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Robustness Analysis applied to Autopilot Design Part 2: Evaluation of a new Tool for μ – Analysis

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Abstract

The mixed structured singular value μ allows the designer to determine the robustness of a control system. The exact calculation of this measure is not possible. Therefore, we confine ourselves to the determination of lower and upper bounds. As lower bound calculation remains still a main interest of research, especially in the case of real parameter uncertainties, a new algorithm for its calculation will be presented.
The new method is based on modal control ideas. The first order variation of the closed loop eigenvalues is used to move the poles towards the imaginary axis in order to determine the smallest destabilizing perturbation Δ. Frequency gridding is not necessary anymore, we will speak of frequency "*sweeping*". For that reason we can not miss a thin peak in the μ–plot.
We apply the new procedure to the Research Civil Aircraft Model (RCAM) that served as the basis for a robust flight control benchmark defined by GARTEUR.
We compare the new lower bound of μ with the classical results. The calculation time is considerably reduced whilst the worst case is always detected. Furthermore, the perturbation matrix Δ is not distorted anymore by adding artificial complex perturbations.

1. Introduction

In industry, the validation of command laws is based on a large number of simulations whilst all the uncertain parameters entering the definition of the considered system are varied. The number of tests is an exponential function of the number of uncertain parameters. In aeronautic and space systems for example, the set of uncertain parameters is so large that the number of considered parameter configurations has to be limited on beforehand, introducing the risk that potential stability problems remain undetected. Until now and with a good physical comprehension, it was possible to minimize, even to avoid such a risk. The actual systems become, for economical reasons, lighter and lighter, therefore still more flexible. The flexibility complicates the robustness analysis considerably. Hence, nowadays more sophisticated validation tools than simulations are necessary.

The structured singular value SSV μ allows the designer to determine the robustness of a control system.[1] For that purpose describe the system in the so–called standard form, a Linear Fractional Transformation LFT description of the model, where all uncertainties are eliminated in the model matrix $M(s)$ and stacked in one matrix $\Delta(s)$, the perturbation matrix (see Fig. 1). The uncertainties can be of pure complex, mixed or pure real character, *i.e.* they describe effects of neglecting dynamics (normally full complex blocks) or respectively uncertain model parameters (real repeated scalars).

The exact calculation of this measure is proved to be NP–hard, *i.e.* the computation time for μ is an exponential function on the problem size, so it is limited to simple problems.[2–4] Therefore, we confine ourselves to the determination of lower and upper bounds.

The case of pure complex perturbations has already been treated in the late eighties. Real repeated uncertainties have been replaced by repeated complex scalars. No problems arise for lower and upper bounds, especially if the number of full complex blocks is high enough.[5,6]

Then the case of mixed perturbations has been handled with. A lot of work has been spent to the upper

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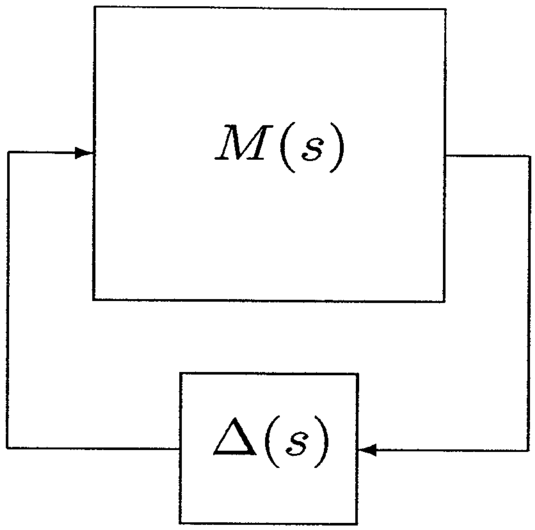

Fig. 1. The standard form

bound computation.[7–10] A good upper bound seems to be found by frequency gridding.

On the other hand the lower bound remains still a main interest of research, especially in the pure real case. The convergence of the so-called standard power algorithm SPA is not sufficient: limit cycles arise during the run. A lot of improvements have been proposed but either the problem is not solved or the calculation time is augmented considerably.[11–14] Some different approaches are limited to certain groups of perturbations.[15,16] Another approach is to determine the worst case of a problem where some complex scalars are added artificially to improve SPA convergence characteristics. Then omit the complex part of this perturbation matrix and reshift the destabilizing eigenvalue on the imaginary axis by minimizing the $\overline{\sigma}(\Delta)$–norm.[17] A problem, common to all cited approaches is frequency gridding. A thin 'peak' in the μ–plot, especially provoked by a flexible mode, can be missed if the number of sample points is not sufficiently high and/or the calculation option is not sufficiently accurate which ends immediately in increasing computation time or workspace problems.

