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BUCKLING AND POSTBUCKLING BEHAVIOUR OF COMPOSITE PANELS

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Abstract

For selected unstiffened and stiffened plane panels and curved unstiffened panels, all made of carbon fibre reinforced epoxy, detailed information on configuration and behaviour under load was made available to the members of a GARTEUR [1]) action group. The members applied their analysis tools to predict the mechanical behaviour of the panels. In this report information on the tools is presented followed by the results of the activity. As a conclusion it is stated that the procedures for the analytical treatment of composite panels have attained a state comparable to that for metallic panels.

The members of the action group came from the four GARTEUR member countries France, the Netherlands, the UK and Germany (FRG). During the phase of work on which this paper is based members were *D.J. Allman, C. Cornuault, C. Czekajski, H. Flüh, B. Geier, J. Locatelli, N.T. Morley, C. Petiau, C. Ridgard, E. Riks, J.H. van der Sloot, M.B. Snell, R. Valid, R.W. West, J.F.M. Wiggenraad, E. Winkler, R. Zimmermann.*

I. Introduction

Thin-walled panels, unstiffened or stiffened, are important elements for light-weight airframes. Frequently they fail by buckling under loads causing compressive or shear stresses. The increasing use of laminated carbon fibre reinforced plastics for structural panels brings about the necessity of establishing procedures for sizing composite panels subjected to such loading conditions.

Depending on dimensions, support conditions and material properties panels may reach their maximum strength at initial buckling, or will fail in the postbuckling regime after having undergone considerable out-of-plane deflections. The analysis of both initial buckling, and postbuckling behaviour, must be based on non-linear kinematics of deformation, i.e. on a geometrically non-linear theory of shells. Uncertainties in defining constitutive laws for composite shells, difficulties in solving the equations of non-linear shell theories and the influence of initial imperfections necessitate careful checking of theoretical results against experimental evidence. Confidence in sizing procedures can only be achieved in this way.

Within GARTEUR[1]) an action group on "Buckling and Postbuckling Behaviour of Composite Panels" was established with the aim of exchanging and improving knowledge on the subject matter with particular emphasis on assessing the capability and reliability of existing analysis tools to predict maximum loads in relation to the test values.

The group selected several panel configurations for which compression or shear test results are available. Various software tools were applied to analyse the buckling and postbuckling behaviour of these panels, and the results were compared and critically evaluated. In this paper the work is summarised and the results obtained are presented. More detailed information is contained in the official report submitted to the authorities of GARTEUR [1].

II. Computational Tools

In the following discussions only the relevant computer programs used by the members of the GARTEUR action group are mentioned. It is obvious that other programs not cited here may be equally well suited for the purpose of analysing the buckling behaviour of composite panels.

Tools for the Computation of Bifurcation Loads

A Formula for Simply Supported Panels.- For the buckling loads of rectangular orthotropic panels subjected to direct membrane stresses, closed-form analytical solutions are available for the classical boundary conditions of simple support. In practice this type of support will hardly occur. Nevertheless, these solutions deserve some attention as they are quick to obtain and well suited to be utilized in optimization studies.

Consider a cylindrical panel of rectangular plan form with length l, width b and radius R. The prebuckling membrane forces N_x, N_y, positive in compression, are constant on the surface of the panel. The lateral membrane force N_y is assumed to be fixed in time while the longitudinal membrane force N_x may take on increasing values. The classical boundary conditions of simple support (SS3 according to a common nomenclature [15]) are assumed to prevail along the four edges.

Given the constitutive law of the laminate in the familiar partitioned matrix form [16]

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix}$$

with
$A_{13} = A_{23} = B_{13} = B_{23} = D_{13} = D_{23} = 0$ (orthotropy) bifurcation loads can be computed as follows:

- Calculate the matrices

$$\mathbf{S} = \mathbf{A}^{-1};\ \mathbf{K} = \mathbf{SB};\ \tilde{\mathbf{D}} = \mathbf{D} - \mathbf{BSB}$$

[1]) GARTEUR = Group for Aeronautical Research and Development in Europe

- Calculate

$$\beta = \frac{m\pi}{l}, \quad \eta = \frac{n\pi}{b},$$

where m, n are the half wave numbers of the buckling mode

$$w = \sin(\beta x) \cdot \sin(\eta y).$$

- Determine

$$G_{11} = S_{22}\beta^4 + 2(S_{12} + \frac{1}{2}S_{33})\beta^2\eta^2 + S_{11}\eta^4$$
$$G_{12} = K_{21}\beta^4 + (K_{11} + K_{22} - 2K_{33})\beta^2\eta^2 + K_{12}\eta^4$$
$$G_{22} = \tilde{D}_{11}\beta^4 + 2(\tilde{D}_{12} + 2\tilde{D}_{33})\beta^2\eta^2 + \tilde{D}_{22}\eta^4 \cdot$$

- Compute the bifurcation loads

$$N_{xc} = \frac{1}{\beta^2}\left(\frac{(G_{12} + \beta^2/R)^2}{G_{11}} + G_{22} - N_y\eta^2\right) \quad (1)$$

- The smallest bifurcation load, with respect to integer variations of the half wave numbers m, n is the buckling load,

$$N_{xb} = \min(N_{xc}) \quad m = 1,2\ldots \quad n = 1,2\ldots \quad (2)$$

The formula for the bifurcation loads, Equ. (1), was derived in refs. [11], [12].

