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A PASSIVE AERODYNAMIC STABILIZATION SYSTEM OF SATELLITES FOR LOW EARTH ORBIT: AN ANALYTICAL APPROACH

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Abstract. For satellites launched into relatively low circular orbit aerodynamic stabilization can be used. It orients the satellite's longitudinal axis along its velocity vector. The purpose of this paper consists of the investigation of the attitude motion of the satellite-rigid body under the action of gravitational and aerodynamic torques. Main directions of study: nonlinear equations of satellite attitude motion, equilibrium orientations of satellite in general case, equilibrium orientations of satellite in special cases.

Equations of motion

We consider the attitude motion of a satellite under the action of gravitational and aerodynamic torques. Equations of the attitude motion of the satellite can be written in such a way[(1)]:

$$\begin{aligned} &Ap_1' + (C-B)q_1r_1 = M_{gx} + M_{ax}, \\ &Bq_1' + (A-C)r_1p_1 = M_{gy} + M_{ay}, \\ &Cr_1' + (B-A)p_1q_1 = M_{gz} + M_{az}; \end{aligned} \qquad (1)$$

$$\begin{aligned} &p_1 = (\alpha' + \omega)a_{21} + \gamma', \\ &q_1 = (\alpha' + \omega)a_{22} + \beta' \sin\gamma, \\ &r_1 = (\alpha' + \omega)a_{23} + \beta' \cos\gamma. \end{aligned} \qquad (2)$$

Here

$$\begin{aligned} &M_{gx} = 3\frac{\mu}{\rho^3}(C-B)a_{32}a_{33}, \\ &M_{gy} = 3\frac{\mu}{\rho^3}(A-C)a_{33}a_{31}, \\ &M_{gz} = 3\frac{\mu}{\rho^3}(B-A)a_{31}a_{32}; \end{aligned} \qquad (3)$$

$$\begin{aligned} &M_{ax} = b_g Z_a - c_g Y_a, \\ &M_{ay} = c_g X_a - a_g Z_a, \\ &M_{az} = a_g Y_a - b_g X_a; \end{aligned} \qquad (4)$$

$$\begin{aligned} &X_a = -Q\left(\frac{V_X}{V}a_{11} + \frac{V_Y}{V}a_{21} + \frac{V_Z}{V}a_{31}\right), \\ &Y_a = -Q\left(\frac{V_X}{V}a_{12} + \frac{V_Y}{V}a_{22} + \frac{V_Z}{V}a_{32}\right), \\ &Z_a = -Q\left(\frac{V_X}{V}a_{13} + \frac{V_Y}{V}a_{23} + \frac{V_Z}{V}a_{33}\right); \end{aligned} \qquad (5)$$

	x	y	z
X	a_{11}	a_{12}	a_{13}
Y	a_{21}	a_{22}	a_{23}
Z	a_{31}	a_{32}	a_{33}

$$\begin{aligned} &a_{11} = \cos\alpha\cos\beta, \\ &a_{12} = \sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma, \\ &a_{13} = \sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma, \\ &a_{21} = \sin\beta, \\ &a_{22} = \cos\beta\cos\gamma, \\ &a_{23} = -\cos\beta\sin\gamma, \\ &a_{31} = -\sin\alpha\cos\beta, \\ &a_{32} = \cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma, \\ &a_{33} = \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma; \end{aligned} \qquad (6)$$

$$\begin{aligned} &V_X = \omega_0 p_*(1 + e\cos\theta) - \Omega\rho\cos i, \\ &V_Y = \Omega\rho\sin i\cos u, \\ &V_Z = \omega_0 p_* e\sin\theta, \\ &V^2 = V_X^2 + V_Y^2 + V_Z^2, \\ &u = \omega_\pi + \theta, \\ &\omega = \omega_0(1 + e\cos\theta)^2, \end{aligned} \qquad (7)$$

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$$\frac{\mu}{\rho^3} = \omega_0^2 (1 + e\cos\theta)^3,$$

