ICAS-98-1,5,2

A98-31461

FLEXIBLE STRUCTURE CONTROL BY MODAL MULTI-MODEL APPROACH : APPLICATION TO A FLEXIBLE AIRCRAFT

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Abstract

This paper presents a generalization to multi-input of a technique used for designing control laws of SISO flexible structures. The first objective is to shift certain modes to the desired direction, generally towards the left in the complex plan, by means of a dynamic controller and a loop gain variation.
The technique is based on the first order development of the variation of the system's eigenvalues. It is used for the stabilization of high frequency modes of a flexible structure which would have been destabilized during a previous loop (stabilizing a rigid body structure).
The method of synthesis can be seen as a generalization to the MIMO case of phase control. Its multi-model feature provides high robustness and the solutions remain very simple. It consists of minimizing a quadratic criterion under linear constraints. The phase control can be combined with the gain control by assigning no motion to some eigenvalues or by minimizing a relevant criterion.
We apply this procedure to the flexible aircraft.

1. Introduction

The evolution in the aeronautical industry leads to high capacity aircraft development. The optimization of the design of these large aircraft, which includes high aspect ratio and new materials like composites, makes them become more flexible. This evolution increases the interaction between aeroelastic dynamics and control laws, known as aeroservoelasticity. Classical synthesis techniques, like eigenstructure assignment with output feedback, may be unsuitable for directly computing flight control laws because of the difficulty of controlling both rigid and flexible modes.(1) Even if several approaches based on eigenstructure assignment for dealing with flexibility have already been proposed in literature,(2–5) the philosophy used therein is very different from ours.
The technique presented in this paper is based on the first order development of the variation of the system's eigenvalues, $\Delta\lambda = u\Delta A v$. Applying this expression to some of the augmented system's eigenvalues (controller and system)(see Figure 1) makes it feasible to express these values according to the variation of the loop gain ρ and the controller's transfer matrix to the given frequency $G_c(\lambda_i)$.
This article is divided into two parts :

- The first part will present the technique.
- The second part, dealing with the problem of flexible aircraft, will demonstrate the multi-model aspect of this method.

2. Problem statement and main result

2.1 Problem statement

We shall consider the following linear system with n states, m inputs, p outputs:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

where y is the vector of measurements, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$. This system may already be in closed loop. For example, if we consider a flexible

21st ICAS Congress
13-18 September 1998
Melbourne, Australia

structure, a controller of the rigid body may be included in (1).

It is well known that variations of the matrix A, say ΔA, induces variation of the eigenvalue λ given at first order by

$$\Delta\lambda = u\Delta A v \qquad (2)$$

where v and u are right and left eigenvectors of A relative to the eigenvalue λ.

Proof.[(6)]

Let define

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c u_c \\ y_c &= C_c x_c + D_c u_c \end{aligned} \qquad (3)$$

$A_c \in \mathbb{R}^{n_c \times n_c}$, $B_c \in \mathbb{R}^{n_c \times p}$, $C_c \in \mathbb{R}^{m \times n_c}$ and $D_c \in \mathbb{R}^{m \times p}$ (n_c will result from a minimal realization of the transfer matrix in (5)). Systems (1) and (3) are connected as follows

$$\begin{aligned} u &= y_c \\ u_c &= \rho y \end{aligned}$$

in which ρ is a scalar gain to be tuned between zero to some positive value.

In this closed loop state space form, A_c, B_c, C_c and D_c are the unknowns that must be chosen in such a way that, when ρ is tuned, the motion of some eigenvalues of A is as desired (see Figure 1).

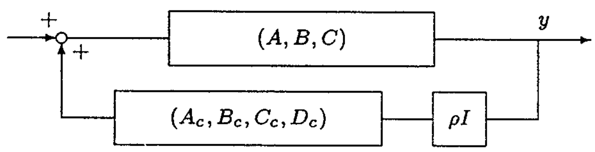

Fig. 1. Generalized phase control scheme, ρ is tuned from zero to some positive value.