Programs Based on the Finite Strip Method.- For long panels the influence of the boundary conditions at the loaded edges can be neglected in regions away from these edges, and the buckling mode may be assumed to be periodic in the longitudinal direction. The solution of the problem can then be reduced to the solution of a system of ordinary differential equations with the transverse in-plane coordinate as independent variable. Exact analytical solutions are established for segments with constant geometry. As these segments represent longitudinal strips the term *finite strip method* was introduced for that solution procedure. It permits the treatment of longitudinal stiffeners as panel branches (as opposed to discrete beams or beam columns), and variations of stiffness properties in the transverse in-plane direction can easily be accounted for.

Computer programs based on this type of analysis are BUCLASP-2 [22], [23] and VIPASA [27]. The program BOSOR4 [8], if applied to panels, can also be regarded to belong to this class, but it uses a finite difference method rather than analytical solutions.

A special finite strip method is used for the program COMBUC developed in the Fokker company. It has the essential feature that overall buckling following short-wave buckling can be computed by using reduced stiffness properties for buckled skin and/or stringer strip elements. In this program the ordinary differential equations are discretized in FEM fashion to give a computationally efficient solution procedure [20].

Programs for Arbitrary Boundary Conditions.- Finally there are codes taking the prevailing boundary conditions along all the four edges into account. An example is BEOS [18]. The most general panel configuration which can be treated by that program is a sandwich shell of parallelogrammic plan form with an anisotropic core and dissimilar anisotropic faces the thickness of which may be of the order of the core height. Discrete stiffeners parallel to the edges can be incorporated and are treated as beam columns. Nearly all physically meaningful boundary conditions can be taken into account including point support conditions, though elastically restrained edges are not considered.

An option allows calculation of frequencies and mode shapes of natural vibrations about the fundamental state. The subspace iteration method is used to extract a prescribed number of eigenvalues and mode shapes at the lower end of the spectrum [10].

In the analysis underlying the program BUCCAL the buckling mode is expanded into known functions with unknown coefficients, and the solutions are determined by the Rayleigh-Ritz method. A variety of boundary conditions can be satisfied. The program is used for quick analysis of buckling loads of plane and curved panels.

Program for Complete Cylinders.- For the buckling loads of highly curved panels a good approximation may be calculated with a program meant for the analysis of complete cylindrical shells, provided the circumferential wave number of the buckling mode is properly selected. One or several half waves should fit closely into the width of the actual panel. The program BACCUS developed at DFVLR was indeed used in this activity. It is capable of calculating buckling loads of perfect cylinders the walls of which are made of arbitrarily layered composite material with complete anisotropy taken into account. The theory and solution procedure (analytical solution) underlying the program are outlined in ref. [13].

Programs for Non-Linear Analysis

The software packages with non-linear analysis capability used by the members of the action group are either general purpose finite element systems or programs particularly dedicated to non-linear plate and shell analysis.

ADINA [7] and NASTRAN belong to the set of general purpose finite element systems, ADINA being applied in the Fokker company and NASTRAN at MBB. Presently the experience within the action group, on the application of these programs to the postbuckling behaviour of panels, is confined to cases with relatively few degrees of freedom (order 10^2).

At AMD, France, the finite element code NLIRAP was developed especially for rapid non-linear structural analysis. It is the nonlinear branch of ELFINI, a multipurpose finite element based computer code which is closely linked to the CAD-system CATIA.

COMPLAN, established at RAE, is a finite element based code for plane plates. Attention was devoted to formulating the theoretical foundations properly in order to account for constant strain and inextensional bending modes of deflexion. The theoretical foundations of the program were layed down in references [2] to [5].

NOLIN [17] computes, for the same configurations as BEOS, the equilibrium states in the pre- and postbuckling range.

STAGS [6] probably is the best-known code for non-linear shell analysis. The version STAGSC-1 is based on the finite element method. A variety of panel configurations can be dealt with, and fully anisotropic material properties are admitted.

Experimental and Theoretical Results for Selected Panels

Unstiffened Plane Panel

Configuration and Loading.- Among the multitude of specimens tested, a panel designed, analyzed and tested in a joint activity of RAE and BAe was selected for the comparisons to be made. It has a rectangular shape with the dimensions as follows: Width between knife edge supports 180 mm, length between end blocks 458 mm, thickness 3.75 mm.

The panel is made from XAS-914C tapes using 30 plies, each with 0.125 mm in thickness. The stacking is $(0°, -45°, 0°, +45°, 0°)_{s3}$, so the panel contains 60% of 0°-layers and 40% of ± 45°-layers.

