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Design of a Flight Control System for a Highly Flexible Aircraft using Convex Synthesis

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Abstract

A simple method is proposed for the synthesis of robust dynamic feedback or feedforward controllers, which satisfy classical time and frequency domains specifications. The only assumption on the plant is to be linear time invariant. The controller is obtained as the solution of a Linear (or Quadratic) Programming problem when only time domain specifications are fixed.(1) The optimization problem becomes a LMI problem when frequency domain specifications are introduced. When synthesizing a feedforward controller, a multimodel synthesis is moreover possible.
The method is then applied to the synthesis of a lateral flight control system for a highly flexible transport aircraft. Trade-offs between various specifications are explored.

1. Introduction

The design of flight control systems for future large body transport aircrafts is a very attractive challenge and a quite new problem. For a classical aircraft, frequencies of the first structural modes are high enough, so that a filtering is sufficient to eliminate interactions between aeroelastic dynamics and rigid control laws. However, for a bigger aircraft, the frequencies of the first bending modes become too close to the control bandwith, so that it becomes necessary to take simultaneously into account flight mechanics and structural dynamics in the control law design.

The convex synthesis technique(2,3) enables to fully take into account time-domain specifications, unlike classical approaches such as LQ, H_2 and H_∞ synthesis. More generally, time and frequency-domain specifications can be directly accounted for in the design either as convex constraints or as convex criteria, which are to be optimized. As a consequence, this technique allows the necessary trade-offs between the various design specifications to be studied in a systematic way.

The basic idea of the method is to remark that many performance specifications as well as some robustness specifications, which are not convex in the space of controllers, can be nevertheless expressed as convex objectives in the space of the achievable closed-loop transfer matrices. The theoretical basis of the method is the parameterization of stabilizing controllers, or more importantly the so-called Q-parameterization of the closed-loop input/output maps achievable with controllers which stabilize the system.(4)

Boyd *et al.*(2) showed that with such an approach, the control design problem becomes a *convex programming problem.* We present in this paper an approach which enables to obtain Linear or Quadratic Programming (LP or QP) problems as approximations of the initial convex programming problem, when time-domain or SISO frequency-domain specifications are fixed. When MIMO frequency-domain objectives are also considered, the initial convex programming problem becomes now a LMI problem.

The method is then applied to the design of a lateral flight control system for a highly flexible transport aircraft. We focus on time-domain responses to typical pilot requirements and also introduce frequency domain specifications, which are used to improve the robustness properties of the controller.

The paper is organized as follows: The basis of convex synthesis is presented in section 2. The design procedures are then detailed in section 3. The flight control system is synthesized in § 4. Concluding remarks end the paper.

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2. Basis of the method

2.1 An affine representation of the closed loop

Consider the augmented plant P of figure 1 where u and y are respectively the control input and sensed output vectors. w and z respectively correspond to the exogenous input vector and to the regulated output vector, on which the specifications are expressed. The transfers H and K respectively denote the dynamic feedforward and feedback controllers.

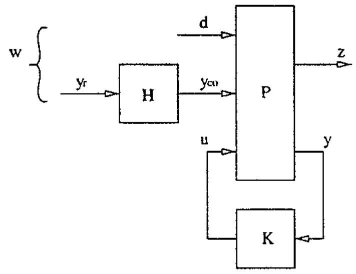

Fig. 1. Augmented plant.

Results 1 and 2 propose an affine representation of the closed loop response as a function of the control design parameters. For the sake of simplicity, the method is presented for the case of scalar inputs and outputs. It is nevertheless straightforward to extend the method to the case of input and output vectors.

- Feedforward design:

Result *1.* The feedback controller K of figure 1 is fixed. The feedforward controller H is parametrized as:

$$H = \sum_{i=1}^{n} \theta_i \, H_i \tag{1}$$

where the transfers H_i are a priori fixed and the scalars θ_i correspond to the design parameters.

A fixed reference input signal $y_r(t)$ is applied to the closed loop. Let $z^i(t)$ the corresponding closed loop response (on output z), which is obtained with the feedforward controller $H = H_i$. Using then the feedforward controller of equation (1), the closed loop response to the signal $y_r(t)$ can be written as :

$$z(t) = \sum_{i=1}^{n} \theta_i \, z^i(t) \tag{2}$$

In the same way, let $G_{bf}(s)$ the closed-loop transfer between y_{co} and z (the feedback K is here again fixed). The frequency response between y_r and z can then be rewritten as:

$$F(e^{jwDT}) = \sum_{i=1}^{n} \theta_i \, G_{bf}(e^{jwDT}).H_i(e^{jwDT}) \tag{3}$$

where DT is the sampling period.

Remarks 1 :
(i) A simple solution is to parametrize the feedforward controller as a Finite Impulse Response (FIR) filter in the above result. Other parametrizations are nevertheless possible: a great deal of work has been devoted to the problem of building orthonormal bases of filters, which generalize the classical Laguerre or Kautz bases[6–8] .
(ii) If the reference input signal is known in advance in the real time application, a non causal feedforward controller H can then be handled in equation (1) (Predictive control).