$$\omega_0^2 = \frac{\mu}{p_*^3},$$

$$Q = \frac{1}{2}\rho_* V^2 S C_x,$$

α, β, γ are the angles of pitch, yaw, roll (Fig. 1), defining the orientation of the central moments of inertia axes of the satellite Ox, Oy, Oz with respect to the orbital coordinate system $OXYZ$ (OY is the normal to the orbital plane, OZ coincides with the local vertical); A, B, C are the central principal moments of inertia of the satellite; p_1, q_1, r_1 are the components of the absolute angular velocity of the satellite in the frame $Oxyz$; M_{gx}, M_{gy}, M_{gz} are the components of the gravitational torque; M_{ax}, M_{ay}, M_{az} are the components of the aerodynamic torque; a_g, b_g, c_g are the coordinates of the center of pressure in the frame $Oxyz$; V_X, V_Y, V_Z are the components of the velocity of the satellite mass center relative to the atmosphere in the frame $OXYZ$; ρ is the distance between the mass centers of the Earth and of the satellite; θ is the true anomaly; e is the eccentricity; i is the inclination of the orbit; ω is the angular velocity of the orbital motion; p_* is the parameter of the orbit; μ is the Earth's gravitational parameter; ω_π is the argument of perigee; Ω is the angular velocity of the Earth's rotation about its axis; Q is the drag of the atmosphere; ρ_* is the atmospheric density; S is the cross-sectional area of the satellite; C_x is the drag coefficient. In equations (1)-(2) the sign$'$ denotes differentiation with respect to time t.

In deriving the expressions (4), (5), (7) it was assumed that the atmosphere is completely carried away by the rotating Earth, the influence of atmospheric drag on the translation motion of the satellite can be ignored, the effect of the atmosphere on the satellite attitude motion is reduced to the aerodynamic drag force applied to the center of pressure and directed against the velocity of the satellite's center of mass with respect to the free air stream.

Equilibria and their stability

Let us consider the attitude motion equations of the satellite at $e = 0$, $\Omega = 0$. Then

$$\begin{aligned}
&A\dot{p} + (C-B)qr - 3(C-B)a_{32}a_{33} = \\
&\bar{h}_2 a_{13} - \bar{h}_3 a_{12}, \\
&B\dot{q} + (A-C)rp - 3(A-C)a_{33}a_{31} = \\
&\bar{h}_3 a_{11} - \bar{h}_1 a_{13}, \\
&C\dot{r} + (B-A)pq - 3(B-A)a_{31}a_{32} = \\
&\bar{h}_1 a_{12} - \bar{h}_2 a_{11}.
\end{aligned} \tag{8}$$

Here

$$\begin{aligned}
&p = p_1/\omega_0 = (\dot{\alpha}+1)a_{21} + \dot{\gamma}, \\
&q = q_1/\omega_0 = (\dot{\alpha}+1)a_{22} + \dot{\beta}\sin\gamma, \\
&r = r_1/\omega_0 = (\dot{\alpha}+1)a_{23} + \dot{\beta}\cos\gamma;
\end{aligned} \tag{9}$$

$$\bar{h}_1 = -Qa_g/\omega_0^2,$$
$$\bar{h}_2 = -Qb_g/\omega_0^2,$$
$$\bar{h}_3 = -Qc_g/\omega_0^2.$$

Equilibrium orientations of the satellite in the orbital coordinate system correspond to the stationary solutions α = const, β = const, γ = const of (8) and are determined by the following system of equations:

$$\begin{aligned}
&(C-B)(a_{22}a_{23} - 3a_{32}a_{33}) = \bar{h}_2 a_{13} - \bar{h}_3 a_{12}, \\
&(A-C)(a_{23}a_{21} - 3a_{33}a_{31}) = \bar{h}_3 a_{11} - \bar{h}_1 a_{13}, \\
&(B-A)(a_{21}a_{22} - 3a_{31}a_{32}) = \bar{h}_1 a_{12} - \bar{h}_2 a_{11}.
\end{aligned} \tag{10}$$

In the systems of equations (8)-(9) the dot denotes differentiation with respect to $\tau = \omega_0 t$.

System (8) possesses the integral of energy

$$\begin{aligned}
&\frac{1}{2}\left(A\bar{p}^2 + B\bar{q}^2 + C\bar{r}^2\right) + \\
&\frac{3}{2}\left[(A-C)a_{31}^2 + (B-C)a_{32}^2\right] + \\
&\frac{1}{2}\left[(B-A)a_{21}^2 + (B-C)a_{23}^2\right] - \\
&\left(\bar{h}_1 a_{11} + \bar{h}_2 a_{12} + \bar{h}_3 a_{13}\right) = h_0 .
\end{aligned} \tag{11}$$

Here

$\bar{p} = p - a_{21}$, $\bar{q} = q - a_{22}$, $\bar{r} = r - a_{23}$.

Using the left-hand part of the integral of energy (11) as Lyapunov function, it is possible to investigate the stability of any stationary solution of system (10).

