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A Mathematical Model of Affine Nonlinear System for Helicopter Flight Dynamics

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Abstract

Preparing for further study of nonlinear system and nonlinear control law of helicopter, a math model of affine nonlinear system (ANSM) for helicopter flight dynamics is established in this paper. In choosing sub-models and giving assumptions, a principle, which all control variables in motion differential equations must be linearized and precision of model must be guaranteed simultaneously, should be abided. The model is used to calculate the conditions of static trim in steady straight flight and dynamic responses to different control inputs for UH-60A helicopter. The results agree with flight test and have considerable precision comparing to Ames GENHEL model. So, the math model are validated, and the theories and methods that have been used in the math model are all proven out.

With better precision and analytic formulas, the affine model in this paper can also be used for real and non-real time digital simulation, and for other studies of helicopter flight dynamics.

1 Introduction

Motion of helicopter is of stronger coupling and nonlinearity. Comparing to the fixed-wing aircraft, helicopter has worse characteristics in stability, controllability and maneuverability and the pilot's work of helicopter is harder. But, the present helicopter flight control system, which based on the linear system theory, can not be used to solve these problems at all. With the development of nonlinear system theory[1,2], especially with fast development of the theories of global linear and decoupling based on the affine nonlinear system (in which all control variables can be expressed in linear forms), it will be possible to design nonlinear control system for aircraft[3~5]. So, it will reduce the labor of pilot intensity, improve control precision and guarantee flying safety more effectively. At present, for advanced study and design of helicopter nonlinear control system, the affine nonlinear system mathematical model with better precision of helicopter flight dynamics need to be set up.

Preparing for further study of nonlinear control system of helicopter, a math model of affine nonlinear system (ANSM) for helicopter flight dynamics is established in this paper. Because of the complexity of rotor and coupling in helicopter, the control variables are hidden in various formulas and can not be showed their linear forms in helicopter equations of motion directly. The math model of affine nonlinear system is a high synthetic work about the sub-models of the helicopter. So, in the process of choosing sub-models and giving assumptions, a principle, which all control variables in differential equations must be linearized and precision of model must be guaranteed simultaneously, should be abided. In order to make all control variables with linear forms in equations, some assumptions, which includes small values about blade flapping angle and air attack angle of blade section, linear aerodynamics about the section of blade, and so on, are used. Meanwhile, in order to improve overall precision of established math model, some measures, such as non-uniform dynamic inflow model, the same comprehensive analysis for dynamics of tail rotor as main rotor, and so on, are also taken.

The math model ANSM in this paper is used to calculate the sample of UH-60A helicopter in some conditions, such as trim in level flight, climb and descend, and dynamic responses for UH-60A to different control inputs at three level flight states.

2 Mathematical Model

2.1 Basic Assumptions

Generic nonlinear system can be written as

$$\dot{X} = f(X, U; t), \quad X \in R^n, U \in R^m, m \le n \tag{1}$$

Here, X is state vector, U is control vector. But, the affine nonlinear system can be written as follow

$$\dot{X} = A(X,t) + B(X,t)U \tag{2}$$

Here, A is $n \times 1$ vector, and B is a array with $n \times m$. To set up Eq. (2) is the target of this paper.

It is important to chose proper assumptions for the affine nonlinear system. The follow assumptions are necessary to get Eq.(2) in the paper.

(1) Linear aerodynamics of airfoil section within small attack angle.

(2) Only considering the basic (constant and one order) modes of flap angle, inflow angle and pitch angle.

(3) Using the cyclic average value of force and moment at hub of rotor and using blade flapping dynamics motion equations with constant coefficients.

If one or more of the above assumptions is not satisfied, the affine nonlinear model may not be got.

2.2 Rotor Force

Blade Flapping angle β, pitch angle θ, and inflow ratio λ of rotor, which are all the function of rotor blade azimuth angle ψ, ignoring two and high order terms, can be written as

$$\beta = a_0 - a_1 \cos\psi - b_1 \sin\psi \tag{3a}$$

$$\theta = \theta_0 - A_{1c} \cos\psi - B_{1c} \sin\psi + (x - \varepsilon - \varepsilon')\theta_t - K_1 \beta \tag{3b}$$

$$\lambda = \lambda_o + \lambda_1 x \cos\psi + \lambda_2 x \sin\psi \tag{3c}$$

Here, θ_0 is pitch of rotor, A_{1c} is lateral cyclic pitch, and B_{1c} is longitude cyclic pitch, θ_t is linear torsion rate, $K_1=\tan\delta_3$, (δ_3 is the fixed deflection angle of flapping hinge of blade), x is the distance to the center of hub, ε is the blade hinge offset from center of rotation, ε' is cutting part from the flapping hinge at blade root.