Due to the affinity of equations (2) and (3), classical time- and frequency-domain specifications upon the transfer between w and z can be expressed as convex constraints or optimization criteria with respect to the design parameters θ_i.[1]

- Feedback design:

Consider now the augmented plant of figure 1 with $H = 1$ and partition the transfer matrix P as :

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

where P_{11} (resp. P_{22}) is the transfer between w (resp. u) and z (resp. y).
The Linear Fractionnal Transformation (LFT) corresponding to the closed-loop transfer matrix between w and z is given by:

$$F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \tag{4}$$

Using the Q-parametrization of stabilizing controllers,[9,10] this closed-loop transfer $F_l(P, K)$ can be parametrized as:

$$F_l(P, K_Q) = T_1 + T_2 Q T_3 \tag{5}$$

where T_1, T_2 and T_3 are stable transfer matrices depending on the plant.

As a matter of fact, many design specifications, which are not convex in the space of controllers K, appear to be convex in the space of closed loop transfer matrices Q,[2] because of the affinity of equation (5) with respect to Q. As a consequence, the idea of the method is first to synthesize a transfer matrix Q which satisfies the design specifications. The corresponding controller K_Q

is then easily obtained using coprime factorization or direct identification.[3, 10]

The Q-parameterization of the stabilizing controllers can be easily obtained using the so-called *modified controller paradigm*.[2] This derivation of the Q-parameterization starts with any controller K_{nom} which stabilizes the plant (see figure 2). The key of this parameterization is that the closed loop map from v to e is *zero* on figure 2, so that Q sees no feedback and can not consequently destabilize the plant.

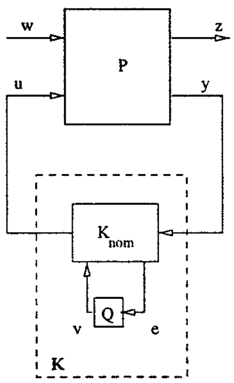

Fig. 2. Q-parameterization as a modification of a nominal controller.

Doyle[11] has especially given a nice interpretation of this Q-parameterization when the nominal controller K_{nom} is an estimated state feedback. Figure 3 shows how the free transfer matrix Q is connected : $e = \hat{y} - y$ corresponds to the output prediction error and v is an auxiliary input signal, which is added to the actuator input signal. The requirement that the closed-loop transfer matrix from v to e be zero is satisfied, because the observer error $x - \hat{x}$, is uncontrollable from v, and the transfer matrix from v to e is thus zero.

In the case of an aircraft, K_{nom} may be chosen to obtain satisfactory closed loop poles and decoupling properties.

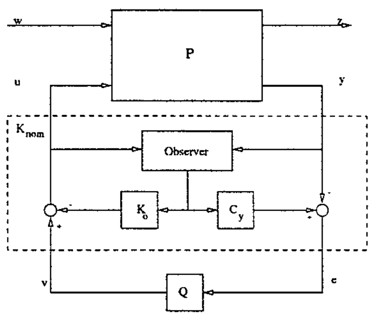

Fig. 3. Q-parametrization in the case of an estimated state feedback nominal controller. K_o is a state feedback gain and $y = C_y x$ where x is the plant state.

Result *2.* The feedforward controller H of figure 1 is now fixed and set equal to 1 for the sake of simplicity. The closed loop transfer $F_l(P, K)$ between the input vector w and output z is parametrized as :

$$F_l(P, K) = T_1 + T_2 Q T_3 \tag{6}$$

Let Q an affine combination of the scalar parameters θ_i:

$$Q = \sum_{i=1}^{n} \theta_i \, Q_i \tag{7}$$

where the transfers Q_i are a priori fixed. A given input signal $w(t)$ is applied to the closed loop. Let :

$$\begin{aligned} z^0(t) &= T_1 \, w(t) \\ z^i(t) &= T_2 Q_i T_3 \, w(t) \end{aligned} \tag{8}$$

With reference to equations (5-8), it is then possible to write the closed loop response to the input signal $w(t)$ as :

$$z(t) = z^0(t) + \sum_{i=1}^{n} \theta_i \, z^i(t) \tag{9}$$

In the same way, it is possible to write the closed-loop frequency response between w and z as:

$$[F_l(P, K)](e^{jwDT}) = F_{l_o}(e^{jwDT}) + \sum_{i=1}^{n} F_{l_i}(e^{jwDT})\theta_i \tag{10}$$

where $F_{l_o} = T_1$ and $F_{l_i} = T_2 Q_i T_3$.

Remarks 2 :

(*i*) Here again, the choice of the fixed filters Q_i (see equation (7)) is the main issue. A first solution is to choose a large base of filters, so as to have a sufficient amount of degrees of freedom (see e.g.[2]). The order of the resulting controller K will be however very large with this method. An alternative method is exposed in section 4.

(*iii*) It is also possible to simultaneously synthesize the feedback and feedforward controllers, so as to obtain a TDF (Two Degrees of Freedom) control law : see section 4.

2.2 Obtaining LP and QP problems

In results 1 and 2, the time-domain response of the closed loop was obtained as an affine combination of the design parameters θ_i, namely $z(t) = z^0(t) + \phi^T(t)\theta$, where $z^0(t)$ and $\phi(t)$ were a priori computed, and θ is the vector of parameters θ_i. In a vector form, let $Z = [z(1), \ldots, z(N)]^T$ the values of the output signal $z(t)$ at $t = 1, \ldots, N$. Z can be written as $Z = Z^0 + \Phi^T\theta$, where the vector Z^0 and the matrix Φ are a priori known.

