of the opposite sign, whose intensity is a function of the variation of circulation around the cylinder:

$$\Gamma_i = -(d\Gamma/dt)\Delta t \tag{12}$$

and it is equal do the difference of the intensities before and after the free moving vortex is introduced:

$$\Gamma_1 = \Delta\Gamma_1 = 4\pi a \cdot V_\infty\left[sin(\alpha - \beta_1') - sin(\alpha - \beta)\right] \tag{13}$$

Now the total complex potential has the value:

$$w(z) = w_0(z) + w_{\Gamma_0}(z) + w_{\Gamma_1}(z) \tag{14}$$

where:

$$w_0(z) = V_\infty z e^{-i\alpha} + V_\infty \frac{a^2}{z} e^{+i\alpha} + i2aV_\infty sin(\alpha - \beta)\ln z$$

$$w_{\Gamma_0}(z) = \frac{i\Gamma_0}{2\pi}\ln(z - z_0) - \frac{i\Gamma_0}{2\pi}\ln\left(\frac{a^2}{z} - \bar{z}_0\right)$$

$$w_{\Gamma_1}(z) = \frac{i\Gamma_1}{2\pi}\ln(z - z_1) - \frac{i\Gamma_1}{2\pi}\ln\left(\frac{a^2}{z} - \bar{z}_1\right)$$

or:

$$\begin{aligned} w(z) = {} & V_\infty z e^{-i\alpha} + V_\infty \frac{a^2}{z} e^{+i\alpha} + i2a \cdot V_\infty sin(\alpha - \beta)\ln z \\ & + \frac{i\Gamma_0}{2\pi}\ln(z - z_0) - \frac{i\Gamma_0}{2\pi}\ln\left(\frac{a^2}{z} - \bar{z}_0\right) \\ & + \frac{i\Gamma_1}{2\pi}\ln(z - z_1) - \frac{i\Gamma_1}{2\pi}\ln\left(\frac{a^2}{z} - \bar{z}_1\right) \end{aligned} \tag{15}$$

Complex velocity is:

$$\begin{aligned} \bar{V} = {} & V_\infty e^{-i\alpha} - V_\infty \frac{a^2}{z^2} e^{+i\alpha} + \frac{i}{z} 2a \cdot V_\infty \cdot sin(\alpha - \beta) \\ & + \frac{i\Gamma_0}{2\pi}\frac{1}{(z - z_0)} + \frac{i\Gamma_0 (a^2/z^2)}{2\pi\left(\frac{a^2}{z} - \bar{z}_0\right)} \\ & + \frac{i\Gamma_1}{2\pi}\frac{1}{(z - z_1)} + \frac{i\Gamma_1 (a^2/z^2)}{2\pi\left(\frac{a^2}{z} - \bar{z}_1\right)} \end{aligned} \tag{16}$$

After a period of time Δt induced velocity at the trailing edge is a consequence of the disposition of both moving and released vortex.

Setting again the condition that a point at the circle is stagnation point and complex velocity at that point equal to zero, we determine a new position of the stagnation point by new angle β'.

This new angle β' defines a new difference $\Delta\beta = \beta - \beta'$. So the new position of the stagnation point is defined by distance:

$$z = a \cdot e^{i\beta'} \tag{17}$$

This again causes generation of a new vortex Γ_2 which is equal do the difference of vortex intensity before and after displacing of the free moving vortex from time t_1 to time t_2.

$$\Gamma_2 = \Delta\Gamma = 4\pi a V_\infty sin(\alpha - \beta') - 4\pi a V_\infty sin(\alpha - \beta) \tag{18}$$

Now the total complex potential has the value:

$$w(z) = w_0(z) + w_{\Gamma_0}(z) + w_{\Gamma_1}(z) + w_{\Gamma_2}(z) \tag{19}$$

or:

$$\begin{aligned} w(z) = {} & V_\infty z e^{-i\alpha} + V_\infty \frac{a^2}{z} e^{+i\alpha} + i2aV_\infty sin(\alpha - \beta)\ln z \\ & + \frac{i\Gamma_0}{2\pi}\ln(z - z_0) - \frac{i\Gamma_0}{2\pi}\ln\left(\frac{a^2}{z} - \bar{z}_0\right) \\ & + \frac{i\Gamma_1}{2\pi}\ln(z - z_1) - \frac{i\Gamma_1}{2\pi}\ln\left(\frac{a^2}{z} - \bar{z}_1\right) \\ & + \frac{i\Gamma_2}{2\pi}\ln(z - z_2) - \frac{i\Gamma_2}{2\pi}\ln\left(\frac{a^2}{z} - \bar{z}_2\right) \end{aligned}$$

Complex velocity is:

$$\begin{aligned} \bar{V} = {} & V_\infty e^{-i\alpha} - V_\infty \frac{a^2}{z^2} e^{+i\alpha} + \frac{i}{z} 2a \cdot V_\infty sin(\alpha - \beta) \\ & + \frac{i\Gamma_0}{2\pi}\frac{1}{(z - z_0)} + \frac{i\Gamma_0 (a^2/z^2)}{2\pi\left(\frac{a^2}{z} - \bar{z}_0\right)} \\ & + \frac{i\Gamma_1}{2\pi}\frac{1}{(z - z_1)} + \frac{i\Gamma_1 (a^2/z^2)}{2\pi\left(\frac{a^2}{z} - \bar{z}_1\right)} \\ & + \frac{i\Gamma_2}{2\pi}\frac{1}{(z - z_2)} + \frac{i\Gamma_2 (a^2/z^2)}{2\pi\left(\frac{a^2}{z} - \bar{z}_2\right)} \end{aligned} \tag{20}$$