The panel was subjected to compression by controlled loading. The loaded and unloaded edges are assumed to be clamped and simply supported, respectively.

Results.- Agreement or disagreement of computational results and of computations and test can most clearly be seen on a plot of load against the out-of-plane displacement at a buckle peak, Figure 1. The programs ADINA, COMPLAN and NOLIN produced almost identical results in the advanced post-buckling range. In the early post-buckling range the results from COMPLAN and NOLIN are still close together and agree remarkably well with the measured values. The results from ADINA are shifted to somewhat lower loads, and the corner at the bifurcation point, between the postbuckling curve and the fundamental path, is rounded off, which obviously is due to assuming too large an initial imperfection.

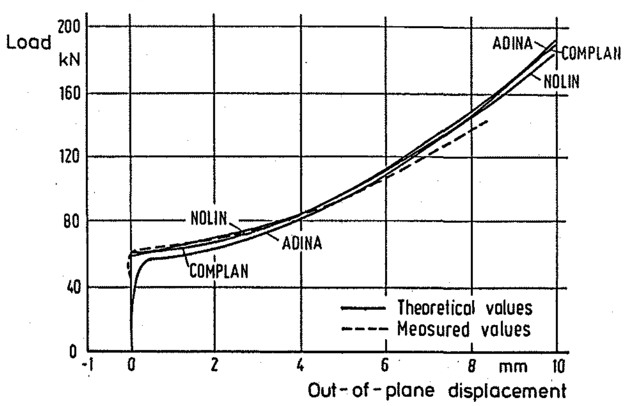

Figure 1. Postbuckling behaviour of the unstiffened plane panel.

The buckling load was determined from the test results by extrapolating records of corresponding strains on opposite sides of the panel. The values predicted by BEOS and COMPLAN differ less than 1.5 % from the test result of 61.5 kN. Both COMPLAN and BEOS yield the correct mode shape with two half-waves along the length and one half-wave across the width.

The failing load was predicted by RAE based on the program COMPLAN and on a simple material failure criterion limiting the fibre strain to 0.00935 (9350 Microstrain). The value obtained was 163.29 kN while the experimental failure load was found to be 167.02 kN. The two values are remarkably close together. However, more similar results would be required before general conclusions can be drawn, with regard to the general applicability of this simple failure criterion.

Blade Stiffened Plane Panels

Configuration and Loading.- Rectangular panels were manufactured by Fokker with the cross-section shown in Figure 2. The loaded ends of the panels were potted in epoxy blocks with a thickness of approximately 20 mm. The free length of the panels between the end blocks was 1060 mm. Two different configurations were designed, typical of a root section and a tip section of a horizontal stabilizer. One panel of each configuration was cut into smaller specimens for short column tests.

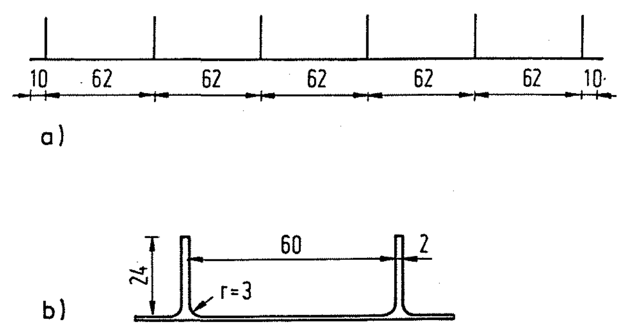

Figure 2. Geometry of blade stiffened plane panels.

The material used was Hexcel F155/T300 unidirectional tape with a lamina thickness of 0.20 mm at a fibre volume content of 54%.

The difference between root and tip configuration is the number of 0° plies in the skin: three for the root section and one for the tip section. The lay-up of the panels is presented in Figure 3 showing that the skin lay-up is slightly different in adjacent skin fields. This is the result of the lay-up procedure in which a + 45° or − 45° layer runs from one stiffener via the skin to the next stiffener, where it changes its sign (cf. Figure 4). In order to keep the *stiffener lay-up* symmetric, every other skin lay-up

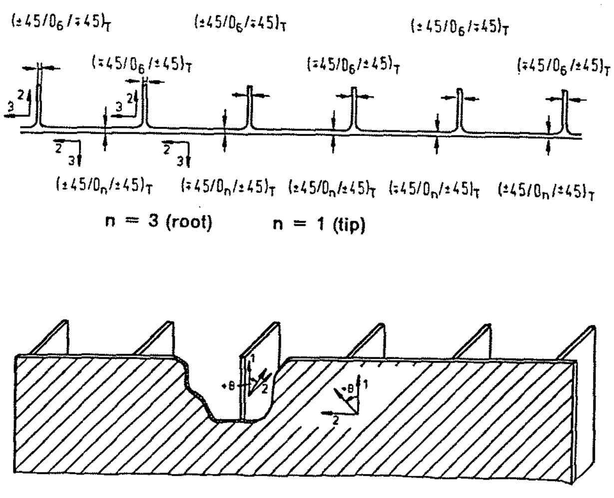

Figure 3. Laminate stacking sequences of blade stiffened panels.

between stiffeners turns out to be non-symmetric. Figure 4 also shows that at the intersection of stiffeners and skin some extra 0°-tape was added.