Likewise, the closed-loop frequency response was obtained as an affine combination of the design parameters θ_i:

$$[F_l(P,K)](e^{jwDT}) = F_{l_o}(e^{jwDT}) + \sum_{i=1}^{n} F_{l_i}(e^{jwDT})\theta_i \quad (11)$$

Using the above affine representations, we would like to translate l_2 and l_∞ classical time-domain objectives as well as some frequency-domain objectives into, either a Linear Programming (LP) problem or, more generally, into a Quadratic Programming (QP) problem:[1]

$$\min_{A\theta\leq b} c^T\theta \qquad \text{or} \qquad \min_{A\theta\leq b}(c^T\theta + \theta^T R\theta)$$

The matrix R must be positive definite, so as to obtain a convex optimization problem.

- Time-domain objectives:

• l_∞ *constraints* : these can be generically written as $z(t) \leq \overline{z}(t)$ or $z(t) \geq \underline{z}(t)$ for $t \in [N_1,\ N_2]$. Consider as an example the case of a unit step on the reference input. A l_∞ constraint may typically correspond to a specification on the maximal allowable overshoot D (i.e. $z(t) \leq (1+D)$ for all $t \geq 0$) or the rising time (e.g. $0.95 \leq z(t) \leq 1.05$ for all $t \geq N$).

• l_∞ *objectives* : consider the problem of minimizing the value of α satisfying for all $t \in [N_1,\ N_2]$:

$$-\alpha \leq z(t) - z_r(t) \leq \alpha \quad (12)$$

where $z_r(t)$ is typically a reference trajectory. This problem can be expressed as an augmented LP problem. Let :

$$\begin{aligned} Z &= [z(N_1),\dots,z(N_2)]^T = Z^0 + \Phi^T\theta \\ Z_r &= [z_r(N_1),\dots,z_r(N_2)]^T \end{aligned} \quad (13)$$

Define then :

$$\begin{aligned} x &= \begin{bmatrix} \theta \\ \alpha \end{bmatrix} \\ c &= [0,\dots,0,1]^T \\ A &= \begin{bmatrix} \Phi^T & -1_m \\ -\Phi^T & -1_m \end{bmatrix} \\ b &= \begin{bmatrix} Z_r - Z_0 \\ Z_0 - Z_r \end{bmatrix} \end{aligned} \quad (14)$$

1_m denotes the unit column vector, of size $m = N_2 - N_1 + 1$. The minimization of α in equation (12) is equivalent to the minimization of $c^T x$, under the constraint $Ax \leq b$.

• l_2 *objectives* : these correspond to the minimization of a criterion of the form $c^T\theta + \theta^T R\theta$. Consider e.g. that the quadratic error between the response z and the reference trajectory z_r is to be minimized, namely:

$$J = \sum_{i=N_1}^{N_2} (z(i) - z_r(i))^2 \quad (15)$$

Note that :

$$(z(t) - z_r(t))^2 = (\phi^T(t)\theta + z^0(t) - z_r(t))^2 \quad (16)$$

so that J can be rewritten as :

$$J = \tilde{Z}^T\tilde{Z} + 2\tilde{Z}^T\Phi^T\theta + \theta^T\Phi\Phi^T\theta \quad (17)$$

where $\tilde{Z} = [z^0(N_1) - z_r(N_1),\dots,z^0(N_2) - z_r(N_2)]^T$.

Remark 3 :
Strictly speaking, a parametric model of the closed-loop system is not required for these time domain specifications. It suffices to be able to measure the closed-loop response of the plant to a given input signal.

- Frequency-domain objectives:

In the frequency domain, it is possible to translate SISO H_2 or H_∞ specifications into a LP or QP problem.

• H_∞ *constraints* : these can be generally written as

$$|[F_l(P,K)](e^{jw_kDT})| \leq I(w_k) \ \text{ for } \ w_k \in [w_1,\dots,w_N] \quad (18)$$

where $I(w_k)$ is the template which is to be satisfied. Note that the above constraint is convex, but not affine with respect to the parameters θ_i.

Let $x(w_k)$ and $y(w_k)$ the real and imaginary part of $[F_l(P,K)](e^{jw_kDT})$, and note that the real functions $x(w_k)$ and $y(w_k)$ are affine with respect to the parameters θ_i. Consider then the following inequality:

$$|[F_l(P,K)](e^{jw_kDT})| = \sqrt{x^2(w_k) + y^2(w_k)} \leq |x(w_k)| + |y(w_k)| \quad (19)$$

and remark that:

$$\begin{aligned} &|x(w_k)| + |y(w_k)| \leq I(w_k), \forall w_k \in [w_1,\dots,w_N] \\ \iff &\begin{cases} -x(w_k) + y(w_k) \leq I(w_k) \\ -x(w_k) - y(w_k) \leq I(w_k) \\ x(w_k) - y(w_k) \leq I(w_k) \\ x(w_k) + y(w_k) \leq I(w_k) \end{cases} \forall w_k \in [w_1,\dots,w_N] \end{aligned} \quad (20)$$

The initial constraints (18) can thus be transformed into linear constraints on the parameters θ_i. Due to the inequality (19), this approach nevertheless introduces some conservatism.