Now we can give general equations of:

- ✓ the stagnation point position:

$$z = a \cdot e^{i\beta'} \tag{21}$$

- ✓ intensity of the released vortex:

$$\Gamma_m = 4\pi a \cdot V_\infty \left[sin\left(\alpha - \beta'_m\right) - sin\left(\alpha - \beta\right) \right] \tag{22}$$

- ✓ total circulation:

$$\Gamma_\Sigma = \Gamma + \Gamma_0 + \sum_1^n \Gamma_m \tag{23}$$

- ✓ circulation inside the cylinder/airfoil:

$$\Gamma_n = \Gamma_s \pm \Gamma_m \tag{24}$$

- ✓ complex potential:

$$w(z) = w_0(z) + \left[w_{\Gamma_0}(z) + \sum_1^n w_{\Gamma_m}(z) \right] \tag{25}$$

or:

$$w(z) = V_\infty z e^{-i\alpha} + V_\infty \frac{a^2}{z} e^{+i\alpha} + \frac{i(\Gamma - \Gamma_m)}{2\pi} \ln z + \frac{i\Gamma_0}{2\pi} \ln(z - z_0) - \frac{i\Gamma_0}{2\pi} \ln\left(\frac{a^2}{z} - \bar{z}_0\right) + \sum_1^n \left[\frac{i\Gamma_m}{2\pi} \ln(z - z_m) - \frac{i\Gamma_m}{2\pi} \ln\left(\frac{a^2}{z} - \bar{z}_m\right) \right]$$

complex velocity:

$$\frac{d\bar{z}_m}{dt} = \bar{V}_m = V_\infty e^{-i\alpha} - V_\infty \frac{a^2}{z^2} e^{+i\alpha} + \frac{i(\Gamma - \Gamma_m)}{2\pi z} + \frac{i\Gamma_0}{2\pi} \frac{1}{(z - z_0)} + \frac{i\Gamma_0}{2\pi\left(z - \frac{z^2}{a^2}\bar{z}_0\right)} + \sum_1^n \left[\frac{i\Gamma_m}{2\pi} \frac{1}{(z - z_m)} + \frac{i\Gamma_m}{2\pi\left(z - \frac{z^2}{a^2}\bar{z}_m\right)} \right] \tag{26}$$

Vortices travel down the flowfield by its velocity so that their position is determined by solving the system of equations:

$$\frac{dx_i}{dt} = \left.\frac{\partial\phi}{\partial x}\right|_i, \qquad \frac{dy_i}{dt} = \left.\frac{\partial\phi}{\partial y}\right|_i. \tag{27}$$

Position of the vortex at a new moment can be determined by:

$$d\bar{z}_m = \bar{V}_m dt \tag{28}$$

and its elementary displacement:

$$\Delta\bar{z}_m = \bar{z}_m^n - \bar{z}_m^s = \Delta t \cdot \bar{V} \tag{29}$$

Now a new distance of each vortex as well as the trajectory of the vortex or any fluid particle is given by:

$$\bar{z}_m^n = \bar{z}_m^s + \Delta t \cdot \bar{V} \tag{30}$$

$$\bar{z}_m^n = \bar{z}_m^s + \Delta t \cdot \left\{ V_\infty e^{-i\alpha} - V_\infty \frac{a^2}{z^2} e^{+i\alpha} + \frac{i(\Gamma - \Gamma_m)}{2\pi z} + \frac{i\Gamma_0}{2\pi} \frac{1}{(z - z_0)} + \frac{i\Gamma_0}{2\pi\left(z - \frac{z^2}{a^2}\bar{z}_0\right)} + \sum_1^n \left[\frac{i\Gamma_m}{2\pi} \frac{1}{(z - z_m)} + \frac{i\Gamma_m}{2\pi\left(z - \frac{z^2}{a^2}\bar{z}_m\right)} \right] \right\}$$

According to the Bernoulli's equation pressure distribution can be calculated and then the pressure coefficient:

$$C_p = 1 - \left(\frac{\bar{V}}{V_\infty}\right) \cdot \overline{\left(\frac{V}{V_\infty}\right)} \tag{31}$$

Vortex wake modeling

Remembering the adequate use of vortex panel method in vortex and separation wake simulation, for 3D problem a discrete model of thin carrying surface and vortex and separation wake models follow.

A concentrated vortex in potential flow field placed in 3D area has been taken into the consideration. The area has a finite size with one negligible dimension in regard to the other two. The surface defined in that way represents a thin vortex surface (fig. 1).

Let the thickness of this thin surface be δn and element length δl.