Thus, a completely new algorithm for the calculation of the lower bound of μ based on first order variations of eigenvalues has been proposed recently.[18] This algorithm is developed to handle with thin μ–peaks of flexible structures by "frequency sweeping", *i.e.* without use of frequency gridding which is demonstrated on a flexible satellite system.[19]

It will shortly be presented in this article and then validated upon the rigid Research Civil Aircraft Model (RCAM), a robust flight control benchmark problem proposed by GARTEUR*.[20] The work reveals that — even if the system is robust and thus the μ–plot is more or less constant over all frequencies and does not show significant peaks — the algorithm detects the maximum value of μ, the worst case. This is a first step towards future robust flight control with flexible aircraft.

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2. Definitions

Let us consider a quadruple in state space form (A, B, C, D) and in transfer matrix form

$$M(s) = C(sI - A)^{-1}B + D$$

A class of uncertainties Δ (which act as a feedback on the system, see Fig. 1) has the following structure:

$$\Delta = \mathrm{diag}(\Delta_1, \ldots, \Delta_r) \qquad (1)$$

in which Δ_i might be a diagonal matrix of the form $\Delta_i = \delta_i I_{n_i}$ with $\delta_i \in \mathbb{R}$ (real repeated scalar block) or $\delta_i \in \mathbb{C}$ (complex repeated scalar block) or a matrix in $\mathbb{C}^{n_i \times n_i}$ (full complex block). In the part relative to the lower bound Δ_i might also be a matrix in $\mathbb{R}^{n_i \times n_i}$ (full real block). Such a matrix Δ will be called an "admissible perturbation".

The real and imaginary part of a complex number are denoted $\Re(.)$ and $\Im(.)$, the maximum singular value $\overline{\sigma}(.)$. A matrix $(.)^*$ is the complex conjugate transpose of $(.)$.

First let us recall a well known result:

Lemma 2.1. If Δ is such that the interconnection (M, Δ) remains well-posed (*i.e.* for all admissible Δ, $\det(I - D\Delta) \neq 0$) and if s_0 does not belong to the spectrum of A:

$$\det\,(I - M(s_0)\Delta) = 0 \qquad (2)$$

$$\Leftrightarrow s_0 \in \text{spectrum}\left(A + B\Delta\,(I - D\Delta)^{-1}\,C\right) \qquad (3)$$

In order to compute the SSV at a complex point λ, we address the problem of assigning λ in the closed loop spectrum with a "feedback" Δ (belonging to the set of admissible perturbations) of minimum "sigma-max norm". Usually, the SSV is defined via Equation (2),[1,21] but in view of Lemma 2.1, the following definition via Equation (3) will be considered.

Definition 2.2. The SSV at point λ is defined as $\mu = 1/\overline{\sigma}(\Delta)$ where Δ is an admissible perturbation such that

(1) λ belongs to the closed-loop spectrum of (A, B, C, D) with feedback Δ

(2) $\overline{\sigma}(\Delta)$ is minimum.

3. The algorithm

3.1 Generalities

A new algorithm which computes a lower bound of the peaks of the μ–curve is proposed. This algorithm is efficient for mixed or pure real uncertainties. From the fact that frequency gridding is not used, the proposed algorithm is very fast. This is quite useful in a design cycle in which it is necessary to detect worst cases.[22]

The idea beyond this technique consists of shifting the eigenvalues towards the imaginary axis with a minimum perturbation. The proposed algorithm is divided into two steps. The first step is used to reach the limit of stability with a perturbation of minimum Frobenius norm. An adaptation of the "pole migration" is used:[23]

Lemma 3.1. The first order approximation $d\lambda$ of the motion of an eigenvalue of the matrix A_0 induced by a gain variation $d\Delta$ of Δ_0 is

$$d\lambda = (uB + tD)d\Delta(Cv + Dw) \tag{4}$$

where v is the right eigenvector of A_0, u is the left eigenvector of A_0 corresponding to the eigenvalue λ and $w = \Delta_0(I - D\Delta_0)^{-1}Cv$, $t = uB\Delta_0(I - D\Delta_0)^{-1}$.

The second step consists of minimizing the "sigma-max norm" of the perturbation obtained after the first step while remaining at the limit of stability.

3.2 Determination of a destabilizing perturbation matrix Δ_F of minimum Frobenius norm

The proposed algorithm is sketched first. Several comments are given afterwards in order to discuss some implementation adaptations that improve the efficiency.