• H_2 *objectives* : these correspond to the minimization of a criterion of the form $c^T\theta+\theta^T R\theta$. Consider e.g. that the quadratic error between the frequency response $[F_l(P,K)](e^{jw_iDT})$ and a reference frequency shape $F_r(e^{jw_iDT})$ is to be minimized, namely:

$$J=\sum_{i=1}^{N}\left|[F_l(P,K)](e^{jw_iDT})-F_r(e^{jw_iDT})\right|^2$$

With respect to equation (10), let $F_{l_{o_{Re}}}$ and $F_{l_{o_{Im}}}$ the real and imaginary parts of F_{l_o}. In the same way, $F_{l_{i_{Re}}}$ and $F_{l_{i_{Im}}}$ respectively denote the real and imaginary parts of F_{l_i}. J can then be rewritten as:

$$\begin{aligned}J=F_{l_{Re}}^T F_{l_{Re}}+F_{l_{Im}}^T F_{l_{Im}}+2\left[F_{l_{Re}}^T\Phi_{Re}^T+F_{l_{Im}}^T\Phi_{Im}^T\right]\theta\\ +\theta^T\left[\Phi_{Re}\Phi_{Re}^T+\Phi_{Im}\Phi_{Im}^T\right]\theta\end{aligned} \tag{21}$$

where:

$$F_{l_{Re}}^T=\begin{bmatrix}[F_{l_{o_{Re}}}-F_{r_{Re}}](w_1)\\ \vdots\\ \vdots\\ \left[F_{l_{o_{Re}}}-F_{r_{Re}}\right](w_N)\end{bmatrix}$$

and:

$$\Phi_{Re}^T=\begin{bmatrix}F_{l_{1_{Re}}}(w_1) & \dots & \dots & F_{l_{m_{Re}}}(w_1)\\ F_{l_{1_{Re}}}(w_2) & \vdots & \vdots & F_{l_{m_{Re}}}(w_2)\\ \vdots & \vdots & \vdots & \vdots\\ F_{l_{1_{Re}}}(w_N) & \dots & \dots & F_{l_{m_{Re}}}(w_N)\end{bmatrix}$$

and $F_{l_{Im}}$ and Φ_{Im} have similar expressions.

2.3 Obtaining LMI problems

The above approach for taking into account H_∞ constraints is a priori conservative. It is more impossible to consider MIMO frequency-domain specifications with this method, so that we now use LMIs(12) to take into account in a non conservative way MIMO frequency-domain specifications, which may typically correspond to robustness requirements on the closed-loop. Before presenting the LMI formulation of our problem, the following lemma is needed.

Lemma 2.1. : Schur Complement
Let M $\in\mathbb{R}^{n\times m}$, R and S two symmetric matrices of dimension $n\times n$ and $m\times m$ respectively, then

$$\begin{pmatrix}R & M\\ M^T & S\end{pmatrix}>0\iff\begin{cases}R>0\\ S-M^TR^{-1}M>0\end{cases} \tag{22}$$

$$\iff\begin{cases}S>0\\ R-MS^{-1}M^T>0\end{cases}$$

Using this lemma, the following well known result is obtained:

Result *3.* Let M $\in\mathbb{R}^{n\times m}$

$$\overline{\sigma}(M)\equiv\sqrt{\lambda_{max}(M^TM)}<\alpha\iff\begin{pmatrix}\alpha I & M\\ M^T & \alpha I\end{pmatrix}>0 \tag{23}$$

Furthermore, let G be a complex matrix $\in\mathbb{C}^{m\times m}$, the following result holds:

$$G>0\iff\begin{pmatrix}Re(G) & Im(G)\\ -Im(G) & Re(G)\end{pmatrix}>0 \tag{24}$$

Consider now a classical H_∞ MIMO specification of the form:

$$\overline{\sigma}([F_l(P,K)](e^{jw_kDT}))\le I(e^{jw_kDT})\ \text{ for }\ w_k\in[w_1,\dots,w_N] \tag{25}$$

Using equations (23) and (24) and the parameterization of the closed-loop transfer $[F_l(P,K)]$ of equation (10), the constraint (25) can be translated into the following LMI problem:

$$\begin{vmatrix}\alpha I & Re(M) & 0 & Im(M)\\ Re(M^*) & \alpha I & Im(M^*) & 0\\ 0 & -Im(M^*) & \alpha I & Re(M)\\ -Im(M) & 0 & Re(M^*)) & \alpha I\end{vmatrix}>0 \tag{26}$$

where the complex matrix $M=[F_l(P,K)](e^{jw_kDT})$ is an affine function of the parameters θ_i.