2.2 Main result

Theorem 2.1. The motion of the eigenvalue λ of A when ρ is tuned in (1) is given at the first order by

$$\Delta\lambda = \rho\, uBG_c(\lambda)Cv \qquad (4)$$

where $G_c(s)$ is defined by

$$G_c(s) = C_c(sI - A_c)^{-1}B_c + D_c \qquad (5)$$

Proof.[(6)]

3. Constraints and criterion

The transfer matrix of the controller is considered under the following form :

$$G_c(s) = \begin{bmatrix} \dfrac{b_{011} + \ldots + b_{q11}s^q}{a_{011} + \ldots + a_{q11}s^q} & \cdots & \dfrac{b_{01p} + \ldots + b_{q1p}s^q}{a_{01p} + \ldots + a_{q1p}s^q} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \dfrac{b_{0m1} + \ldots + b_{qm1}s^q}{a_{0m1} + \ldots + a_{qm1}s^q} & \cdots & \dfrac{b_{0mp} + \ldots + b_{qmp}s^q}{a_{0mp} + \ldots + a_{qmp}s^q} \end{bmatrix}$$

In order to obtain linear inequality constraints, the denominator coefficients are chosen *a priori*.[(6)] This assumption is not very restrictive because if we take into account roll-off requirements and the bandwidth in which control effect is expected there does not remain so much freedom. The free design parameters are the numerator coefficients denoted b_{ijk}.

Constraints to reassign the direction of certain modes

We use result (4). $\Re$ and $\Im$ denote respectively the real and imaginary part. Let Δ_i be the matrix of parameters that characterizes the model considered for controlling the motion of λ_i, u_i and v_i are left and right eigenvectors of the matrix A_{Δ_i} corresponding to the eigenvalue λ_i. From Theorem 2.1, the inequalities that are to be solved for $G_c(\lambda_i)$ are as follows.

- To move λ_i of Model (i) to the left:

$$\Re(u_i B_{\Delta_i} G_c(\lambda_i)\ C_{\Delta_i} v_i) \le \delta_R \qquad (6)$$

- To control the vertical motion of λ_i:

$$\delta_{I1} \le \Im(u_i B_{\Delta_i}\ G_c(\lambda_i)\ C_{\Delta_i} v_i) \le \delta_{I2} \qquad (7)$$

- For variations bounded inside a sector.

$$\begin{aligned} &\delta_{I1}\Re(u_i B_{\Delta_i}\ G_c(\lambda_i)\ C_{\Delta_i} v_i) \le \\ &\Im(u_i B_{\Delta_i}\ G_c(\lambda_i)\ C_{\Delta_i} v_i) \le \\ &\delta_{I2}\Re(u_i B_{\Delta_i}\ G_c(\lambda_i)\ C_{\Delta_i} v_i) \end{aligned} \qquad (8)$$

Usually, it is necessary to combine the first inequality with equations of the second or the third kind, for example $\delta_{I1} = \delta_{I2} = 0$:

- It is also possible to fix simultaneously the real and imaginary motion

$$u_i B_{\Delta_i}\ G_c(\lambda_i)\ C_{\Delta_i} v_i = \delta_R + j\delta_I \qquad (9)$$

Constraints for controller structure.

The structuring of the dynamic controller allows for the fixing of different dynamics and roll-off on each of the inputs and outputs. Several kinds of constraints that are often encountered,(7) for example

- entries set to zero.
- scalar entries. In this case inequality constraints relative to the sign or to the magnitude might be of interest.
- degree of the numerator.
- entry proportional to some filter.

Quadratic criteria.

To define efficient design procedure it is necessary to minimize a criterion. Here we shall only consider criteria of the form

$$J = \sum_{l} \text{trace } (G_c(j\omega_l)G_c(j\omega_l)^*) \qquad (10)$$

This criterion is quadratic in the unknowns. The most natural criterion concerns the static gain of the transfer matrix $G_c(s)$. Similar criteria can be considered, for example to impose roll off at a frequency ω_H.