Equilibrium orientations: General case

Let $A \neq B \neq C$. Substituting the expressions for the direction cosines from (6) into (10), we obtain three equations with three unknowns α, β, γ. A second approach for closing equations (10) is to add the following six orthogonality conditions for the direction cosines:

$$\begin{aligned}
&a_{11}^2 + a_{12}^2 + a_{13}^2 = 1,\\
&a_{21}^2 + a_{22}^2 + a_{23}^2 = 1,\\
&a_{31}^2 + a_{32}^2 + a_{33}^2 = 1,\\
&a_{11}a_{21} + a_{12}a_{22} + a_{13}a_{23} = 0,\\
&a_{11}a_{31} + a_{12}a_{32} + a_{13}a_{33} = 0,\\
&a_{21}a_{31} + a_{22}a_{32} + a_{23}a_{33} = 0.
\end{aligned} \tag{12}$$

Equations (10) and (12) form a closed system with respect to the direction cosines, which also specifies the equilibrium positions of the satellite under influence of gravitational and aerodynamic torques.

Projecting equations (10) onto axes of the orbital coordinate system OXYZ, we obtain the equivalent system of equations:

$$\begin{aligned}
&Aa_{21}a_{31} + Ba_{22}a_{32} + Ca_{23}a_{33} = 0,\\
&3\left(Aa_{11}a_{31} + Ba_{12}a_{32} + Ca_{13}a_{33}\right) +\\
&\bar{h}_1 a_{31} + \bar{h}_2 a_{32} + \bar{h}_3 a_{33} = 0,\\
&\left(Aa_{11}a_{21} + Ba_{12}a_{22} + Ca_{13}a_{23}\right) -\\
&\bar{h}_1 a_{21} - \bar{h}_2 a_{22} - \bar{h}_3 a_{23} = 0.
\end{aligned} \tag{13}$$

The system of equations (12), (13) can be solved for a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23} [(2,3)] in the form

$$\begin{aligned}
a_{11} &= \pm\frac{(I_3 - A)a_{31}}{\sqrt{D}},\\
a_{12} &= \pm\frac{(I_3 - B)a_{32}}{\sqrt{D}},\\
a_{13} &= \pm\frac{(I_3 - C)a_{33}}{\sqrt{D}};
\end{aligned} \tag{14}$$

$$\begin{aligned}
a_{21} &= \pm\frac{(B - C)a_{32}a_{33}}{\sqrt{D}},\\
a_{22} &= \pm\frac{(C - A)a_{33}a_{31}}{\sqrt{D}},\\
a_{23} &= \pm\frac{(A - B)a_{31}a_{32}}{\sqrt{D}}.
\end{aligned} \tag{15}$$

Here

$$\begin{aligned}
D = {}&(B - C)^2 a_{32}^2 a_{33}^2 + (C - A)^2 a_{33}^2 a_{31}^2 +\\
&(A - B)^2 a_{31}^2 a_{32}^2,\\
I_3 = {}&Aa_{31}^2 + Ba_{32}^2 + Ca_{33}^2.
\end{aligned} \tag{16}$$

It is easy to show that direction cosines (14)-(15) will satisfy the equations (12/1), (12/2), (12/4) - (12/6), (13/1).

Using the expressions for a_{1i} from (14) we can show that

$$Aa_{11}a_{31} + Ba_{12}a_{32} + Ca_{13}a_{33} = \mp\sqrt{D}. \tag{17}$$

Then the second and the third equations of the system (13) can be written in the form

$$\begin{aligned}
&\pm 3\sqrt{D} = \bar{h}_1 a_{31} + \bar{h}_2 a_{32} + \bar{h}_3 a_{33},\\
&\left(Aa_{11}a_{21} + Ba_{12}a_{22} + Ca_{13}a_{23}\right) -\\
&\left(\bar{h}_1 a_{21} + \bar{h}_2 a_{22} + \bar{h}_3 a_{23}\right) = 0.
\end{aligned} \tag{18}$$

Squaring the first equation of the system (18) and eliminating a_{1i}, a_{2i} from the second equation by means of (14) and (15) we obtain three equations

$$
\begin{aligned}
&9\left[(B-C)^2 a_{32}^2 a_{33}^2+(C-A)^2 a_{33}^2 a_{31}^2+\right.\\
&\left.(A-B)^2 a_{31}^2 a_{32}^2\right]=\left(\bar{h}_1 a_{31}+\bar{h}_2 a_{32}+\bar{h}_3 a_{33}\right)^2,\\
&3(B-C)(C-A)(A-B) a_{31} a_{32} a_{33}-\\
&\left[\bar{h}_1(B-C) a_{32} a_{33}+\bar{h}_2(C-A) a_{33} a_{31}+\right.\\
&\left.\bar{h}_3(A-B) a_{31} a_{32}\right]\left(\bar{h}_1 a_{31}+\bar{h}_2 a_{32}+\bar{h}_3 a_{33}\right)=0,\\
&a_{31}^2+a_{32}^2+a_{33}^2=1
\end{aligned}
\tag{19}
$$

for determination of a_{31}, a_{32}, a_{33}. It is possible to show [3] that system (19) (and system (12) - (13)) have no more than 24 solutions in the general case.