Principle of the algorithm. Let λ denote one of the eigenvalues of $A+B\Delta_0(I-D\Delta_0)^{-1}C$. It is intended to find $d\Delta$ (a variation of Δ_0) that shifts (first order approximation) λ to a vertical line which is distant of a small amount denoted $\Re_i$. A motion from λ to the vertical line defined by $\Re_i$ is performed as follows. Equation (4) is a linear constraint on $d\Delta$

$$\Re((uB + tD)d\Delta(Cv + Dw)) = \Re_i \tag{5}$$

$d\Delta$ satisfying (5) will be computed such that the Frobenius norm

$$J_1 = ||\Delta_0 + d\Delta||_F^2 \tag{6}$$

is minimum. This is a problem of quadratic optimization under linear constraints. *Such a problem will be solved at each iteration of the proposed algorithm.* In order to avoid initialization problems due to the fact that, when the criterion J_1 is minimum for large values of $d\Delta$, the first order approximation of Lemma 3.1 is no longer valid, we shall minimize a combination of criteria J_1 and J_0, where J_0 is defined by

$$J_0 = ||d\Delta||_F^2 \tag{7}$$

At the beginning of the algorithm, J_0 is considered, then a combination of J_0 and J_1 that becomes equal to J_1 (see (9)). Usually, $N = 20$ is enough.

Algorithm 1.

Step 1 - Initialization. Choose the initial open-loop eigenvalues that are to be moved towards the imaginary axis. Choose also the expected number of steps (say N) that will be used in order to shift each initial eigenvalue to the target. For each initial eigenvalue λ, perform the following steps. Set $i = -1$ and $\Delta_0 = 0$.

Step 2 - Compute u, v, w, t and solve (5), for $d\Delta$ having the admissible structure, for a variation of $\Re(\lambda)$ given by $\Re_i = -\Re(\lambda)/(N - i)$ *i.e.*

$$\Re((uB + tD)d\Delta(Cv + Dw)) = \Re_i \tag{8}$$

such that

$$\frac{(N - i - 1)||d\Delta||_F^2 + (i + 1)||\Delta_0 + d\Delta||_F^2}{N} \tag{9}$$

is minimum.

Step 3 - Set $i = \min(i+1, N-1)$, $\Delta_0 = \Delta_0 + d\Delta$. After Δ_0 is updated, select the new closed loop eigenvalue that is the closest to $\lambda + \Re_i$ that will become the new λ. If λ is close enough to the imaginary axis, stop, otherwise go to step 2.

Comments relative to Algorithm 1. In order to reduce the computing time it is useful to apply the algorithm only to a subset of the poles of $M(s)$. The use of the bandwidth knowledge of the system behavior can help. It is also possible to apply a controllability measure.

Equation (8) is a linear constraint relative to the entries of $d\Delta$ and (9) is a quadratic criterion. An intermediate step which consists of writing the entries of Δ_0 and $d\Delta$ as real vectors ξ_0 and $d\xi$ (in which uncertainties are not repeated) is necessary. Equation (8) can be written

$$A d\xi = b \tag{10}$$

and J_0 and J_1 in (9) become respectively

$$\begin{aligned} J_0 &= d\xi^T \boldsymbol{H}_0 \; d\xi \\ J_1 &= d\xi^T \boldsymbol{H}_1 \; d\xi + 2c_1 \; d\xi \end{aligned} \tag{11}$$

Limitations come from the fact that just first order approximations are applied and that Equation (4) is only valid for non repeated eigenvalues.[19]

3.3 Determination of a destabilizing perturbation matrix Δ of minimum $\overline{\sigma}(\Delta)$ norm

After having used the algorithm of the previous section, we have at our disposal a matrix Δ_0 which assigns a pole λ_t on the imaginary axis. But the norm that was minimized for obtaining this result is not the right one. So, a second algorithm is proposed: the assignment of $\Re(\lambda_t)$ is preserved while the convergence towards a matrix "Δ" with minimum sigma-max norm is performed.

Principle of the proposed algorithm. Considering the singular value decomposition (s.v.d.) of $\Delta_0 = USV^*$ and denoting V_1 the first column vector of V, the maximum singular value of Δ_0 is

$$\overline{\sigma}(\Delta_0) = \sqrt{V_1^* \Delta_0^* \Delta_0 V_1} \tag{12}$$

so we have to minimize (12). For that purpose a new criterion

$$J_2 = V_1^*(\Delta_0 + d\Delta)^*(\Delta_0 + d\Delta)V_1 \tag{13}$$

is defined in which V_1 is relative to Δ_0. As it is expected to consider small variations of Δ_0 we shall have

$$J_2 \approx \overline{\sigma}(\Delta_0 + d\Delta)$$

It is this approximation J_2 of the maximum singular value that will be considered.

Algorithm 2.

Step 1 - Initialization. Perform Algorithm 1. Let Δ_0 denote the resulting admissible perturbation and λ the eigenvalue which is approximatively on the imaginary axis. Choose the number of iterations N. Set i = 0.