With quasi-linear assumption about the aerodynamics of airfoil section of blade, through integrating the force of airfoil section from blade root to tip, the cyclic average of force and moment at rotor hub can be got, which are all the analytic expressions of basic parameters of rotor and blade, motion parameters of helicopter body, and the parameters in Eq.(3). There are some terms of product and square in these analytic expressions of force and moment of rotor, which are all nonlinear. But, the control variables θ_0, A_{1c}, B_{1c} are all of linear expressions, which is very important to get the equations of affine nonlinear system model.

In order to improve the precision of affine nonlinear model, the force at the hub of tail rotor is taken the same as the main rotor. The aerodynamics and induced velocity of fuselage and empennage within large scope is come from the test data[6].

2.3 Dynamics Equations

The follow equations describe the motion of helicopter (motion of mass center, rotation around the mass center, and others of geometry relationship) at body axes.

$$\dot{u} = -qw + rv + \sum X / M_h - g\sin\Theta \tag{4a}$$

$$\dot{v} = -ru + pw + \sum Y / M_h + g\cos\Theta\sin\Phi \tag{4b}$$

$$\dot{w} = -pv + qu + \sum Z / M_h + g\cos\Theta\cos\Phi \tag{4c}$$

$$\dot{p} = \{I_z[\sum L + (I_y - I_z)qr + I_{zx}pq] + I_{zx}[\sum N + (I_x - I_y)pq - I_{zx}qr]\} / (I_x I_z - I_{zx}^2) \tag{4d}$$

$$\dot{q} = [\sum M + (I_z - I_x)rp + I_{zx}(r^2 - p^2)] / I_y \tag{4e}$$

$$\dot{r} = \{I_x[\sum N + (I_x - I_y)pq - I_{zx}qr] + I_{zx}[\sum L + (I_y - I_z)qr + I_{zx}pq]\} / (I_x I_z - I_{zx}^2) \tag{4f}$$

$$\dot{\Psi} = (r\cos\Phi + q\sin\Phi)/\cos\Theta \tag{4g}$$

$$\dot{\Phi} = p + q\tan\Theta\sin\Phi + r\tan\Theta\cos\Phi \tag{4h}$$

$$\dot{\Theta} = q\cos\Phi - r\sin\Phi \tag{4i}$$

Here, Mh is the total mass of helicopter, ($\Sigma X, \Sigma Y, \Sigma Z$) and ($\Sigma L, \Sigma M, \Sigma N$) are the forces and moments at body mass center. (u, v, w) and (p, q, r)

are velocity and angular rate about body axes. Θ is pitch angle, Φ is roll angle, Ψ is yaw angle.

The motion equations of the rotor flapping are two-order differential equations with cyclic coefficients naturally. But here, through using the assumption of constant coefficients, the equations can be written as

$$\begin{bmatrix}\ddot{a}_0\\ \ddot{a}_1\\ \ddot{b}_1\end{bmatrix}_M + \boldsymbol{D}_M\begin{bmatrix}\dot{a}_0\\ \dot{a}_1\\ \dot{b}_1\end{bmatrix}_M + \boldsymbol{K}_M\begin{bmatrix}a_0\\ a_1\\ b_1\end{bmatrix}_M = \boldsymbol{P}_M\begin{bmatrix}\theta_0\\ \theta_t\\ A_{1c}\\ B_{1c}\end{bmatrix}_M + \boldsymbol{Q}_M\begin{bmatrix}p\\ q\\ \dot{p}\\ \dot{q}\end{bmatrix}_M + \boldsymbol{R}_M\begin{bmatrix}\lambda_0\\ \lambda_1\\ \lambda_2\end{bmatrix}_M + \boldsymbol{G}_M \tag{5}$$

Here, subscribe 'M' expresses the main rotor, the coefficient matrixes are all longer(see reference 7), then omitted here. The flapping motion equations are linear forms, and control variables (θ_0, A_{1c}, B_{1c}) are also linear forms, which is satisfied with the requirement of the affine nonlinear system model. Otherwise, the coefficient matrixes also include nonlinear factors, for example, there is coupling items of body motion parameters in the coefficient matrix $\boldsymbol{G}_M$.