Resolution.

Finally, the problem to solve turns out to be a problem of *quadratic optimization under linear constraints.*

4. Application to a large flexible aircraft

In this example, we consider the lateral dynamics of a large flexible aircraft. The model includes the standard rigid-body states (angle of side slip, roll rate, yaw rate, roll angle), the actuators states, and the states for modelling of aeroelastic modes. The model's order is seventy one. The inputs are the aileron and rudder deflections. The four rigid-body states are measured. We have to consider six models corresponding to six cases of mass. In Figure 2, the pole maps of the six considered models and in Figure 3 the corresponding step responses in open loop are depicted.

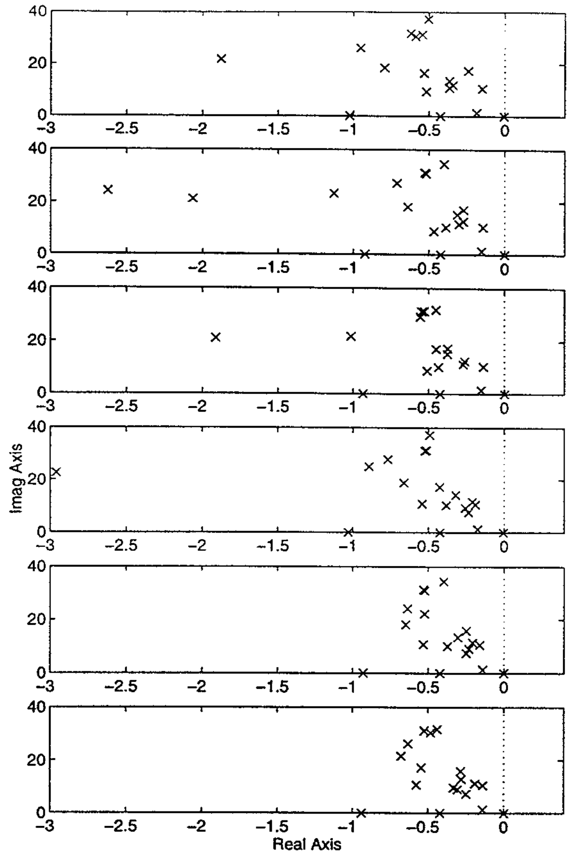

Fig. 2. Pole maps for six models in open loop.

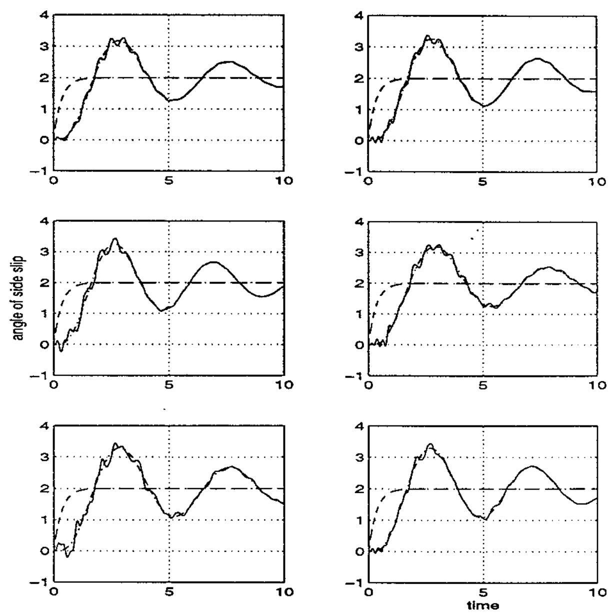

Fig. 3. Step responses in open loop for six models.

4.1 Design procedure

Initialization

First we have to assign the eigenstructure of the rigid body subsystem. In order to limit the effect of the rigid body feedback to the flexible modes, we use a filter (see Figure 4).