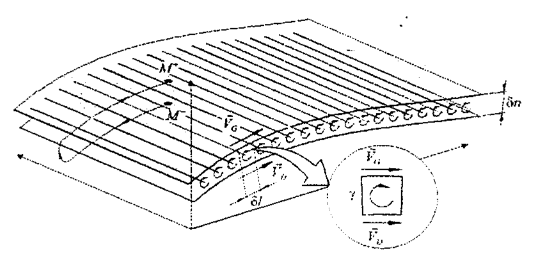


Fig. 1

Circulation around such element can be determined as contour integral (32)

$$\Delta\Gamma = \oint_L \vec{V} \cdot d\vec{l} = V_{G_t} \delta l + \left(-V_{D_t}\right) \delta l = \left(V_{G_t} - V_{D_t}\right) \delta l$$

So that the local intensity of vortex surface equals increment of tangential velocity through surface.

$$\gamma = \left(V_{G_T} - V_{D_T}\right) \tag{33}$$

A pressure coefficient for unsteady flow can be calculated from following expression:

$$C_P = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = 1 - \frac{V^2}{V_\infty^2} - \frac{2}{V_\infty^2}\frac{\partial \Phi}{\partial t} \quad (34)$$

If the difference between pressure coefficients on upper and lower side is presented by following expression:

$$\Delta C_P = -\frac{V_G^2 - V_D^2}{V_\infty^2} - \frac{2}{V_\infty^2}\frac{\partial}{\partial t}(\Phi_G - \Phi_D) \quad (35)$$

and the border of vortex surface is defined as impermeable then following expression can represent the subtraction of squarer velocities in the first approximation:

$$V_G^2 - V_D^2 \approx -2V_\infty \gamma \quad (36)$$

In that way the final expression for reduced pressure coefficient of thin carrying surface is:

$$\Delta C_P = -\frac{2}{V_\infty^2}\left(V_\infty \gamma + \frac{\partial}{\partial t}\int_{NI}^{M} \gamma \, dl\right) \quad (37)$$

Vortex wake modeling of blade considered as carrying surface

A simplification of the thin carrying surface model used for simulation of a blade, as a lift surface is possible by using discretization based on the panel method principle.

The basis of the simulation is distributed vortices discretized in a finite number of vortices in form of concentrated closed quadrangle. The number of vortices equals the number of panels.

One side of a vortex line is placed on the first quarter of panel chord and represents carrying vortex adjoined to that panel. The opposite side is attached to the trailing edge. The other two sides represent beginning of vortex wake, meaning they are streamwise (fig. 2) .

Now, only discrete model of n panels placed in one chord will be considered.

Since a circulation surrounding this discrete vortex surface does not equals zero; this surface can not be considered as carrying one. An additional vortex wake in form of closed quadrangle vortex placed in the airflow is necessary to make this surface a carrying one. One side of this line is attached to the trailing edge and the opposite side is in infinity. The other two sides representing the vortex wake are streamwise.

The vortex intensity of such vortex equals the sum of vortices of all carrying vortex fibers placed at corresponding chord, but in opposite direction. That way the vortex fiber effects are mutually annulled. There fore a circulation surrounding vortex surface does not equals zero, that is it equals the sum of circulations of carrying vortex fibers on panels on corresponding chord.

That way set up vortex surface and vortex wake models are in accordance with the panel schemes formed for steady flow.

Whatever, it can be easily developed for unsteady flow.

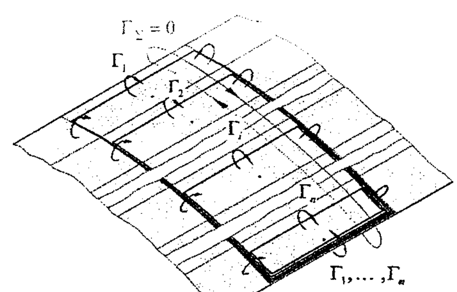


Fig. 2

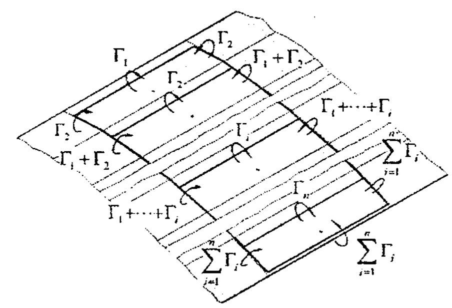


Fig. 3

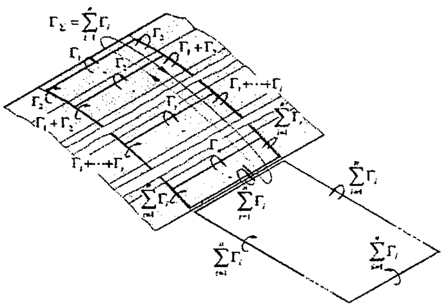


Fig. 4

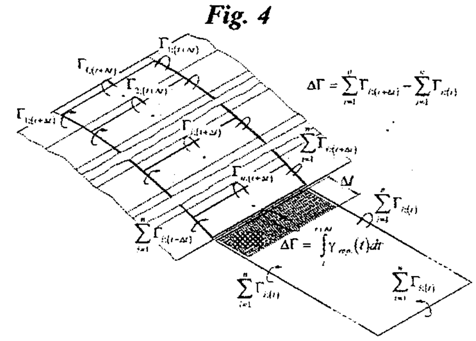


Fig. 5

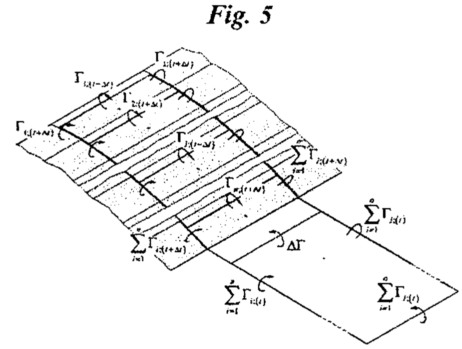
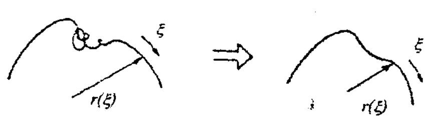