The panels were loaded in compression by controlled end displacement.

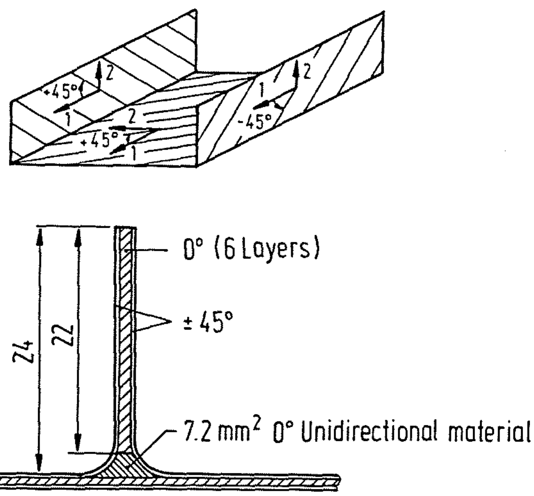

Figure 4. Details of stiffener and skin of blade stiffened panels.

Results for Full Scale Configurations.- In Figure 5 and Figure 6 the test results of blade stiffened plane panels are presented together with selected analytical results obtained with STAGS.

One of the objectives of the research program based on these panels was to establish the effects caused by the coupling terms B_{16}, B_{26}, D_{16}, D_{26} of the stiffness matrix, which are non-zero for these panels, but are usually neglected in the analysis. According to the design of the panels these terms are alternately zero and non-zero in the skin fields between the stringers. In the diagram of load vs. end shortening the curves marked with a plus sign were computed taking the coupling coefficients correctly into account whereas the curves marked by a minus sign were computed using a simplified elasticity law in which the coupling terms are ignored. The diagrams clearly indicate that inclusion of the coupling terms leads to lower computed buckling loads. Neglection yields unconservative results. However, the magnitude of the effect was small for the panels considered.

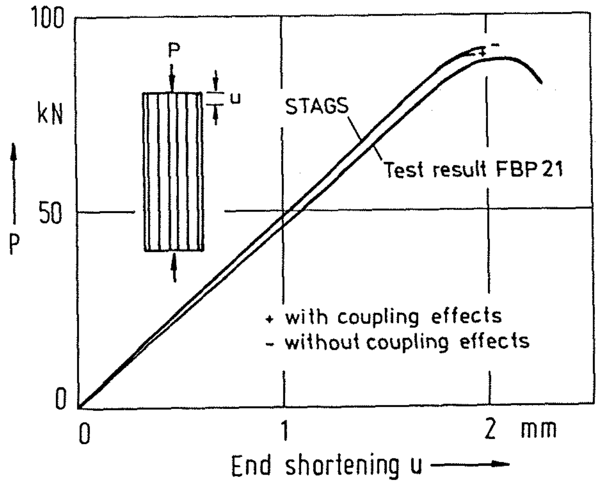

Figure 5. Results from test and analysis for blade stiffened panel, root configuration.

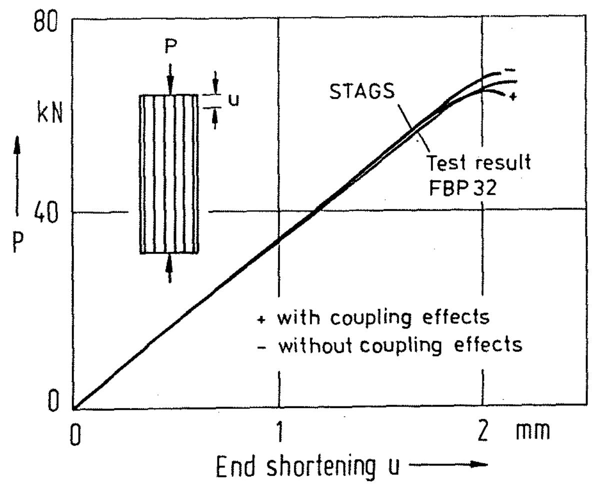

Figure 6. Results from test and analysis for blade stiffened panel, tip configuration.

The computational results from STAGS are in excellent agreement with the test results. The two root panels tested buckled at 87 kN and failed at 88 kN. The buckling load prediction of STAGS was 89 kN. The two tip panels buckled at 62kN and 65kN and failed at 63 kN and 66 kN, respectively. STAGS predicted 62 kN. Excellent agreement was also stated for the deformation modes [26]. They should be computed without neglecting the coupling terms in order to be in accordance with tests.