The n-dimensional state space of the open loop system will be denoted $\mathcal{X}$ and the n_f-dimensional filter extension for feedback will be denoted $\mathcal{X}_f$. The right eigenvectors of the system connected to the dynamic feedback belonging to $\mathcal{X} \oplus \mathcal{X}_f$ are denoted (v_i, v_{fi}). The state space is extended ($\mathcal{X} \to \mathcal{X} \oplus \mathcal{X}_f$).

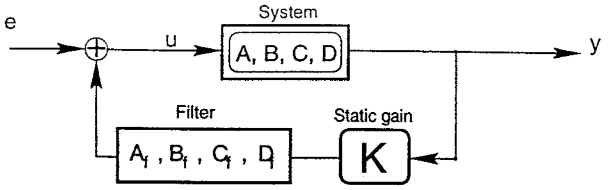

Fig. 4. Output filter of "rigid body controller".

The design of the rigid body control loop take into account the filter. If $D_f = 0$, the closed loop matrix is as follow :

$$\begin{bmatrix} A & BC_f \\ B_f KC & A_f + B_f KDC_f \end{bmatrix} \tag{11}$$

If we apply the standard design procedure of eigenstructure assignment when K is a constant matrix, we obtain the following result :

Proposition 1. Consider $\lambda_i \in \mathbb{C}$ and $[v_i, v_{fi}] \in \mathcal{X} \oplus \mathcal{X}_f$, which satisfies for some vector $w_i \in \mathbb{C}^m$:

$$\begin{bmatrix} A - \lambda_i I & BC_f & 0 \\ 0 & A_f - \lambda_i I & B_f \end{bmatrix} \begin{bmatrix} v_i \\ v_{fi} \\ w_i \end{bmatrix} = 0$$

The vector $[v_i, v_{fi}]$ is assigned by the static gain K if and only if

$$K\ (Cv_i + DC_f v_{fi}) = w_i \tag{12}$$

The "rigid" control loop is computed which permits us to assign the eigenstructure of all models. So, for each model k, we have to solve equation (12). In order to increase the robustness, we shall select the eigenvectors to be assigned by using orthogonal projection.[8] This projection is realized for each model k.
Finally we have to solve :

$$K = \left[W_1 \dots W_k \right] \left[\overline{V_1} \dots \overline{V_k} \right]^{-1} \tag{13}$$

With :

$$\overline{V_k} = C_k\, V_k + D_k\, C_f\, V_{fk}$$

For this application, the filter order is 2 for each input. After closing the first loop, the rigid body states are correctly assigned, for the six models, but this controller, in spite of the filter, has a destabilizing effect on some flexible modes (see Figure 5).

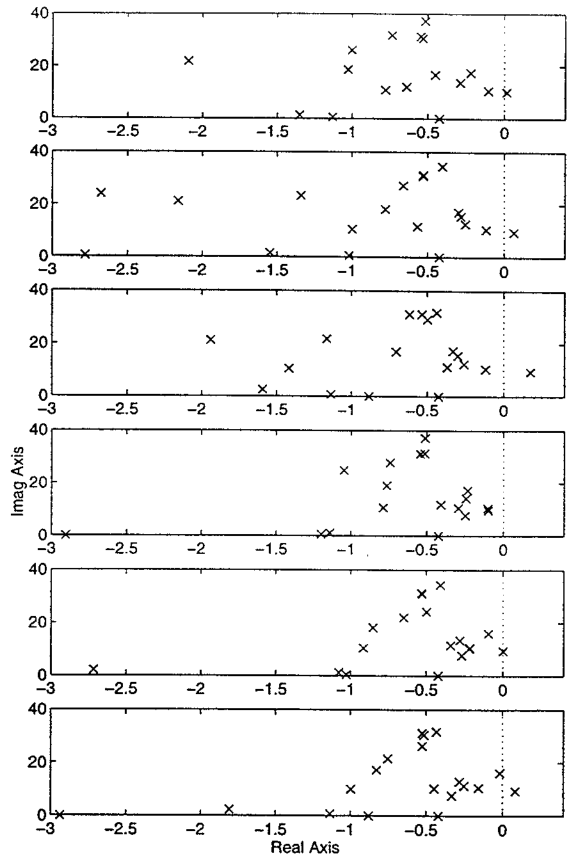

Fig. 5. Poles map for six models after closing the first loop.