Fig. 6

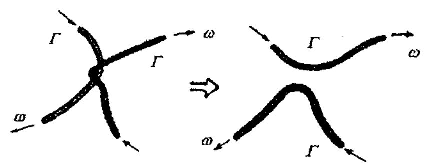
If unsteady flow model forming is considered, it is necessary to set up the model of releasing vortices.
Time change of position of carrying surface results in time change of lift, that is, in change of circulation around carrying surface. On basis of Kelvin's theorem of conservation of vortices, change of circulation around carrying surface must also implicate change of circulation around particles forming vortex wake.
The constant circulation is kept along curve surrounding always the same fluid particles (carrying surface and vortex wake) by successive releasing of vortices in free flow. Modeled carrying surface rests in peace until the time t when it starts to move relatively to the undisturbed airflow. The continually released vortices form a vortex surface which intensity is $\gamma(t)$ behind carrying surface.
There fore, behind carrying surface exists a vortex wake formed by released vortex fibers in any next moment $t+\Delta t$ (fig 5). A circulation around such vortex wake equals the difference between circulations around carrying surface in times $t+\Delta t$ and t .
Discretization of such vortex wake is based on replacement with quadrangle vortex fiber whose one side is attached to the trailing edge, and the opposite side is placed behind trailing edge. The position of this side will be determined in computer procedure. The final form of unsteady flow model is shown at fig 6.
For flow over helicopter blade simulation model is developed and applicated to calculation in following manner. A helicopter blade is discretized to a certain number of segments (fig 7 to 10). Characteristics in particular sections are used for defining flow over whole helicopter rotor blade. Viscous effects are also adapted to the flow conditions.
On that basis unsteady forces acting upon blade are defined. The forces are resulting in extremely unsteady aerodynamic flow field over rotor.

3D simulations

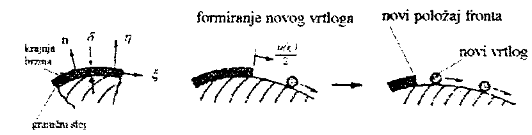
In aim of the most accurate 3D simulation following conditions should be satisfied:
✓ a method of finite cores is used in aim of better tracing of continually distributed vortices and because of local vortex fiber extension; basic characteristics of this method are approximate volume conservation and vortex reinforcement in constant core model;
✓ effects of expression in equations of vortex transport $\nu\nabla^2\omega$ for viscous fluid can be correctly treated by using Gauss distribution; $\gamma_i(\vec{x}-\vec{r}_i)=\dfrac{\exp\left[-|\vec{x}-\vec{r}|^2/\sigma_i^2\right]}{\pi^{3/2}\sigma_i^3}$
✓ viscosity depending on fiber is proportional to the circulation;
✓ knot forming is avoid;



✓ a great number vortex wake component is formed;
a fine net is used;
✓ a change of distribution and directions of closed vortex nooses and its separation;



✓ vortices are formed at solid boundary surfaces;
✓ a vortex net per unit of surface within boundary layer is vertical in regard to velocity at the end of boundary layer; that velocity is constant and tangent to the body surface;



✓ a circulation within boundary layer which passes through given point at the surface in unit of time is streamwise and given by equation:

$$\frac{d\Gamma}{dt}=-\int_0^b\left|\vec{n}\times\vec{\omega}(\eta)\right|_t u_t(\eta)\,d\eta=\frac{\left|u(\vec{r}_c)\right|^2}{2}$$

Helicopter blade vortex wake modeling

Helicopter blade vortex wake modeling includes each previous consideration. That way the changed discrete vortex wake model is consistent with:

- ✓ a panel model,
- ✓ a viscous effects model,
- ✓ a boundary layer simulation model,
- ✓ a streamline separation model,
- ✓ a vortex wake model

Therefore this modeling concept is reasonably adequate for engineer practice. Vortex wake composed of net of vortex line segments with in general, different vortex intensities is approximated in following manner:
✓ a change of circulation around carrying surface is balanced by releasing vortex fibers from carrying surface into vortex wake;
✓ a new line of vortex line segments occurs in radial direction in each time;
✓ a change of circulation in radial direction around carrying surface induces vortex fibers in chord direction;
✓ these vortex fibers can also be approximated by vortex line segments;
✓ knot points of vortex net are taken for characteristic points of vortex wake;
✓ vortex wake deformation is accomplished by displacing characteristic points of vortex wake during time;
✓ characteristic point velocity is equal to the sum of undisturbed flow field velocity and velocity induced by other vortex elements of flow field;