Step 2 - Compute u, v, w, t and solve, for $d\Delta$ having the admissible structure,

$$\Re((uB + tD)d\Delta(Cv + Dw)) = -\Re(\lambda) \tag{14}$$

such that

$$\frac{(N-i)\|\Delta_0 + d\Delta\|_F^2}{N} + \frac{iV_1^*(\Delta_0 + d\Delta)^*(\Delta_0 + d\Delta)V_1}{N} \tag{15}$$

is minimum, in which V_1 is relative to the s.v.d. of Δ_0.

Step 3 - Set $i = \min(i+1, N-1)$, $\Delta_0 = \Delta_0 + d\Delta$. After Δ_0 is updated, select the new closed loop eigenvalue that is the closest to $\Im(\lambda)$ that will become the new λ. If the value of J_2 becomes stationary, stop, otherwise go to step 2.

Comments relative to Algorithm 2. The linear constraint in Equation (14) can be expressed like in Equation (10) and the quadratic criteria J_2 of Equation (15) obviously like J_1 in Equation (11).

The criterion in (15) is a combination of J_1 and J_2. At the first step, $i = 0$, J_1 is dominant in the composite criterion. When i increases, the part of J_2 increases. Note that J_2 is not minimized without considering a small amount of J_1 because J_2, unlike J_1, is not a definite quadratic form. Usually, $N = 20$ is enough.

If the algorithm is run as above, the optimization will end when the two leading singular values of Δ become equal. It is worth noting that Algorithm 2 tends to tune the Frobenius optimal perturbation by reducing the leading singular values. Coalescence of two or more leading singular values is therefore usual. In order to optimize further after the two leading singular values are equal it suffices testing that some singular values become equal and then to adding some rows to the linear equation that corresponds to Equation (14). A number of additional iterations depending on the number of singular values which become equal has to be performed. Globally, 80 iterations are enough for most systems.[19]

4. Application to a civil transport aircraft

In the following we will consider the Research Civil Aircraft Model RCAM which served as a robust flight control benchmark problem in the GARTEUR† framework. It is a twelve state model for longitudinal and lateral motion. The challenge was to design flight

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controllers robust to variations in airspeed V_A, mass m, time delay τ and position of the horizontal and vertical center of gravity X_{cg} and Z_{cg} during cruising, capture of glidepath and localizer beam and final approach. 12 controllers have been proposed. Performance, safety, passenger comfort and control activity criteria have been evaluated in an automated procedure. Robustness has been assessed by tools from the standard MATLAB μ–toolbox.[20,21,24]

The RCAM is a rigid model, however this example is not less interesting as the controlled aircraft should be more or less robust and the μ–plot should not show significant peaks. The fact that an algorithm developed for flexible structures with their pregnant thin peaks does also work under such conditions is remarkable. Its performance with flexible systems is demonstrated on a flexible satellite.[19]

4.1 Modelling under standard form

For analysis in the μ–framework the aircraft has to be modeled in the so–called standard form M_p–Δ_p (Linear Fractional Transformation LFT), *i.e.* the aircraft is given at a nominal configuration (if possible the algebraic average of the parameter ranges) in the transfer function M_p and all (normally norm–bounded) uncertainties are stocked outside the aircraft model in a perturbation matrix Δ_p. This work is detailed in Part I and Part III of this article in the same proceeding.[21,24] At first, airspeed V_A is not considered as uncertain parameter. It results

$$\Delta_p = \begin{bmatrix} \delta_m I_{17} & 0 & 0 \\ 0 & \delta_{xcg} I_{15} & 0 \\ 0 & 0 & \delta_{zcg} I_3 \end{bmatrix}$$

With airspeed as uncertain parameter and after renaming of current combinations of the uncertain parameters m and V_A we get

$$\Delta_p' = \begin{bmatrix} \delta_m I_{34} & 0 & 0 & 0 \\ 0 & \delta_{xcg} I_{29} & 0 & 0 \\ 0 & 0 & \delta_{zcg} I_7 & 0 \\ 0 & 0 & 0 & \delta_{V_A} I_{92} \end{bmatrix}$$

After adding the LFT description of the control system's delay time M_τ–Δ_τ and the actuator dynamics, it suffices here to close the loop by the different controllers K, see Fig. 2.