There is no doubt that STAGS is an adequate analysis tool for problems like this, but its computational expense is high. Therefore, the programs BEOS, COMBUC, BUCLASP and BOSOR4 were also applied to predict the buckling loads of the blade stiffened panels. The results are given in Table 1.

BEOS yields higher buckling loads than the other codes, although it admits a more realistic modelling of boundary conditions at the loaded edges. However, the stiffeners are treated as beam columns, and weakening of their effective stiffness due to lateral buckling is not taken into account. This inability to deal adequately with stiffeners tending to lateral buckling might be responsible for the unconservative results in this specific example.

The distinguishing feature of COMBUC is its ability to reduce automatically the effective stiffness of elements buckled locally. It is this feature which enables COMBUC to predict very closely the load carrying capacity of panels like the ones under discussion, i.e. of panels that deform into a short wave buckling pattern and subsequently fail in an overall mode. Corresponding results are cited on the line "overall buckling, reduced stiffness". A slightly more detailed discussion with regard to the behaviour of the test panels compared to the COMBUC results was given by J.H. van der Sloot [21].

		BEOS (DFVLR)	COMBUC (Fokker)	BUCLASP (NLR)	BOSOR4 (DFVLR)
Root panel FBP2	short wave buckling	120.1	99.3	104	96.9
	overall buckling	111.7	89.0	101	93.8
	overall buckling reduced stiffness		—	—	—
Tip panel FBP3	short wave buckling	83.4	68.0	73	58.9
	overall buckling	87.4	76.0	81	74.2
	overall buckling reduced stiffness	74.9	63.0	—	—

Table 1. Analytical buckling loads of blade stiffened panels values in kN.

Of the programs BUCLASP and BOSOR4 the latter yields lower buckling loads. Both programs predicted bifurcation loads for overall buckling which are higher than the experimental values, since no stiffness reduction accounting for short-wave buckling was applied.

More information on the tests and the computations with BUCLASP and STAGS can be found in J.F.M. Wiggenraad's papers [24], [26].

Results for Short Columns.- The short columns cut from one of the root panels had the full width of 330 mm, i.e. they comprised six stiffeners, but their length was only 305 mm (compared to 1060 mm of the full size panels). The ones cut from the tip configuration were 290 mm long and only 142 mm wide comprising three blades. For these panels the bifurcation loads corresponding to short-wave and overall buckling modes are much farther apart than for the full scale panels.

In Figure 7 for a short root section panel, the test recording of load vs. end shortening is reproduced and compared to analytical results from STAGS [26]. The buckling loads can better be read from plots of load vs. lateral displacement (not reproduced here). They are presented in Table 2 for root and tip type panels and are compared with computational buckling loads obtained with STAGS, BUCLASP and BEOS. No failure loads were computed but the experimental failure loads are also presented in the table. They exceed the buckling loads by more than 80%. It is obvious that means for an appropriate failure analysis for that type of panels is urgently needed.

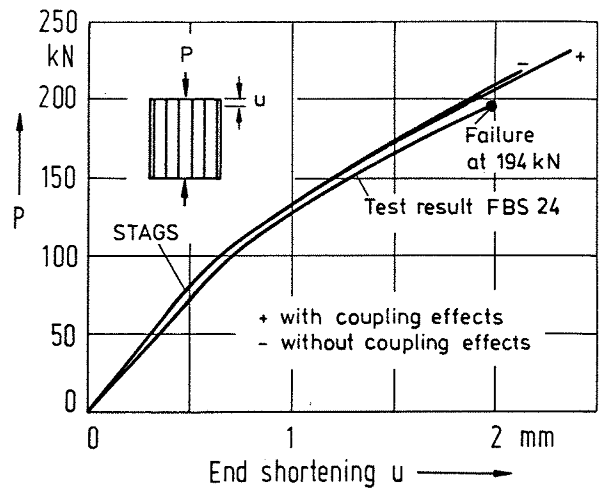

Figure 7. Results from test and analysis, blade stiffened short panel, root configuration.

Regardless of the inability of BUCLASP and BEOS to cope with stiffness terms which change sign in every other skin field the computed buckling loads are very close to the ones found in the experiments.

The ratio *(width between stiffeners)/(panel thickness)* for the two different panels amounts to 44 and 62, respectively. The good agreement of experimental and analytical buckling loads indicates that for ratios of that order the usual Navier-type plate theories which neglect the influence of transverse shear flexibility are sufficiently accurate.

Panel	Test		Analytical buckling loads		
	Buckling load	Maximum load	STAGS (NLR)	BUCLASP (NLR)	BEOS (DFVLR)
Root panel FBS23	100	181	102	104	107
Root panel FBS24	100	194	102	104	107
Tip panel FBS31	38	75	–	36	36
Tip panel FBS32	40	72	–	36	36

Table 2. Results for blade stiffened short panels values in kN

Stiffened Plane Shear Web

Configuration and Loading.- From a variety of shear webs with different geometries tested at MBB [14] one without holes was selected for the purposes of the action group.