Therefore, vortex wake shape corresponding to reality is accomplished.
Viscosity is, also, taken into the consideration by additional modeling of vortex wake rolling up process, taking place on the edges of vortex wake in real conditions.
Vortex line segments change its positions during deformation process. In that case there is a possibility that knot points can be found very close to the other vortex line segments, especially in areas of highly deformed vortex wake such as roll up area and reverse flow area.
If vortex line segment is modeled as potential vortex fiber, it will induce in very close point extremely high-induced velocities.
Therefore, for vortex wake modeling it is necessary to use vortex line segments with vortex shell. This vortex shell is used for elimination of singularities and has no physical meaning. Then velocities in points of flow field close to vortex line segment have finite values. The edges of vortex wake are modeled by vortex line segments with smaller vortex shell radius, while internal vortex elements are modeled by greater radius elements, not as presentation of real physical effects, but as good approximation of vortex surface with continually distributed velocities. Is taken as $r_c = 0.00275R$ a value for vortex shell radius of elements at the ends of vortex wake, where R is rotor radius. Vortex shells avoid big differences between velocities at the rotor blade in areas close to the vortex wake.
Vortex wake effects at certain distance from the blade can be neglected. It is understandable that size of deformation area depends on regime of helicopter flight. Therefore, as a competent parameter, a coefficient of rotor work regime can be adopted. The area of calculation of vortex wake deformation can be defined through number of revolutions made by rotor as $\mu = 0.4 \cdot m^{-1}$, where m is number of revolutions for which it is necessary to calculate vortex wake deformation. After that vortex wake is ' frozen' in achieved state and continues to flow downstream. This means that their knot points have the same velocity as undisturbed flow. Therefore induced velocities in these points is not necessary to compute. "Frozen" part of vortex wake, however, still effects vortex wake deformation area that it borders on until the distance does not exceed certain value. Furthermore, "frozen" part can, also be neglect by simple elimination from model. In that way, achieved discrete model is consistent with panel model and releasing vortices model.

Vorticity in incompressible viscous flow

In many cases viscosity of the fluid can be neglected. If it has to be taken into account, some approximations and simplifications can be done, without affecting remarkably final results.
Incompressible fluid flow is governed by Navier-Stokes equation:

$$\Delta \mathrm{u} = 0 \qquad (38)$$

$$\frac{\partial \mathrm{u}}{\partial \mathrm{t}} + (\mathrm{u} \cdot \nabla)\, \mathrm{u} = -\nabla p + \nu \nabla^2 \mathrm{u} \qquad (39)$$

The vorticity of the flow is defined as:

$$\omega = \nabla \times \mathrm{u} \qquad (40)$$

By taking the curl of equation (39) we obtain:

$$\frac{\partial \omega}{\partial \mathrm{t}} + (\mathrm{u} \cdot \nabla)\omega = \omega \cdot \nabla \mathrm{u} + \nu \nabla^2 \omega \qquad (41)$$

For two-dimensional flows $\omega \cdot \nabla \mathrm{u} = 0$ and equation (40) reduces to:

$$\frac{\partial \omega}{\partial \mathrm{t}} + (\mathrm{u} \cdot \nabla)\omega = \nu \nabla^2 \omega \qquad (42)$$

We can say that if ω is known than u can be computed using Biot-Savart law . Thus:

$$\mathrm{u}(\mathrm{x}) = \frac{1}{2\pi} i_\omega \times \int \frac{\mathrm{x} - \mathrm{x}'}{|\mathrm{x} - \mathrm{x}'|^2} \omega(\mathrm{x}) dx' \qquad (43)$$

The essence of the inviscid-vortex method is to replace by:

$$\omega(\mathrm{x}) = \sum \Gamma_i \delta(\mathrm{x} - \mathrm{x}_i) \qquad (44)$$

where δ are functions approximating the Dirac δ - function and Γ_i is circulation of the i-the vortex. It has to satisfy the kinematic condition:

$$\frac{d\Gamma}{dt} = -\frac{(u'_x - u''_x)}{2} \qquad (45)$$

Where:

- ✓ $d\Gamma/dt$ - is the change of circulation in the shear layer moving over the stagnation point
- ✓ u_x', u_x'' - are velocities on the upper and lower side of the shear layer

In order to satisfy the equation of motion of inviscid flow:

$$\frac{D\omega}{Dt} = (\omega \cdot \nabla) \cdot V + \nu \quad \nabla^2 \omega = 0 \qquad (46)$$

the velocity of every vortex must be determined by the value of flow velocity at its instant position:

$$\frac{dx_i}{dt} = u(x_i, t) \qquad (47)$$

Let us assume that rather small number of vortices per unit length represent the free shear layer. The known assumption of no-slip condition at the wall (airfoil) leads to the model of wake simulation. If the wake is properly simulated, it will leave airfoil at the trailing edge. In order to obtain proper simulation of viscous effects, it is necessary to assume that the size of the vortex is proportional to the thickens of the wake $\sigma \approx f\delta$. If the wake length is L then number of vortices can be defined as:

$$n \approx \frac{\delta \cdot l}{\sigma^2} \approx \frac{l}{\delta \cdot f^2} \approx \frac{\sqrt{R_e}}{f^2} \qquad (48)$$

Where R_e is Reynolds number

$$R_e = \frac{V_\infty l}{\nu}$$

And ν - kinematic viscosity (ν = μ / ρ).