The flapping motion equations of tail rotor is resemblance to main rotor Eq.(5), but uses uniform inflow assumption.

$$\begin{bmatrix}\ddot{a}_0\\ \ddot{a}_1\\ \ddot{b}_1\end{bmatrix}_T + \boldsymbol{D}_T\begin{bmatrix}\dot{a}_0\\ \dot{a}_1\\ \dot{b}_1\end{bmatrix}_T + \boldsymbol{K}_T\begin{bmatrix}a_0\\ a_1\\ b_1\end{bmatrix}_T = \boldsymbol{P}_T\begin{bmatrix}\theta_0\\ \theta_t\end{bmatrix}_T + \boldsymbol{Q}_T\begin{bmatrix}p\\ q\\ \dot{p}\\ \dot{q}\end{bmatrix}_T + [r\lambda_0]_T + \boldsymbol{G}_T \tag{6}$$

Here, subscribe 'T' expresses the tail rotor.

Assuming that the induce velocity on rotor disk plane rotor is distributed as

$$v = v_o + v_c x\cos\psi + v_s x\sin\psi$$

In the paper, one-order Pitt/Peters dynamic inflow model[8] is adopted as follow

$$\frac{1}{\Omega}\Gamma\begin{bmatrix}\dot{v}_0\\ \dot{v}_s\\ \dot{v}_c\end{bmatrix}_M + \begin{bmatrix}v_0\\ v_s\\ v_c\end{bmatrix}_M = \Pi\begin{bmatrix}C_t\\ C_l\\ C_m\end{bmatrix}_M \tag{7}$$

Here, the matrix Γ and Π are shown in reference 8, C_t, C_l and C_m are the lift coefficient, roll moment coefficient and pitch moment coefficient of the main rotor respectively. For the tail rotor, we have (here, the subscribe 'T' is omitted)

$$\dot{v}_0 + 0.75\pi\sqrt{\mu^2+\lambda_0^2}\,v_0 = 0.375\pi C_t \tag{8}$$

2.4 Setting up Affine Nonlinear System Model

With the force and moment of each part of helicopter into Eq.(4)~Eq.(8), we can get the following one-order nonlinear differential equations

$$\dot{\boldsymbol{X}} = \boldsymbol{g}(\dot{\boldsymbol{X}},\boldsymbol{X},\boldsymbol{U};t) \tag{9}$$

Here, state vector $\boldsymbol{X}$ and control vector $\boldsymbol{U}$ are

$$\boldsymbol{X} = [u,v,w,p,q,r,\Phi,\Theta,\Psi,v_{0_M},v_{c_M},v_{s_M},\\ v_{0_T},a_{0_M},a_{1_M},b_{1_M},\dot{a}_{0_M},\dot{a}_{1_M},\dot{b}_{1_M},a_{0_T},a_{1_T},\\ b_{1_T},\dot{a}_{0_T},\dot{a}_{1_T},\dot{b}_{1_T}]^{\mathrm{T}}$$

$$\boldsymbol{U} = [\theta_{0_M},\theta_{0_T},A_{1c},B_{1c}]^{\mathrm{T}}$$

The item including $\dot{\boldsymbol{X}}$ at the right of Eq.(9) can be written as a linear form, so Eq.(4) becomes

$$\dot{\boldsymbol{X}} = \boldsymbol{B}_1(t)\dot{\boldsymbol{X}} + g_1(\boldsymbol{X},\boldsymbol{U};t)$$

So, we can get the standard form of one-order differential equations

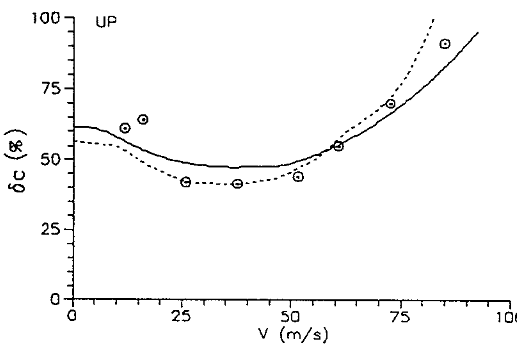
$$\dot{\boldsymbol{X}} = [\boldsymbol{I}-\boldsymbol{B}_1(t)]^{-1}g_1(\boldsymbol{X},\boldsymbol{U};t) = f(\boldsymbol{X},\boldsymbol{U};t)$$