We do not have to add 5 artificial complex scalar uncertainties as it was done in Part I to make the SPA (for the lower bound of the μ–toolbox) to converge.[21] Hence, we do not disturb our results by that regulation. Normally, a perturbed system is not yet on the limit of stability if the artificial uncertainties are just omitted,

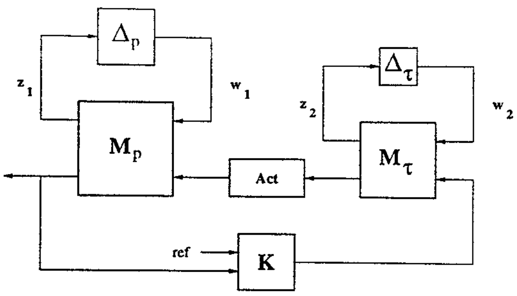

Fig. 2. Interconnection of the subsystems

so the perturbation matrix has to be adapted. Thus, we save some computation time.

Finally, the structure of Fig. 1 is obtained with

$$\Delta = \begin{bmatrix} \Delta_p & 0 \\ 0 & \Delta_\tau \end{bmatrix}$$

or Δ_p' respectively.

4.2 Robustness analysis of one controller

In Fig. 3 the μ–plot of controller HI–09 is depicted. It is an $\mathcal{H}_\infty$ controller with 26 states which served as an example controller for all participating groups, so that it was not optimized in sense of robustness. This fact explains the peak at about $0.7\,rad/s$ of $\overline{\mu} = 0.91$ in the standard μ–toolbox upper bound (continuous line). The lower bound is given with $\underline{\mu}_{SPA} = 0.83$ by the SPA (dashed line). The second strong peak at $0.3\,rad/s$ is not detected by the SPA, neither the peak at $0.1\,rad/s$. On the other hand, the new algorithm computes 4 peaks of the lower bound (*) where the critical one is $\underline{\mu} = 0.88$. In comparison with the corresponding upper bound a gap of 3.3% results which is acceptable in contrast to a gap of 8.7% using the SPA.

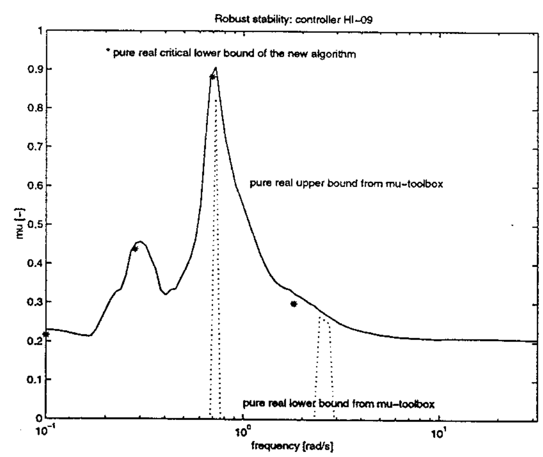

Fig. 3. Comparison between the classical pure real μ – plot and the proposed lower bound for controller HI-09

For further validation of the calculated lower bound we confine ourselves to the result of the highest peak in the μ–plot. At first, the stability domain is plotted in Fig. 4. The system is perturbed by a set of matrices Δ where the parameters ξ vary in between -1.5 and 1.5. The configuration where the system remains stable is plotted with a dot $\cdot$, the unstable ones are represented with a white background. Normally, we would get a hyper–space of dimension 4 or 5 (if V_A is also uncertain), for readability purposes we just present the cut of the plane ξ_1 and ξ_2. If our algorithm works well, we should find configurations on the limit of the dotted region.

After algorithm 1 we dispose of a destabilizing matrix Δ_{Frob} with minimum Frobenius norm. This is expressed by the parameter set A_{Frob} on the "smallest" hyper–ball (in 2D a circle) around the nominal point which can be inscribed in the dotted region. After algorithm 2 we finally have the destabilizing matrix $\Delta_{\sigma_{max}}$ with minimum σ_{max} norm, in other words with magnitude of μ. That is the "smallest" hypercube (here square) which can be inscribed. The movement of 70% of point A highlights the importance of algorithm 2.

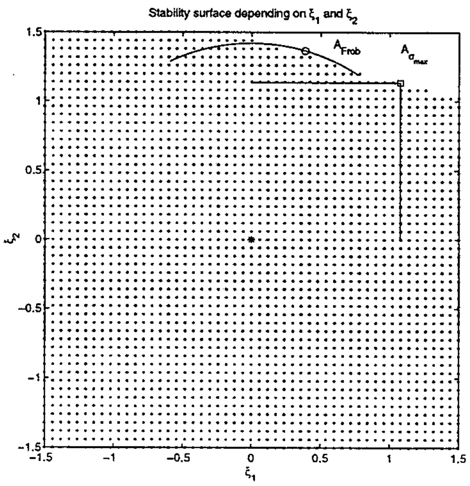

Fig. 4. Stability domain of the system with controller HI–09

Another test, possibly more common, is a pole map plotted during the iterations of algorithms 1 and 2. One pole is shifted to the imaginary axis during algorithm 1 and remains there during algorithm 2. We are indeed on the limit of stability.