Active flexible control design

From now on, models considered will include the "rigid" control loop. So, we can point out which flexible modes have to be shifted back by the second loop, and where.

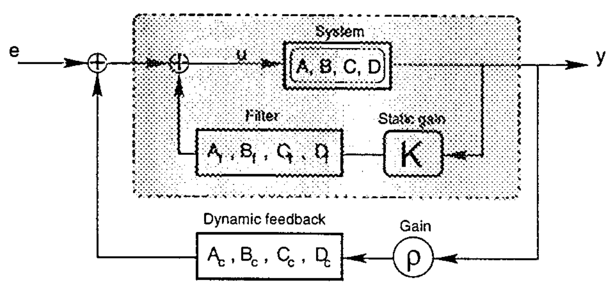

Fig. 6. Active flexible control design

We must assign the motions of the poles which moved in Figure 5 in order to push them back to their open loop location. For that, we define several inequalities of the form (6), (7), (8) or (9). These constraints are considered for several flexible modes corresponding diverse models. For each model we considered the flexible mode that is the most destabilized (see Figure 7).

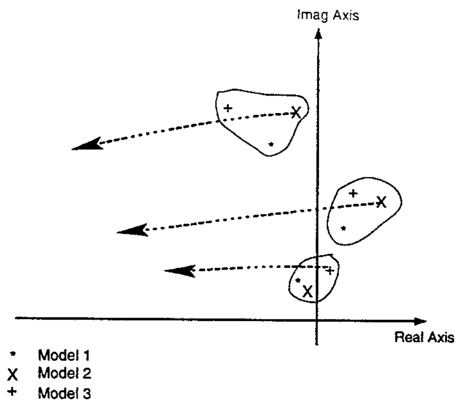

Fig. 7. Choice of flexible modes to deal with among several models

Furthermore, we do not want to move the rigid-body poles. Depending on the choice of the denominator of the feed-forward transfer, the second loop has or not a negligible effect on the low frequency design. In other words, the minimization of the static gain as in (10) is not sufficient. To prevent this effect, it suffices to fix to zero real and imaginary parts of low frequency poles.

For our application, ten modes are to be dealt with, five rigid modes and five flexible modes. The constraints are as follows :

$$
\begin{array}{l}
\Re(u_1 B_{\Delta_1} G_c(\lambda_1)\ C_{\Delta_1} v_1) \leq 0 \\
\Re(u_2 B_{\Delta_1} G_c(\lambda_2)\ C_{\Delta_1} v_2) \leq -0.1 \\
\Re(u_3 B_{\Delta_2} G_c(\lambda_3)\ C_{\Delta_2} v_3) = -0.1 \\
\Im(u_3 B_{\Delta_2}\ G_c(\lambda_3)\ C_{\Delta_2} v_3) = 0 \\
\Re(u_4 B_{\Delta_3} G_c(\lambda_4)\ C_{\Delta_3} v_4) \leq -0.1 \\
\Re(u_5 B_{\Delta_3} G_c(\lambda_5)\ C_{\Delta_3} v_5) \leq -0.5 \\
\Re(u_6 B_{\Delta_4} G_c(\lambda_6)\ C_{\Delta_4} v_6) \leq -0.1 \\
\Re(u_7 B_{\Delta_5} G_c(\lambda_7)\ C_{\Delta_5} v_7) \leq 0 \\
\Re(u_8 B_{\Delta_6} G_c(\lambda_8)\ C_{\Delta_6} v_8) = 0 \\
\Re(u_9 B_{\Delta_6} G_c(\lambda_9)\ C_{\Delta_6} v_9) \leq 0 \\
\Re(u_{10} B_{\Delta_6} G_c(\lambda_{10})\ C_{\Delta_6} v_{10}) = 0
\end{array}
\qquad (14)
$$

It remains to choose the denominator of $G_c(s)$. Its order is set to 2, sufficiently high to have enough degrees of freedom to solve the system of equations relative to the eigenstructure assignment. The poles of the denominator of the transfer matrix of the controller are chosen in the neighborhood of the frequency filter of the first loop.The loop gain ρ should be about 1 to recover open loop flexible dynamics. Finally the order of the state space realization of the proposed controller is 2, including the first loop.