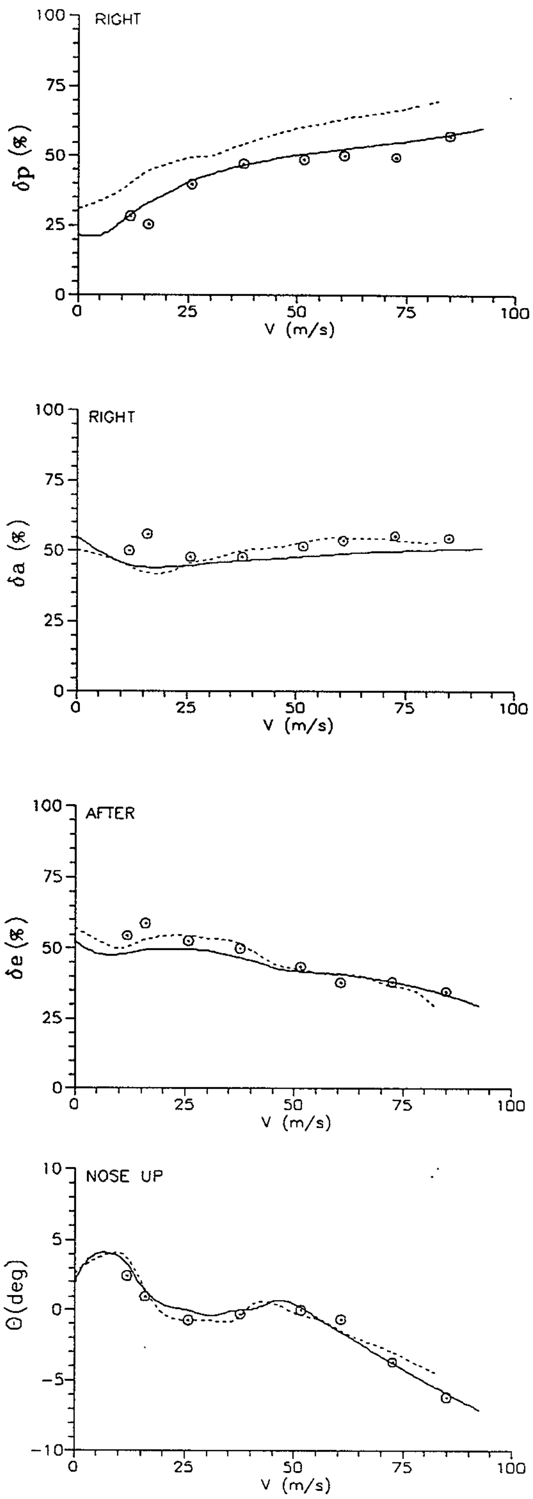
Here $\boldsymbol{B}_1$(t) and $\boldsymbol{I}$ are all $n\times n$ matrixes. Because the axes transition from the axes of each part of helicopter to the body axes is normal transition, the linear or nonlinear form of forces and moments are saved, especially the linear form of control variables are not varied. So, the above equations can be written as an affine nonlinear system like Eq.(2), which includes 25 one-order equations and is called ANSM model. In Eq.(2), $\boldsymbol{B}(X,t)$ and $A(X,t)$ are the nonlinear function of state variables and are longer (see reference 7).

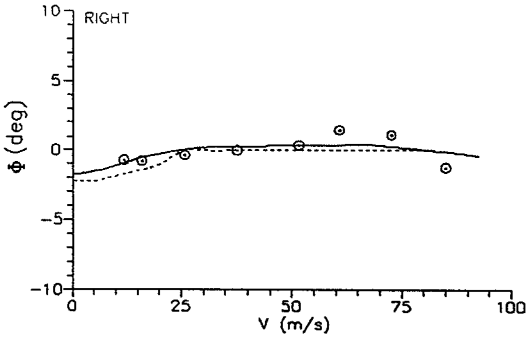
3 Result and Validation of Model

The Black Hawk UH-60A helicopter is the sample. According to the condition of flight test of UH-60A, we calculate some static trim of straight flight and dynamic responses to control inputs of pilot using ANSM. The calculation results will compare with the flight test[9] and with the calculation result[9] of Ames GENHEL model in Ames center. Ames GENHEL model is a generic nonlinear system (like Eq.(1)) model, which is a real-time engineering simulation model for helicopter flight dynamics.