3. Design procedures

On the basis of the above results, we can now summarize the design procedures of dynamic feedforward and feedback controllers as follows:

Synthesis of a dynamic feedforward controller:

- *Step 1* : choose an affine parametrization of the feedforward controller :

$$H=\sum_{i=1}^{n}\theta_iH_i \tag{27}$$

- *Step 2* : let $H=H_i$. Compute the closed loop response $w^i(t)$ to the reference input signal $y_r(t)$.
- *Step 3* : when using the feedforward controller of equation (27), the closed loop response $z(t)$ is obtained as:

$$z(t)=\sum_{i=1}^{n}\theta_iz^i(t) \tag{28}$$

and the closed-loop frequency response is given by:

$$F(e^{jwDT}) = \sum_{i=1}^{n} \theta_i \, G_{bf}(e^{jwDT}) \, H_i(e^{jwDT}) \quad (29)$$

Using the results of the previous section, it is then possible to translate the time and frequency domain specifications into a LP, QP or LMI problem.
- *Step 4* : the θ_i's are directly obtained as the solution of the optimization problem, which is to be solved.

The procedure for synthesizing a dynamic feedback controller essentially follows the one for synthesizing a dynamic feedforward controller.

Synthesis of a dynamic feedback controller :

- *Step 1* : choose an affine parametrization of Q :

$$Q = \sum_{i=1}^{n} \theta_i Q_i \quad (30)$$

The closed loop system is then obtained as $T_1 + T_2QT_3$.
- *Step 2* : Let a given input signal $w(t)$. With reference to equation (8), compute the closed loop responses $z^0(t)$ and $z^i(t)$ for i=1, … ,n.
- *Step 3* : when using the affine parametrization of Q (see equation (30)), the closed loop response $z(t)$ is obtained as:

$$z(t) = z^0(t) + \sum_{i=1}^{n} \theta_i z^i(t) \quad (31)$$

and the closed-loop frequency response is given by:

$$[F_l(P,K)](e^{jwDT}) = F_{l_o}(e^{jwDT}) + \sum_{i=1}^{n} F_{l_i}(e^{jwDT})\theta_i \quad (32)$$

Here again, use the results of the previous section to translate the time and frequency domains specifications into a LP, QP or LMI problem.
- *Step 4* : the θ_i's are first obtained as the solution of the optimization problem, which is to be solved. The corresponding transfer matrix Q is computed. See[3,11] for the obtention of a state-space model of K from a state-space model of Q.

Remarks 4:
(*i*) Results 1 and 2 and the above design procedures can be easily extended to the continuous-time case : as done in section 4, it suffices indeed to handle the values of the signals at each point of a time- and frequency-domain griddings.
(*ii*) Standard routines are available in MATLAB for solving the LP or QP problems. Note however that more efficient algorithms may be used if the size of the problem (i.e. the dimension of matrix A in the constraint $A\theta \leq b$) is very large : see e.g.[13] and included references. This may be especially the case when considering MIMO systems with a small sampling period.
(*iii*) The LMI problems are solved using the LMI Control Toolbox of MATLAB.[14]

4. Application: synthesis of a flight control system

Our complete aircraft lateral model is a high order ($\sim$ 60) linear time invariant transfer matrix. It especially contains the three lateral rigid modes (spiral, roll and dutch roll) and 15 structural modes. The measured outputs correspond to the lateral acceleration n_y, the roll rate p, yaw rate r and roll angle ϕ at different locations on the body. Given a sensor selection, the first step of the design is to build a standard augmented plant and to synthetize an initial controller (an estimated state feedback for example).
A static transfer matrix Q is first used, so that the order of the controller is equal to the order of the augmented plant. A dynamic transfer matrix Q is then used to have more degrees of freedom for tuning the controller. Note that the dimensions of Q are the same as the dimensions of K.

We first emphasize that the convex synthesis techique can largely improve the performance of the initial controller. We show then that the convex synthesis techique enables us to study in a systematic way various trade-offs between the specifications of our problem.

4.1 Aircraft model and performance specifications

A lateral model of a flexible transport aircraft is considered. Two lateral control surfaces, namely one aileron δ_p and one rudder δ_r, are used to pilot this aircraft. The outputs, which will be used by the feedback controller, are chosen as the lateral acceleration n_y, the roll and yaw rates p and r and the roll angle ϕ, measured at the center of gravity of the aircraft.

The synthesis model is a linear time invariant transfer matrix of order 16 (a fourth aerodynamical model, a first order actuator on the rudder and the aileron and 5 structural modes).

When building the augmented plant of figure 1, additional outputs are considered in the design. These regulated outputs, which are not used by the control law, are introduced to express the design specifications.

Performance specifications The inputs and outputs of the augmented plant of figure 1 are defined as follows :

• The exogeneous input vector w contains the reference inputs β_c and Φ_c on the sideslip and roll angles β and Φ. Because of the small gain constraint exposed in the following, it also contains perturbation inputs on the measured outputs n_y, p, r and ϕ.

• The control input vector u contains the actuator inputs δ_p, δ_r.

• The regulated output vector z contains :
- the 2 actuator outputs.
- the 2 actuator rate outputs.
- the sideslip and roll angles β and Φ and the derivative $\dot{\phi}$ of ϕ with respect to time.
- the 2 actuator inputs (which are used to introduce the small gain constraint).

• The measured output vector y finally contains the lateral acceleration n_y, the roll and yaw rate p and r and the roll angle ϕ.