Equilibrium orientations: Special case

Solutions (14)-(15) exist if $D \neq 0$. Let us investigate the problem when $D=0$. If $A \neq B \neq C$, $D=0$ in the next three cases:

$$a_{31}=0, \quad a_{32}=0; \tag{20}$$
$$a_{32}=0, \quad a_{33}=0; \tag{21}$$
$$a_{31}=0, \quad a_{33}=0. \tag{22}$$

In the case (20)

$$a_{31}=0, a_{32}=0, a_{33}^2=1, a_{13}=0, a_{23}=0. \tag{23}$$

Then from systems (12)-(13) it follows that $\bar{h}_3=0$ and

$$
\begin{aligned}
&A a_{11} a_{21}+B a_{12} a_{22}-\bar{h}_1 a_{21}-\bar{h}_2 a_{22}=0,\\
&a_{11}^2+a_{12}^2=1,\\
&a_{21}^2+a_{22}^2=1,\\
&a_{11} a_{21}+a_{12} a_{22}=0.
\end{aligned}
\tag{24}
$$

The solution of the system (23)-(24) have the form:

$$
\begin{aligned}
&a_{11}=\frac{\bar{h}_1 x}{(A-B)x+\bar{h}_2}, && a_{12}=x, \quad a_{13}=0,\\
&a_{21}=-x, && a_{22}=a_{11}, \quad a_{23}=0,\\
&a_{31}=0, && a_{32}=0, \quad a_{33}=1;
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&a_{11}=\frac{\bar{h}_1 x}{(A-B)x+\bar{h}_2}, && a_{12}=x, \quad a_{13}=0,\\
&a_{21}=x, && a_{22}=-a_{11}, \quad a_{23}=0,\\
&a_{31}=0, && a_{32}=0, \quad a_{33}=-1.
\end{aligned}
\tag{26}
$$

Here x is the real root of the algebraic equation

$$
\begin{aligned}
&(A-B)^2 x^4+2(A-B)\bar{h}_2 x^3+\\
&\left[\bar{h}_1^2+\bar{h}_2^2-(A-B)^2\right]x^2-\\
&2(A-B)\bar{h}_2 x-\bar{h}_2^2=0.
\end{aligned}
\tag{27}
$$

Since the number of real roots of equation (27) does not exceed 4, the satellite in a circular orbit for the special case (20) can have at most 8 equilibrium orientations in the orbital coordinate system.

A similar way can be used to solve the special cases (21) and (22). If $D \neq 0$ it is possible to use the general approach - the formulas (14), (15) and the system (19).

Aerodynamic stabilization system

The attitude motion of satellites under the influence of gravitational and aerodynamic torques was investigated in many papers.

The practical implementation of the aerodynamic stabilization system has been effected in the russian satellites "Space Arrow" (Cosmos-149, Cosmos-320) [4-10]. These satellites were placed in low orbits to investigate physical processes in the Earth's atmosphere and to determine atmospheric parameters. This stabilization system is of special interest, since it represents the first (and unique up to the present) use in space technology of the aerodynamic principle of satellite control in pitch and yaw.

In the last few years an application of a passive aerodynamic stabilization for small satellites was studied also by NASA Goddard Space Flight Center engineers [11-13].

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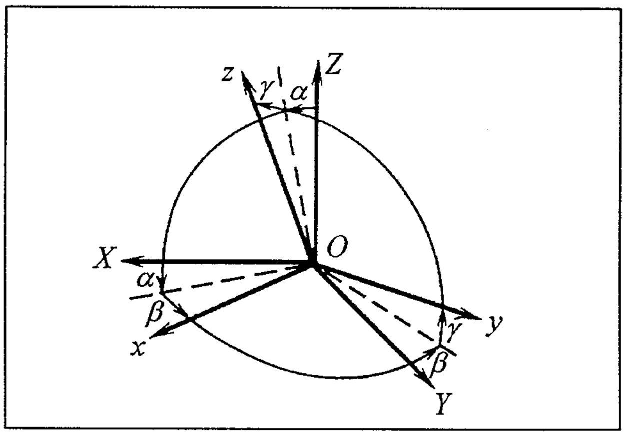

Fig.1. Orientation angles pitch (α), yaw (β), roll (γ)