The planform is nearly square. Three blade stiffeners are arranged parallel to the shorter edges; their height is 25 mm. The skin and the stiffeners are 2.04 mm and 3.04 mm thick, respectively. A sketch of the web is shown in Figure 8.

The panels were fabricated from T300 fibres in a Hexcel epoxy matrix. Prepregs with unidirectional fibre arrangement yielding .25 mm thick layers, and with bidirectional fabric yielding .34 mm thick layers, were used in the fabrication process. The average fibre content achieved was 60% by volume.

The panel was clamped into a frame. It was loaded in shear by applying a tensile force across one diagonal of that frame (cf. Figure 8).

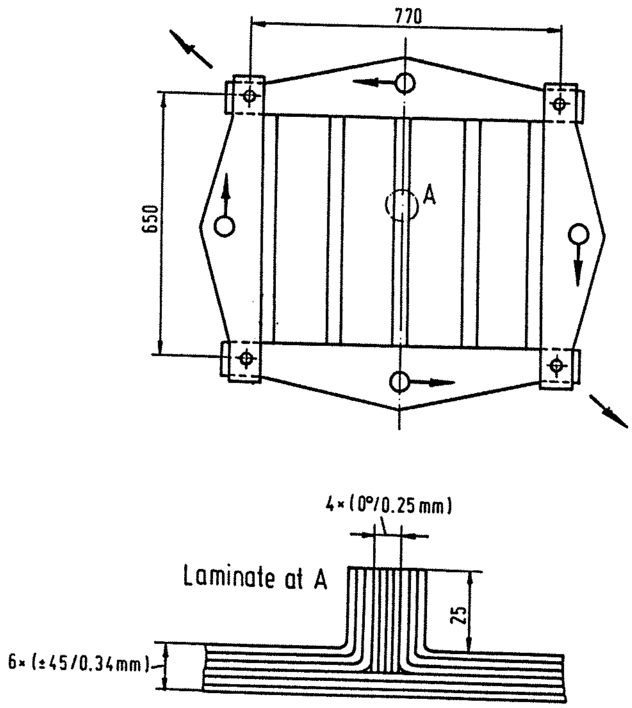

Figure 8. **Sketch of shear web and loading frame.**

Results.- When the tensile force applied across the diagonal of the load frame was as high as 120 kN the first waves were observed. Strain gauge recordings from opposite sides of the panel indicated, however, that buckling actually had started earlier, at a load of approximately 95 kN. Failure occurred at 300 kN when the skin separated from the stiffeners [14].

As the configuration comprising the test frame and the stiffened panel is statically indeterminate a relation between the applied force and the shear flow in the panel had to be established. This was done both by a calculation using NASTRAN, and by measuring the shear strain. Calculations of the buckling load were performed at MBB with NASTRAN SOL 65 and BEOS. Another run of BEOS was performed at DFVLR. The results for the critical shear flow are compiled in the following list.

Experiment	N_{xy} = 65.08 N/mm,
NASTRAN SOL 65	N_{xy} = 78.10 N/mm,
BEOS (MBB)	N_{xy} = 76.73 N/mm,
BEOS (DFVLR)	N_{xy} = 87.07 N/mm.

It can be seen that all theoretical results are above the experimental value read from the strain gauges. The high buckling load obtained at DFVLR is probably due to assuming too small an unsupported panel area, viz. the window of the load frame.

Curved Panels

Configuration and Loading.- Reference will be made to 6 highly curved unstiffened rectangular panels, A1 to A6. Fabrication and testing were performed by BAe and RAE. The panels have the same overall dimensions, viz. width between supports (arc length) 420 mm, length 540 mm, radius of curvature 250 mm, thickness 2 mm. They differ with respect to their lay-ups.

The panels are built up from 16 plies of XAS-914C prepregs, each ply having a thickness of .125 mm. Their angular lay-ups are illustrated in Table 3

No	Lay-up	Description
A1	(+, -, -, +, 0, 0, 0, 0)$_s$	blocked lay-up, 50% 0°, 50% ±45°
A2	(+, -, +, -, +, -, +, -)$_s$	distributed, 100% ±45°
A3	(+, 0, -, 0, +, 0, -, 0)$_s$	distributed, 50% 0°, 50% ±45°
A4	(+, 0, 0, 0, 0, -, 0, 0)$_s$	75% 0°, 25% ±45°
A5	(+, 90, 0, 0, 0, -, 0, 0)$_s$	62.5% 0°, 25% ±45°, 12.5% 90°
A6	(+, -, +, 0, -, +, 0, -)$_s$	25% 0°, 75% ±45°

Table 3. **Lay-up of highly curved panels.**

The specimens are subjected to compression. The curved edges were loaded and straight edges were knife-supported. Hence the curved edges may be assumed to be clamped, and the straight edges may be considered simply supported.

Results for Bifurcation Buckling Loads.- The buckling loads of the curved panels were computed with several programs. In Table 4 the results are presented and compared with experimental values.