Mapping of a circular cylinder to airfoil

Mapping of the circular cylinder to airfoil is done by Joukowsky transformation:

$$\zeta = z + \frac{a^2}{z} \tag{49}$$

Where:

$$z = \varepsilon e^{i\delta} + \rho e^{i(\theta - \sigma)} \tag{50}$$

is the parametric equation of the function of mapping of θ and mapping derivative:

$$\frac{d\zeta}{dz} = 1 - \frac{a^2}{z^2} \tag{51}$$

Calculation

According to the whole mentioned analysis a numerical model was established. This model is based on the following:

- ✓ vortices move along the flowfield by the flowfield velocity
- ✓ trajectory of every vortex is defined
- ✓ for every point in the flowfield it is necessary to determine value of complex potential and complex velocity
- ✓ at a certain moment in a defined initial point a free moving vortex is simulated
- ✓ after every time interval Δt moving vortex changes the stagnation point position which must be compensated by introduction of new vortex which brings stagnation point back to its proper place
- ✓ new positions of all vortices are calculated by multiplying local velocities and adding these values to previous
- ✓ viscous effects should be simulated by generating vortices on the airfoil to satisfy no-slip condition; so introduced vortices must be moved by velocity defines by inviscid part of the equation of motion
- ✓ boundary condition of impermeability of the airfoil must be fulfilled
- ✓ vortex diffusion is simulated by variation of the vortex size and arbitrary step
- ✓ computer program is made so that flow parameters can be calculated for different angles of attack.
- ✓ The result of complete calculation for 3D simulation is shown at figs. 21-29

Analysis of the Calculation

Program was run first for characteristic cases of flow around the cylinder (Figs 7, 8 & 9) and flow around airfoil (Figs. 10-20).
Due to computer limitations, a rather small number of streamlines is shown. Model is calculated for different angles of attack (Figs. 11,12,14 & 18), different flowfield velocities (Figs. 14 & 15), different intensities of the moving vortex and its starting position (Figs. 14 & 15 ; 18,19 & 20).

Conclusion remarks

Analysis of results achieved by unsteady lift modeling and viscous effects simulation method shows that they can be used with sufficient accuracy in rotor analysis and construction.
The aim of this particular simulation is to use advantages of vortex methods. For example, vortex methods use the description of flow field of the smallest range; aerodynamic forces can be obtained with small number of vortices. On the other hand, singular vortex distribution can be accurately determined by using data obtained in small time range. Vortex methods, also, permit boundary layer simulation at large Reynolds's number by local concentration of computational points.

Achieved results imply the direction for the further development of this program

- ✓ the model should be expanded for transonic flow in the aim of blade tip analysis
- ✓ a computational dispersion and gradual disappearing of vortex wake are possible by using viscous vortex shell, as more elegant solution than violent elimination of vortex wake used in this program
- ✓ an inclusion of curved vortex elements in aim of achieving better results from vortex wake selfinduction aspect
- ✓ from elastic point of view blade should be modulate as deformable by using a finite element method

On the basis of analysis of presented model and program package for viscous effects and unsteady lift simulation it can be concluded that this paper presents an original scientific contribution, applicable in aerodynamic analyses of practical problems in helicopter rotor projecting.

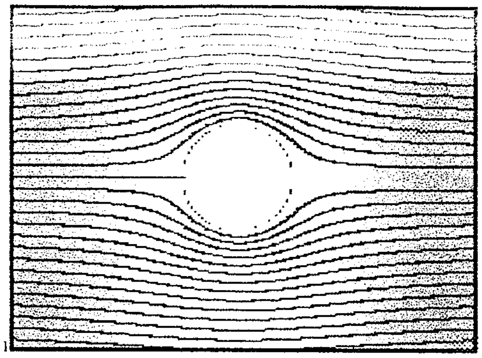


Fig. 7

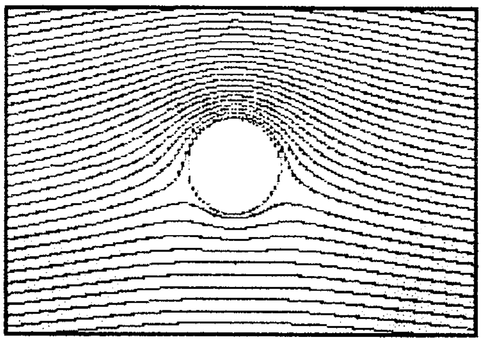


Fig. 8

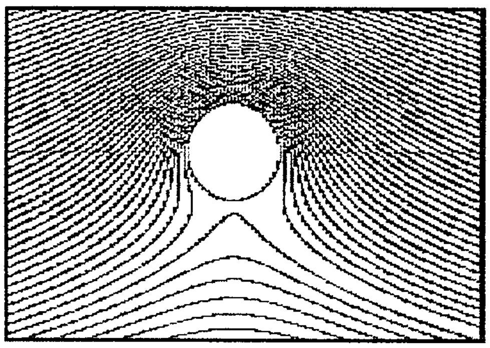
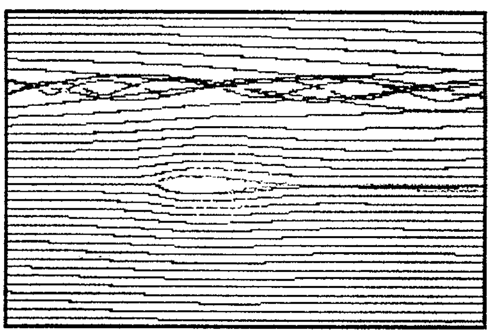


Fig. 9



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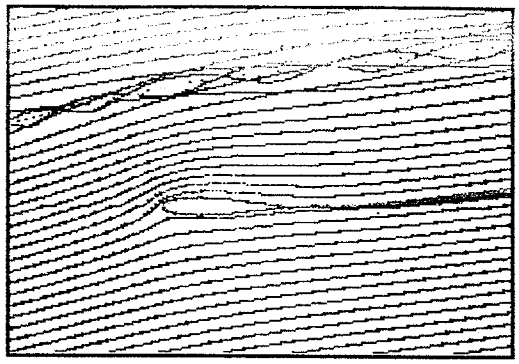