In static trim (in level flight, climb and descend) calculations, the rotor shaft speed Ω is 27 *rad/s*, flight height is 1600.2*m*, flight airspeed is from 0 m/s to 92.6 m/s (i.e. $\mu = 0 \sim 0.42$), and there is no yaw. Here, only gives the results of level flight, see Fig.1, which is compared with the flight test and with the calculation results of Ames GENHEL model. Fig.1 show the positions of longitudinal cyclic (δ_e), lateral cyclic (δ_a), collective stick (δ_c), pedal position (δ_p) with airspeed and Θ, Φ.

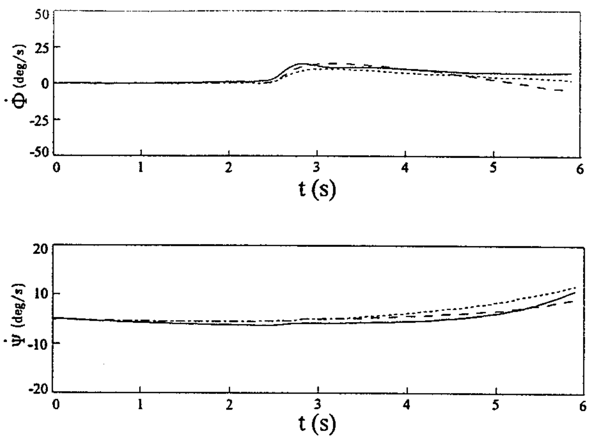






⊙ Flight Test -----Ames GENHEL ——ANSM

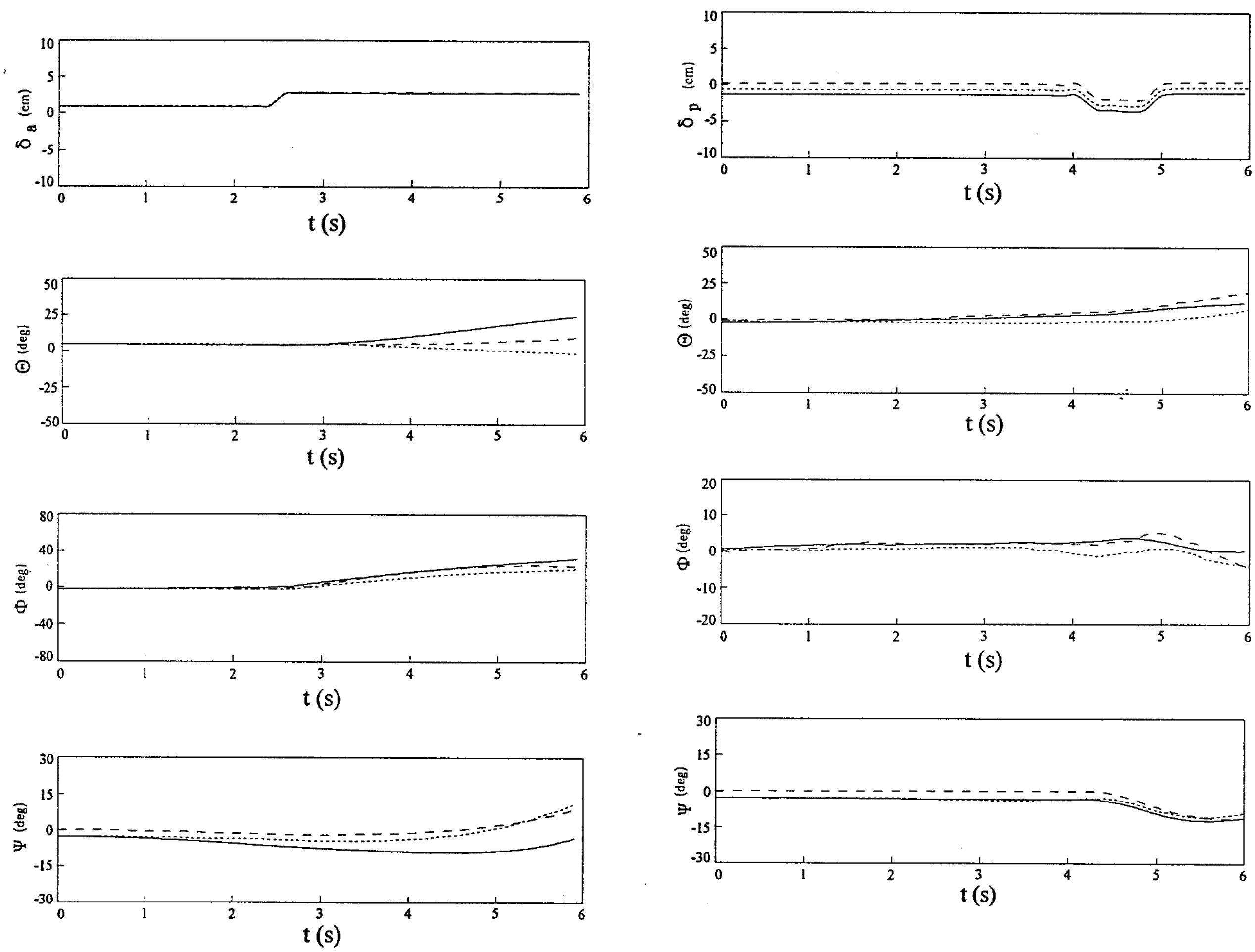
Fig.1 UH-60A Black Hawk level flight static trim