Panel	A1	A2	A3	A4	A5	A6
Test	134	79	134	117	131	130
STAGS	149.2	123.5	173.2	124.9	142.8	155.8
BEOS	149.8	128.5	179.2	131.2	150.7	159.2
VIPASA	141.0	137.4	130.2	105.2	118.8	151.2
BUCCAL	169.1	106.7	149.0	125.8	148.0	150.1
Equ.(1)	147.3	125.7	176.7	163.0	176.4	156.6
BOSOR	150.6	120.3	171.1	158.1	176.1	159.7
BACCUS	145.3	122.3	173.8	126.2	144.6	153.8

Table 4. **Curved panels under axial compression: Bifurcation buckling loads (in kN).**

The results of the calculation are presented from top to bottom in the order of decreasing computational effort, except for the last two rows. For the following discussion the results obtained with STAGSC-1 are considered as the best ones, since in that program all effects, viz. boundary conditions, non-uniform fundamental state due to edge effects and bending-twisting coupling are correctly taken into account. For ease of comparison the results of the other programs are normalized with the STAGSC-1 results in Table 5.

Panel	A1	A2	A3	A4	A5	A6
BEOS	1.004	1.040	1.036	1.050	1.055	1.022
VIPASA	0.945	1.113	0.752	0.842	0.818	0.970
BUCCAL	1.133	0.864	0.860	1.007	1.036	0.963
Equ.(1)	0.987	1.018	1.020	1.305	1.235	1.005
BOSOR	1.009	0.974	0.988	1.266	1.233	1.025
BACCUS	0.974	0.990	1.003	1.010	1.013	0.987
Test	0.898	0.640	0.774	0.937	0.917	0.834

Table 5. Curved panels under axial compression: Bifurcation buckling loads (in kN) normalized with STAGSC-1 results.

BEOS yields consistently higher buckling loads than STAGSC-1, but the differences keep below 5.5%. They reflect the influence of prebuckling edge effects which were not taken into account in BEOS.

VIPASA mostly yields lower results than STAGSC-1. It might be supposed that the treatment of boundary conditions at the curved edge in the finite strip method is responsible for the deviations, but part of them are certainly due to modelling the curved panel by plane strips.

The results of BUCCAL are fairly close to those of STAGSC-1 and BEOS except for panels A2 and A3, for which BUCCAL produces substantially lower buckling loads. However, lower buckling loads are not expected to be obtained with BUCCAL, since that program is based on the Rayleigh-Ritz method and assumed global trial functions. It should yield, therefore, upper limits to the buckling loads.

Equ. (1) was evaluated with a pocket calculator. Except for panels A4 and A5, the results are surprisingly good in spite of the fact that in the derivation of that equation simple support was assumed for both the loaded and unloaded edges, and bending-twisting coupling was neglected. Most probably the discrepancies for shells A4 and A5 are due to the neglection of bending-twisting coupling terms.

The program BEOS was used to calculate the buckling modes. They reveal a tendency of the panels A1, A2 and A6 to buckle with one wave across the width and many short ripples along the panel length, cf. Figure 9, example A1. For the panels A3, A4 and A5 the buckling mode appears similar but the ripples are skew, cf. Figure 9, example A3. The modes suggest that the boundary conditions at the unloaded longitudinal edges

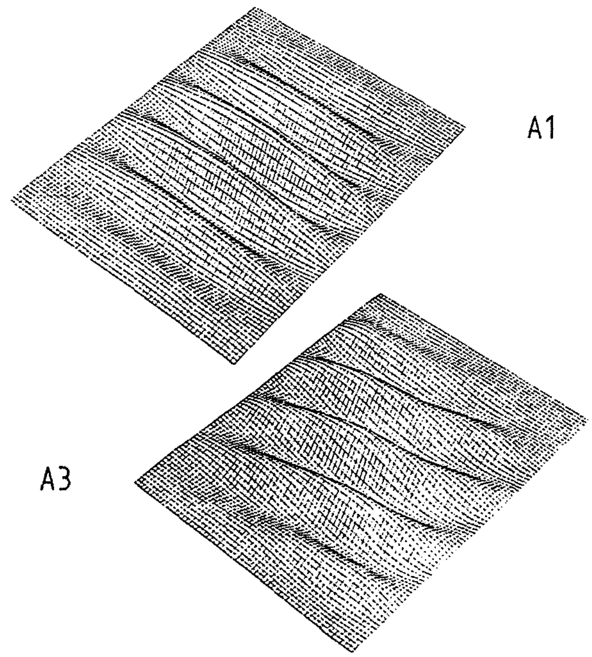

Figure 9. Computed buckling modes of highly curved panels.

might have little influence. A complete cylinder of the same wall construction and radius should therefore buckle at nearly the same membrane force. The results obtained with the programs BOSOR4 and BACCUS confirm this supposition. Moreover the differences between their predictions for panels A4 and A5 clearly point upon a strong influence of bending-twisting coupling that is neglected in BOSOR4, but taken into account in BACCUS.