Airspeed V_A is omitted in the analysis of Part I as the LFT is too big for acceptable computation time. The calculation of μ (lower and upper bounds) takes for example $18\,h$ for controller HI-09 if airspeed is considered as an uncertain parameter. Of course, no lower bound is obtained. This is really unacceptable as the lower bound gives the worst case configuration and not the upper bound. The upper bound serves just for validation purposes. On the other hand, the proposed algorithm takes just $37\,min$ which is still reasonable for

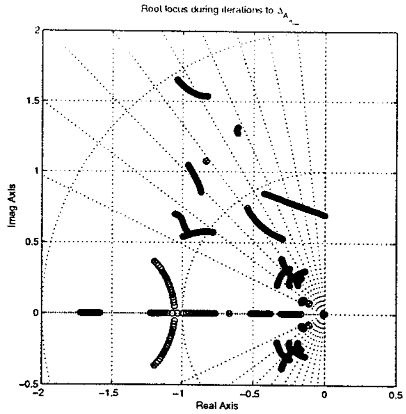

Fig. 5. Root locus of the system perturbed by Δ

such a problem size. The peak is detected with 10% precision(see Fig. 6). Hence, the proposed algorithm still works well whilst other algorithms (for example SPA or LMI based upper bounds) do already not work anymore.

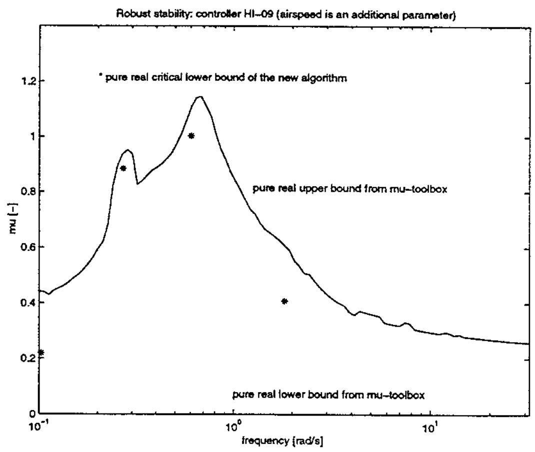

Fig. 6. Comparison between the classical pure real μ – plot and the proposed lower bound for controller HI-09 with airspeed V_A as additional uncertain parameter

4.3 Robustness analysis of all controllers

In Table 1 the μ lower bounds of the critical peak from the new algorithm are compared with those of the SPA. Often, the SPA does even not offer a result. For validation of the computed lower bounds the standard upper bound from the μ–toolbox is determined. The biggest gap is of 7%.

For controller MS–19 the lower bound is greater than the upper bound which is due to a very thin peak. During a first analysis of all controllers described in Part I this peak was not detected at all, neither with a gridding of 50 points. For the determination of a good upper bound make a zoom around $15\,rad/s$,

Controller [1]	New algorithm		μ–toolbox [2]		
	Frequency [rad/s]	lower bound [–]	lower bound [–]	upper bound [–]	Frequency [rad/s]
HI–09	0.69	0.88	0.83	0.91	0.72
MS–11	0.60	0.45	0.44	0.48	0.61
MM–12	7.99	0.33	0	0.35	7.83
CC–13	0.87	0.48	0	0.49	0.86
LY–14	0.75	0.54	0.53	0.56	0.77
FL–15	5.82	0.41	0	0.43	5.86
MO–16	6.37	0.33	0	0.34	6.58
EA–18	6.96	0.77	0	0.81	6.97
MS–19	15.06	1.31	0	1.25	14.85
HI–21	1.28	1.49	1.35	1.50	1.29
EA–22	0.51	0.37	0.37	0.38	0.51
MF–25	0.65	0.63	0.62	0.64	0.64

[1] **Type:** FL = Fuzzy Logic, MM = Modal Multi-Model Synthesis, MO = Multi-Model Multi-Objective Optimization, EA = Eigenstructure Assignment, CC = Classical Control, MS = μ–Synthesis, MF = Model Following, HI = H_∞, LY = Lyapunov; **Number:** corresponds to GARTEUR report number

[2] gridding with 100 points is imperative for accuracy

Table 1. Comparison between the worst–cases computed by the classical approach and by the proposed algorithm

i.e. define a small frequency interval around $15\,rad/s$ and then compute for example 100 points. The upper bound peak increases.

Concerning computation time, generally the new algorithm takes just a third of the corresponding time with the standard μ–tool of MATLAB. Hence, the new algorithm is also useful for an initial analysis to detect critical configurations where to apply deeper–going analysis.