Fig. 11

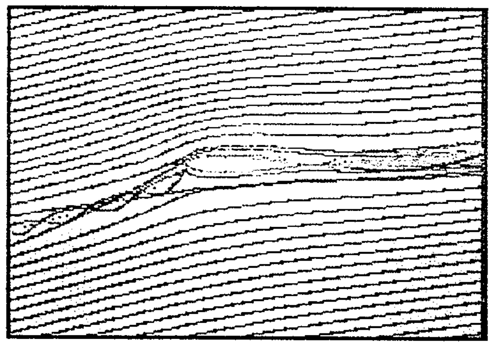


Fig. 12

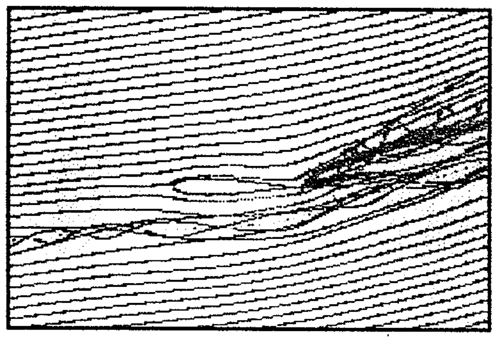
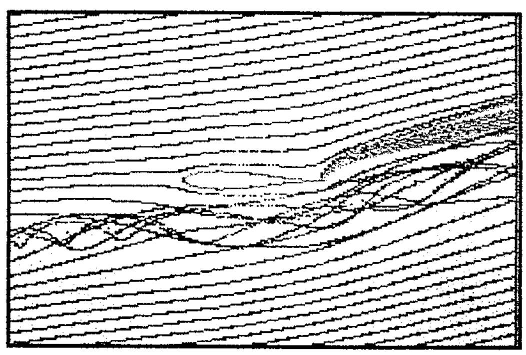


Fig. 13



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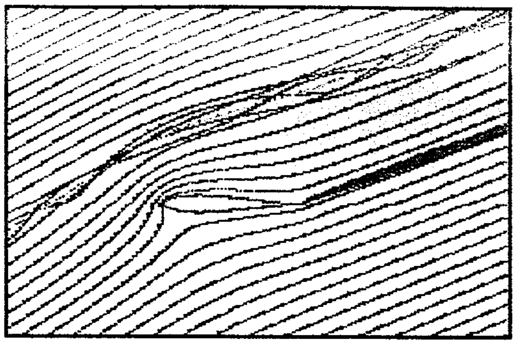