We now present the main design specifications. As a first point, we consider the step response of a first order filter, which represents our first reference input signal. This filtered step is applied to the reference sideslip angle β_c. The specifications are :

- The sideslip angle β must track the reference input signal with a rise time of less than 6 seconds * , an overshoot of less than 20 % and a zero static error.
- the roll angle must remain between two extremal values :

$$\phi_{min} \leq \phi(t) \leq \phi_{max} \quad \forall t \qquad (33)$$

- The steady-state value ϕ_s of the roll angle must satisfy $\phi_s \sim -\beta_s$, where β_s is the steady-state value of the sideslip angle.

As a second point, a step is now applied to the roll rate output p. The specifications are essentially the same as above:

- The roll rate output must track the reference input signal with a rise time of less than 6 seconds and a zero static error.
- the sideslip angle must remain between two extremal values :

$$-\varepsilon \leq \beta(t) \leq \varepsilon \quad \forall t \qquad (34)$$

The following specification is also requested for both responses above, to take into account saturation constraints :

- The actuator outputs must remain between ± 30 degrees.
- The actuator rate outputs must remain between ± 50 degrees/s.

As a final point, the following robustness specification is considered:
A small gain constraint is added to ensure the robustness property with respect to the bending modes, which were not included in the design model and which are thus considered as neglected dynamics:

$$\overline{\sigma}([F_{z_1 w_1}](jw)) \leq \gamma.I(w) \ \text{ for } \ w \in [w_1, \ldots, w_N]$$

where the vector z_1 corresponds to the actuator inputs and w_1 corresponds to the perturbation inputs on n_y, p, r and ϕ.

The value γ is either fixed or to be minimized.

4.2 Synthesis of the flight control system

As a preliminary, an initial estimated state feedback K_{nom} and an initial static feedforward H_{nom} are synthesized. A modal method is used to ensure satisfactory closed loop poles and nearly perfect decoupling with the feedback K_{nom}. On the other hand, the static feedforward is used to respect the classical steady state design specifications (see above):[15]

$$\begin{bmatrix}\beta \\ \phi\end{bmatrix} = \begin{bmatrix}1 & 0 \\ -1 & 1\end{bmatrix}\begin{bmatrix}\beta_c \\ \phi_c\end{bmatrix} \qquad (35)$$

We present in figures 6, 7 the time-domain responses on the sideslip angle and roll rate output obtained with this nominal controller.

The LMI method is then applied to the augmented plant of figure 4. Note in this figure that the reference inputs y_r are included in the measured outputs y, so that a TDF (Two-degrees-of-freedom) control law is synthesized : the feedback controller K and the feedforward controller H are indeed simultaneously synthesized, and K and H are two transfer matrices in figure 4, which share the same dynamics.

A static transfer matrix Q is first chosen, the order of the control law remains equal to the order of the augmented plant, namely 16. Consequently, 12 parameters θ_i are to be optimized, namely 8 for the feedback part of the control law and 4 for its feedforward part † .
We would like to study the trade-off between the small

* The rise time t_r is defined here as $0.95 \leq y(t) \leq 1.05$ for all $t \geq t_r$, if $y(t)$ is the unit step response of the closed loop.

† 8 = 4 * 2, since the aircraft model contains 2 inputs and 4 outputs.
4 = 2 * 2, since the aircraft model contains 2 inputs and there are 2 reference inputs.

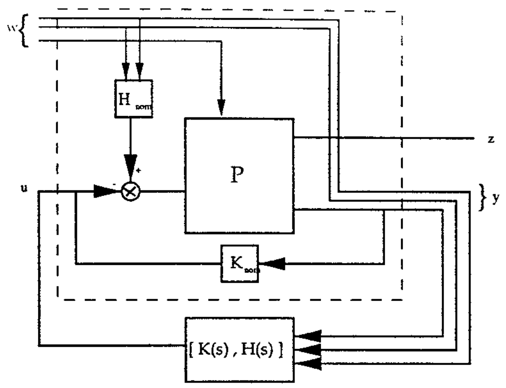

Fig. 4. Augmented Plant with initial feedback K_{nom} and feedforward H_{nom}

gain constraint and the reduction of the influence on the closed loop time domain responses of the bending modes, which are included in the synthesis model. To this aim, a value of γ is fixed and the minimization of the difference $|\beta - \beta_r|$ between the step-response on the sideslip angle on the output β and a reference signal β_r - which corresponds to a rigid response - is chosen as the optimization criterion. By this way, we directly minimize the contribution of bending modes on this output.
The other design specifications are expressed as l_∞ constraints.

The optimization problem, which is to be solved is an LMI problem, as exposed in section 2. Due to the sampling griddings and lengths of the responses considered, the LMI problem to solve contains 7000 LMIs and 3 variables, corresponding to the parameters θ_i and γ. The problem is solved using MATLAB in 5 to 15 minutes.
Various controllers have been designed for several values of γ. The difference $|\beta - \beta_r|$ is minimized in each case. As expected, note that the less the small gain specification is constraining, the more the bending modes contribution can be reduced. Figure 5 presents the trade-off between γ value and this contribution.