In Fig. 7 – Fig. 18 the μ–plots corresponding to all proposed controllers are depicted in the same manner as in Fig. 3. Generally speaking, all critical peaks are detected by the new algorithm while the SPA misses several peaks. What is more, the new algorithm detects almost all smaller peaks as well. Just with controller EA–22 it misses peaks. The problem is that the controller is robust, so that the variations of eigenvalues of the system should be very small and the first order approximations are not valid anymore (especially at the beginning of algorithm 1).

With controller MO–16 it detects a peak at almost $0.7\,rad/s$, but the magnitude of the peak is not correct. Remark the fact that the SPA has converged as well to a lower bound comparable to our one. So, the question arises if it is not the upper bound which is too conservative. A zoom in the frequency interval of $0.3\,rad/s$ and $1\,rad/s$ with 100 points and application of the more precise calculation option have reduced the upper bound peak from 0.32 to 0.26. Hence, the gap is in fact not too bad.

5. Conclusions

A new powerful algorithm has been presented and applied to a certain number of robust flight controllers. The results are very promising. For validation purposes of the lower bound we need an upper bound. To draw advantage from the fast lower bound computation and to get rid of the time consuming and not very reliable calculation of the upper bound by frequency gridding with the standard μ–toolbox, we are actually working on a LMI based upper bound based on the use of piecewise constant scalings.[19]

Having both tools by hand, we can go one step further and attack robust controller synthesis problems by self–scheduled gains.

6. Acknowledgements

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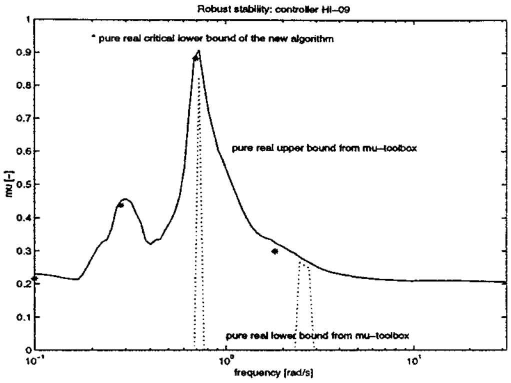

Fig. 7. $\mathcal{H}_\infty$-synthesis (HI-09)

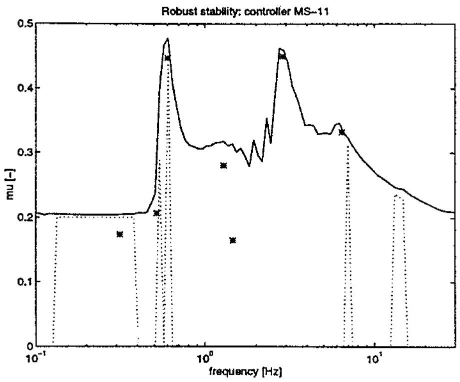

Fig. 8. μ-synthesis (MS-11)

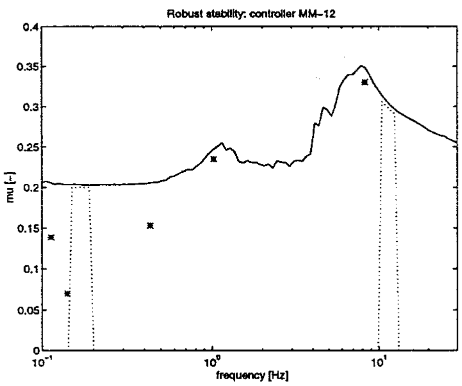

Fig. 9. Modal-multimodel approach (MM-12)

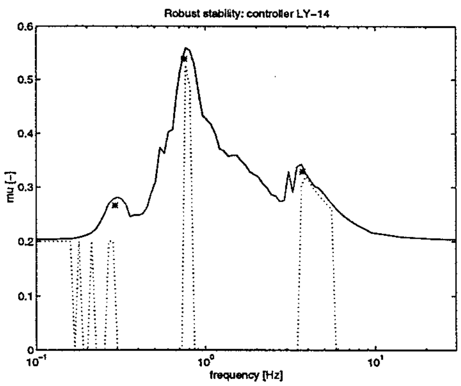

Fig. 10. Quadratic stability (LY-14)

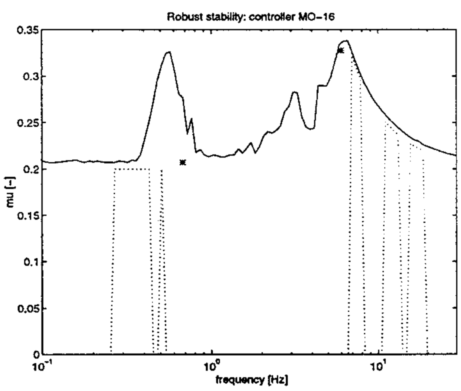

Fig. 11. Multi-objective optimization (MO-16)