The maximal loads carried by the panels are considered as experimental buckling loads. They are put into perspective with the buckling loads obtained by STAGSC-1 in the last row of Table 5.

All experimental buckling loads stay below the theoretical ones. If we disregard the very low buckling load of panel A2 the ratio between experimental maximum loads and calculated bifurcation loads ranges from 0.774 to 0.937 with an average of 0.87. Compared to corresponding ratios found with complete cylinders they are well within the range of composite shells, but considerably above the average if we include isotropic cylinders under axial compression, cf. Fig. 3.18 of [9].

Results for the Postbuckling Behaviour.- For the analysis of the postbuckling behaviour the program STAGSC-1 was applied [19]. Figure 10 the curves of load vs. end isplacement for panel A1. The postbuckled equilibrium states at loads below the buckling load can be seen. Their existence is responsible for the fact that the theoretical buckling load is not attained in the tests.
Theoretical analyses using measured imperfection shapes (lateral bulges) deliberately introduced into the test panels did not achieve close agreement of computed load maxima with test results, but non-linear analysis using small imperfections in the form of the first or second eigenmodes gave closer agreement.

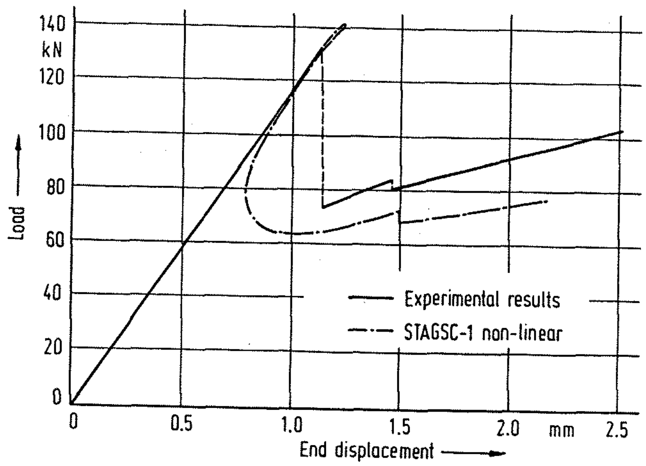

Figure 10. Postbuckling behaviour of panel A1.

IV. Conclusions

The primary aim of the studies was to find out whether certain computer programs are able to predict the actual behaviour of composite panels adequately. The results admit the general conclusion that the state of knowledge on the buckling and postbuckling behaviour of composite panels has achieved a standard that is comparable to that for metal structures.

The panels considered had planform dimensions which were at least an order of magnitude larger than the panel thicknesses, and for these structures Kirchhoff-Love-type plate and shell theories, together with the classical lamination theory for defining the constitutive law, were found adequate.

Most of the programs were used to predict bifurcation buckling loads, although bifurcation had not really been observed in any of the tests as a clearly distinguishable event. It could be shown that the calculated buckling loads give good indication of the load level for accelerated growth of out-of-plane displacements.

For short stiffened panels it was found, as expected, that primary buckling normally occurs in a short-wave mode that merely reduces the stiffness in the buckled areas, but does not limit the load carrying capacity. The corresponding buckling loads were adequately predicted. There was strong evidence that blade stiffeners should be modelled as shells branching off the skin. Modelling them as beams does not take into account their tendency towards lateral buckling, and yields too high buckling loads.

One of the programs, COMBUC, allows to reduce the stiffness properties of locally buckled parts, and to proceed with the calculation until buckling into an overall mode is found. The corresponding buckling loads showed good agreement with the experimental failure loads in the examples considered.

For highly curved cylindrical panels the load carrying capacity was found to be lower than the calculated bifurcation loads. The observed buckling behaviour resembles the known behaviour of complete cylinders. It was found that the reduction factors that have to be applied on calculated buckling loads in order to arrive at the failure loads of highly curved composite panels, are higher than the factors usually recommended for isotropic shells [25], i.e. the composite shells show a more favourable behaviour in this respect.

Short plane panels, unstiffened or stiffened, exhibit considerable ability to carry loads in the postbuckled states. Some of the computer programs were applied for analysing these highly nonlinear states. For the highly curved panels the postbuckling behaviour was studied, too, although the postbuckled states in that case are of no importance for practical applications. The results agreed fairly well with the measured behaviour.

No failure criterion can presently be offered for stiffened panels. Most of them will fail in the postbuckling range by skin-stringer separation. It was clearly indicated that the problem of utmost importance is to find a criterion for predicting this type of failure. Therefore, more test results on stiffened plane and curved panels are desirable.

The fact that skin-stringer separation is the predominant failure mode of stiffened composite panels suggests that it might be worth while to reconsider the application of sandwich construction.

The analysis of the postbuckling behaviour requires a large amount of computation time and cost. This feature prohibits its application in the early design phase or in optimization cycles. Hence, it is recommended for the future to pay more attention to the development of approximate, but quick analysis methods and tools.

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