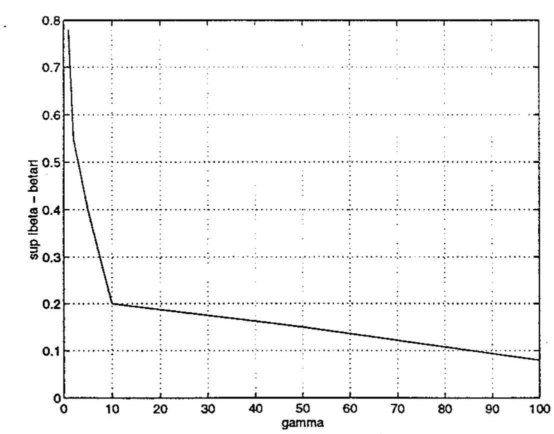

Fig. 5. Trade-off between $sup|\beta - \beta_r|$ and γ.

Next, a dynamic transfer matrix Q is then used to have more degrees of freedom for tuning the controller. We consider a base of filter which contains only gain filters and one filter of order one. 24 parameters θ_i are now to be optimized and the order of the matrix Q obtained is equal to 2. Consequently, the order of the control laws is then equal to 18.
Using this dynamic matrix Q, we can both easily reduce bending modes contribution and satisfy a good small gain constraint ($\gamma = 1$).

We present in figures 8, 9, 10 and 11 the time-domain responses on the sideslip angle and roll rate output for different cases : The time-domain responses 8 and 9 are obtained for a value of $\gamma = 1$ using a static transfer matrix Q.
The time-domain responses 10, 11 are obtained for the same value γ, using the dynamic transfer matrix Q. Note that the additional degrees of freedom, which are provided by the use of a dynamic transfer Q, enable to achieve a much better trade-off : using a static matrix Q, the same level of reduction of the bending modes contribution would require to increase γ up to $\gamma = 100$.

5. Conclusion

We have proposed a new approach for synthesizing feedforward and feedback controllers, which allows time- and frequency-domain specifications to be accounted for as convex objectives. Consequently, trade-offs between the various specifications can be studied in a systematic way. Moreover, the proposed method enables LP, QP or LMI problems to be obtained as approximations of the initial convex programming problem.

As an application example, we have considered the case of a highly flexible transport aircraft. We have especially optimized the trade-off between a small gain constraint, related to some robustness property with respect to unmodeled bending modes, and the reduction of the influence on the closed loop time domain responses of the bending modes. The use of a dynamic transfer Q provides more degrees of freedom for tuning the controllers and satisfying the set of specifications. Another main trade-off concerns the control bandwith. On the one hand, we would like to minimize this bandwith, so as to maximize the robust stability properties with respect to unmodeled structural modes. On the other hand, we would like to maximize this control bandwith, so as to ensure a fast rejection of exogenous perturbations (especially wind disturbance). This trade-off will be further studied in the context of a flight control system, which actively controls some of the bending modes.

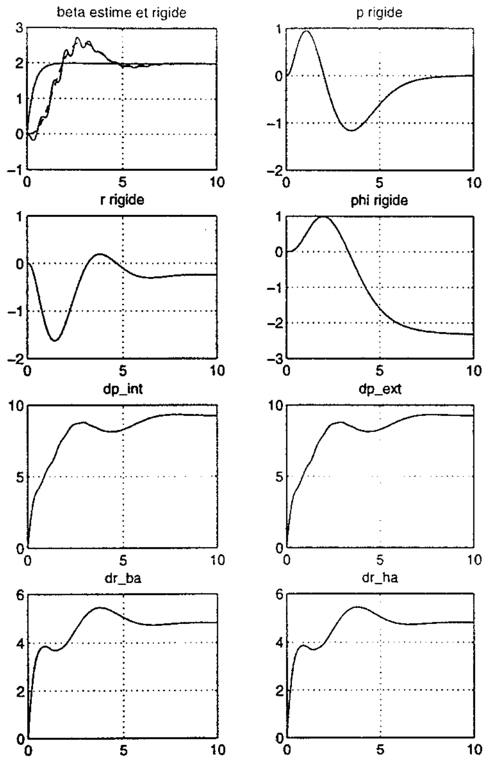

Fig. 6. Step response on the sideslip angle - Nominal controller.

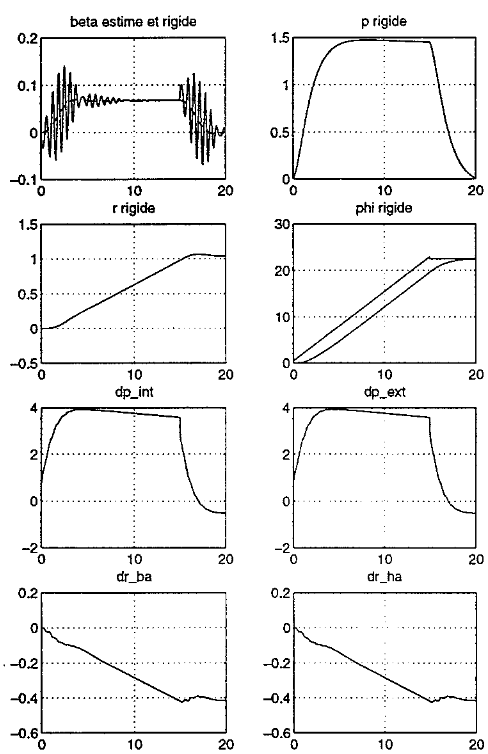

Fig. 7. Step response on the roll rate output - Nominal controller.