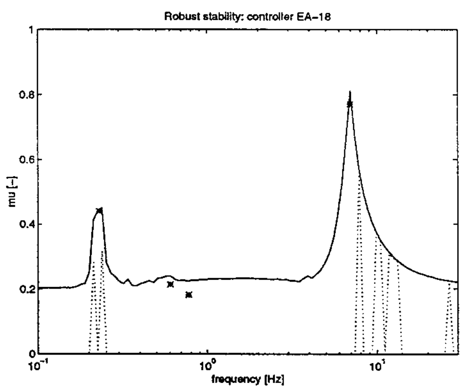

Fig. 12. Eigenstructure assignment (EA-18)

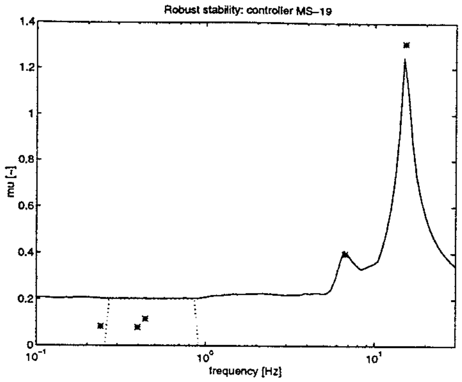

Fig. 13. μ-synthesis (MS-19)

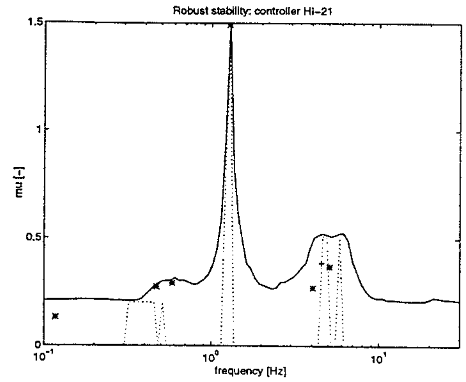

Fig. 14. $\mathcal{H}_\infty$-synthesis (HI-21)

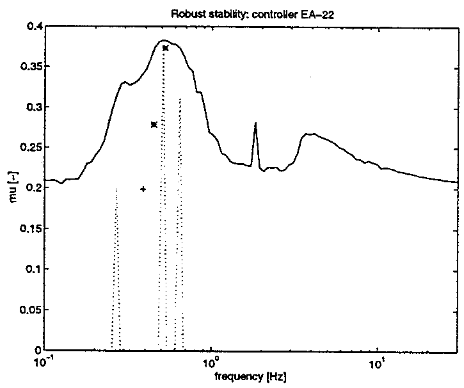

Fig. 15. Eigenstructure assignment (EA-22)

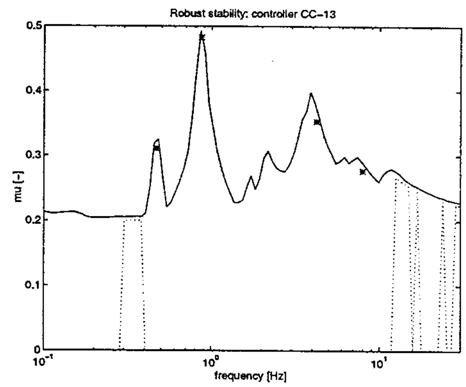

Fig. 16. Classical control (CC-13)

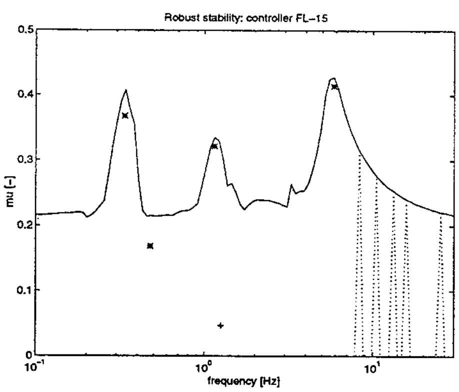

Fig. 17. Fuzzy control ("linearized") (FL-15)

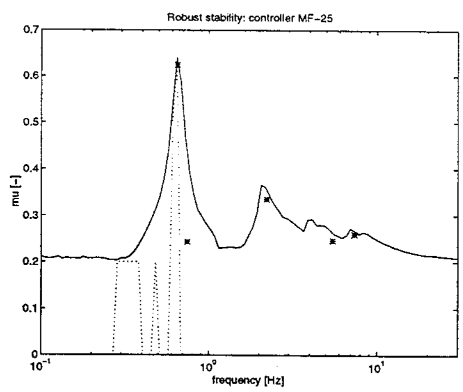

Fig. 18. Model of reference (MF-25)