The results are analyzed by means of the root locus given in Figure 8. It appears that rigid modes are unchanged (as expected). The flexible modes have been shifted to the left, almost at this open loop location, without undesirable effects on other poles. From Figure 9 derives that the contribution at the flexible modes is not more important than in the open loop (refer to Figure 3).

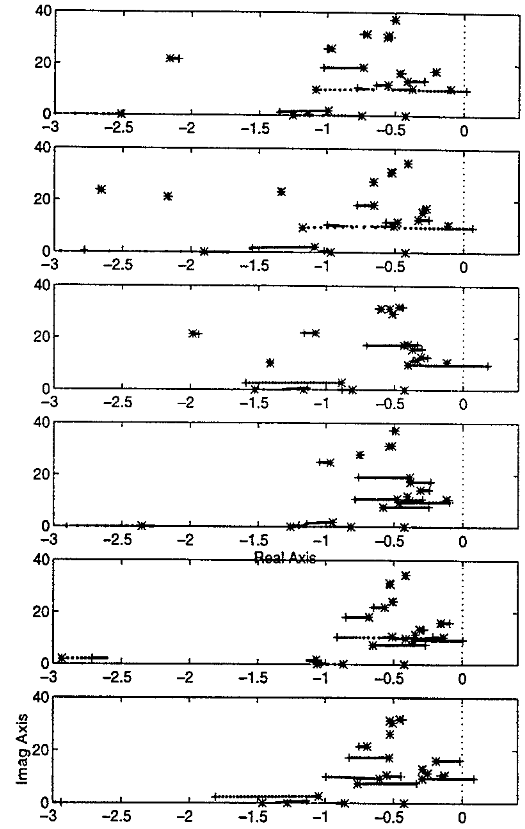

Fig. 8. Root locus for six models. "+"denotes "open loop" poles after closing the first loop and "*" denotes the poles after closing the second loop.

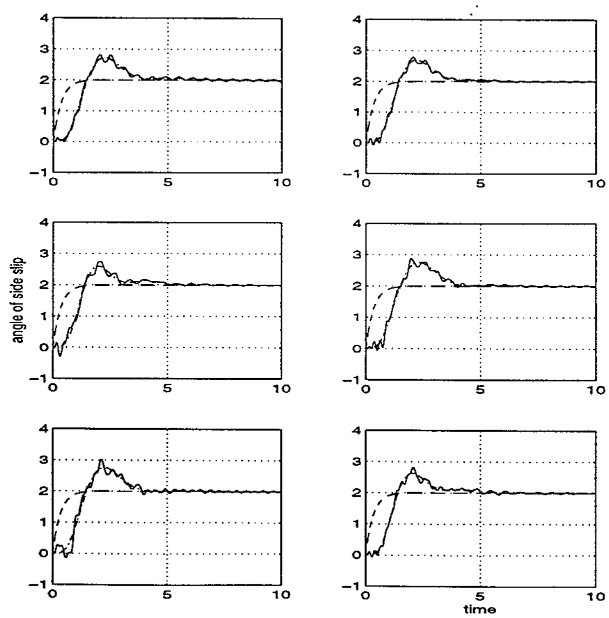

Fig. 9. Step responses in closed loop for six models.

5. Conclusion

After the presentation of the theoretical background the interactive design procedure is detailed onto a practical auto-pilot design of a flexible aircraft. The technique used allows to assign the eigenstructure of the rigid body subsystem without destabilize the flexible modes. The multi-model approach provides high robustness.

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