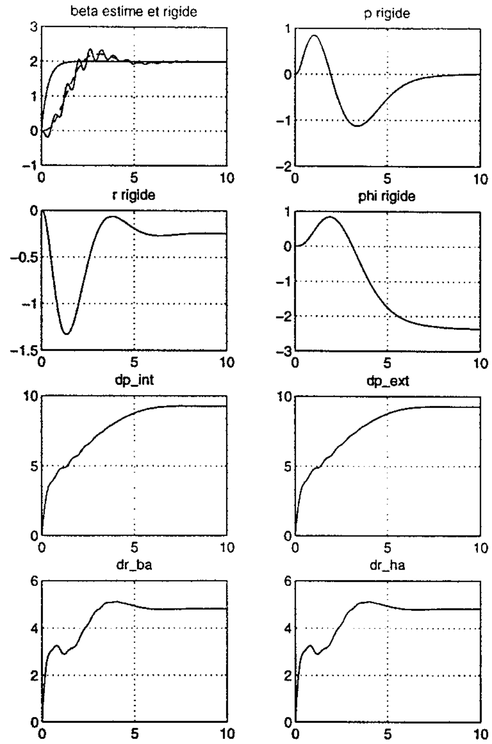

Fig. 8. Step response on the sideslip angle - $\gamma = 1$, Q static.

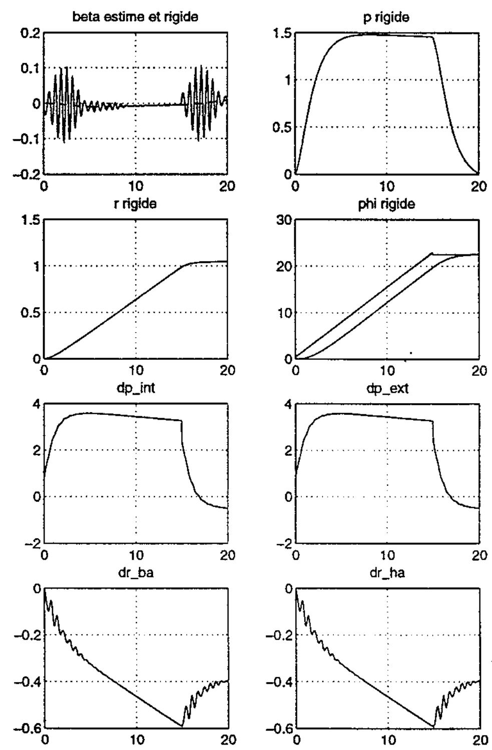

Fig. 9. Step response on the roll rate output - $\gamma = 1$, Q static.

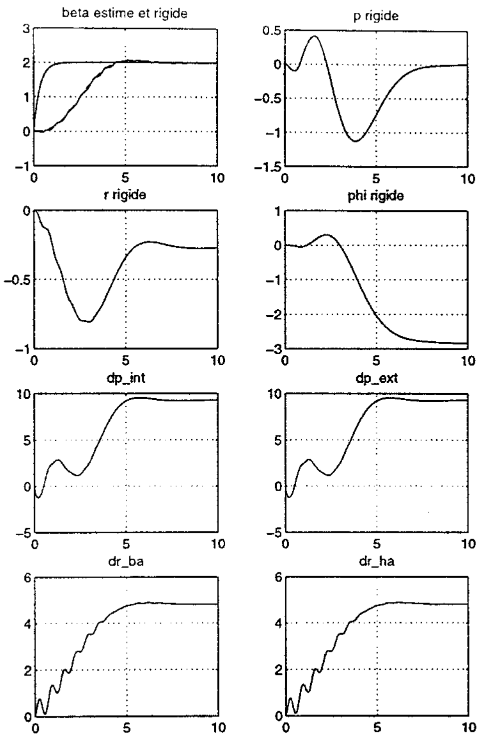

Fig. 10. Step response on the sideslip angle - $\gamma = 1$, Q dynamic.

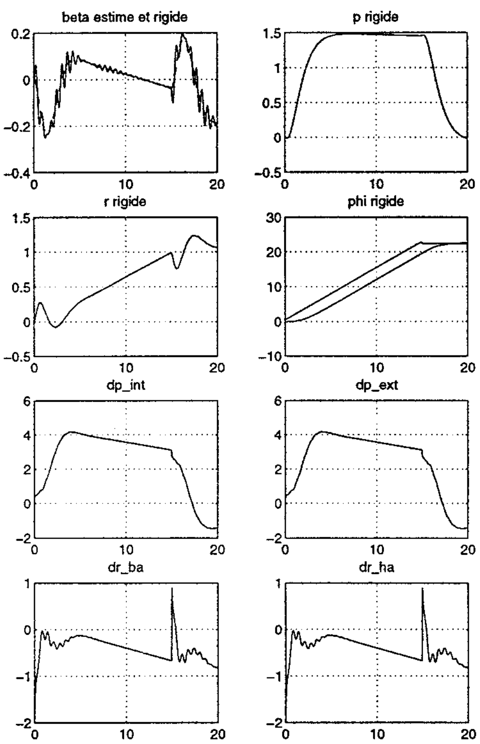

Fig. 11. Step response on the roll rate output - $\gamma = 1$, Q dynamic.

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