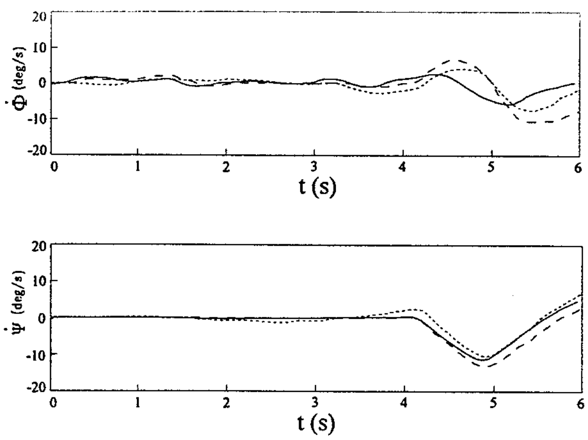


···Flight Test - - Ames GENHEL ——ANSM

Fig.2 Dynamic response calculations and comparison of UH-60A
(at hover, 2.54*cm* right cyclic input)

Otherwise, the dynamic responses results and comparing with UH-60A to control inputs of pilot at μ=0, 0.14, 0.23 calculated using ANSM. Here gives two results of attitude angle(Θ, Φ, Ψ) and their variations with time, see Fig.1 and Fig.2.





···Flight Test - - Ames GENHEL —— ANSM

Fig.3 Dynamic response calculations and comparison of UH-60A (μ=0.14, hover, 2.54*cm* left pedal input)

Fig.1~Fig.3 show that, on the whole, the calculation results using the ANSM model have good agreement with flight test and with the calculation using Ames GENHEL. And, there is better result in static trim of pedal position (i.e. the collective of tail rotor) than the Ames GENHEL model, because of considering the lateral induced velocity at tail rotor. Using some simple assumption to get the affine nonlinear system may reduce the precision of model, but the non-uniform dynamic inflow model, tail rotor model similar to the main rotor, and considering lateral induced velocity at tail rotor, etc., can improve the precision of ANSM model.

4 Conclusion

The math model (ANSM) is used to calculate the sample of UH-60A helicopter in some conditions, such as static trim in level flight, climb and descend, and dynamics responses for UH-60A to different control inputs in three level flight states. The results agree with available UH-60A flight test and the Ames GENHEL. So, ANSM model are validated, and the theories and methods that have been used in the math model are all proven out. Meanwhile, the works to improve the precision of math model are also proven out.

Further study of ANSM model is to design nonlinear flight control law. With better precision and analytic formulas, the affine model in this paper can also be used for real and non-real time digital simulation, and for other studies of helicopter flight dynamics.

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