Fig. 15

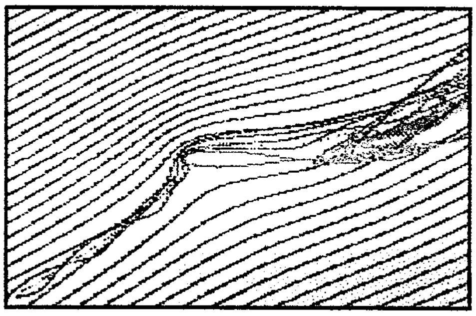

Fig. 16

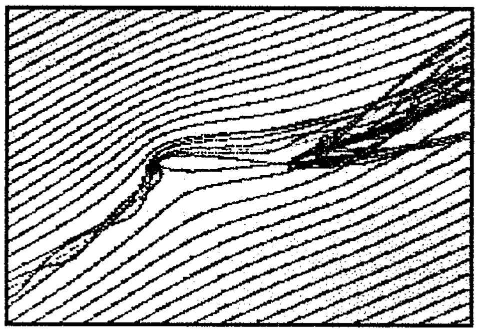

Fig. 17

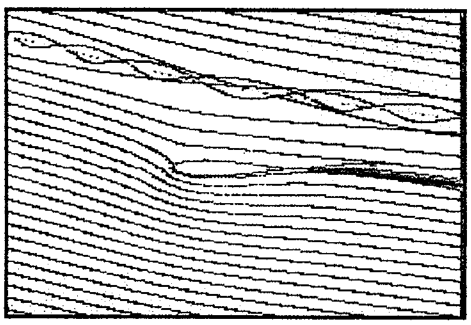

Fig. 18

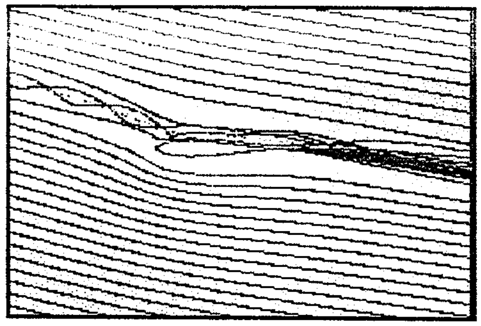

Fig. 19

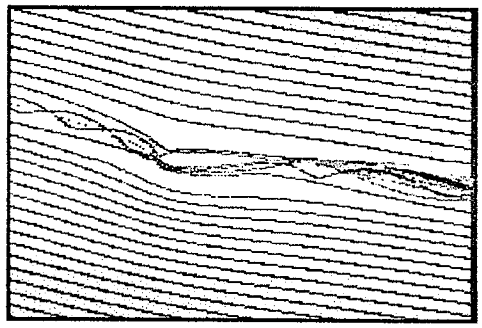

Fig. 20

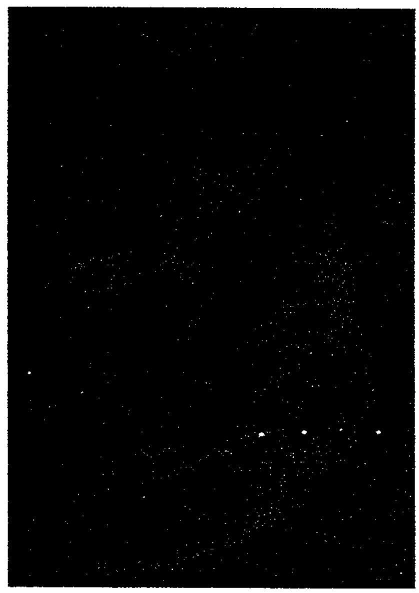

Fig. 21

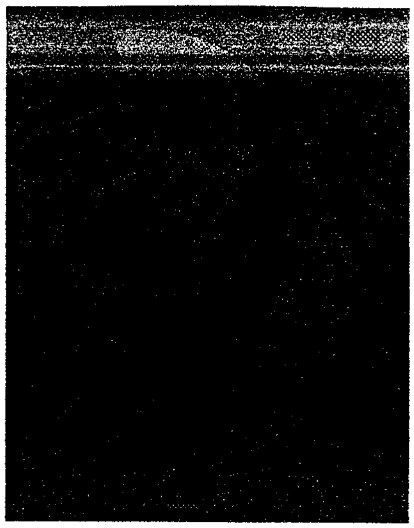

Fig. 22

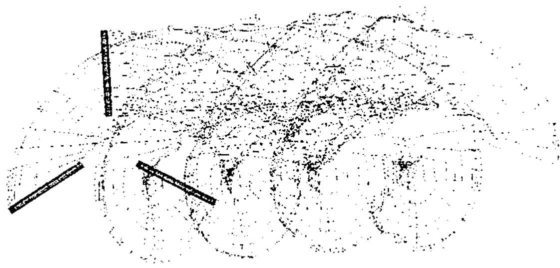

Fig. 25

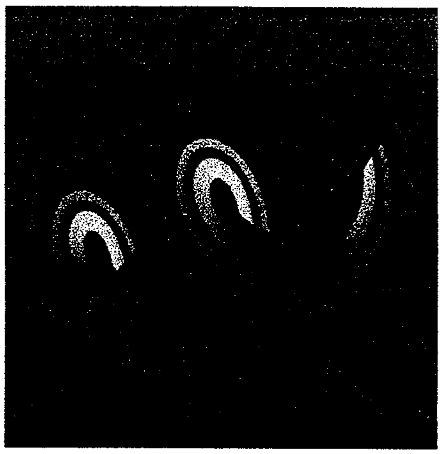

Fig. 23

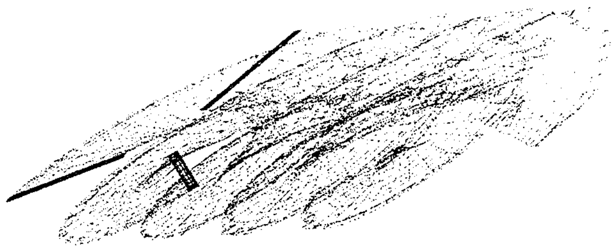

Fig. 26

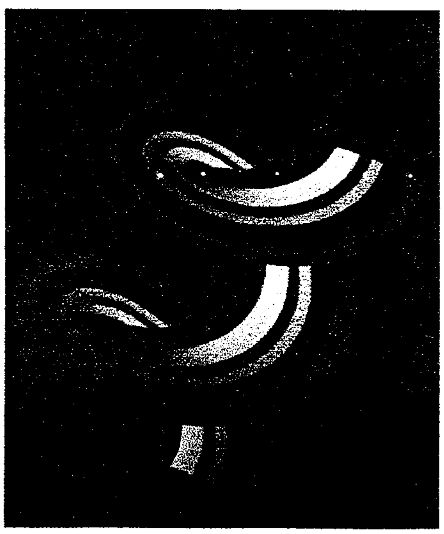

Fig. 24

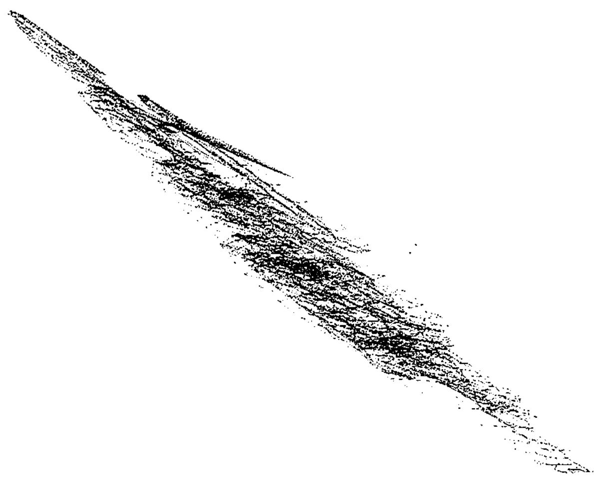

Fig. 27

Fig. 28

Fig. 29

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