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DYNAMIC STABILITY OF TRANSVERSELY ISOTROPIC VISCOELASTIC FLAT PLATES

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Abstract

This paper deals with a dynamic stability analysis of transversely-isotropic viscoelastic flat plates subjected to in-plane bi-axial edge load systems. In deriving the associated governing equations a Boltzmann hereditary constitutive law was used and in addition, transverse shear deformation, transverse normal stress and rotatory inertia effects were incorporated. The integro-differential equations governing the stability of simply-supported flat plates is solved by using the integral transform technique. Its solution concerns the determination of the critical in-plane edge loads yielding with the asymptotic instability of flat plates. While studying this problem the general dynamic stability solutions are compared with the ones based on the first order transverse shear deformation theory and with their quasi-static counterparts. Numerical applications are presented and pertinent conclusions are formulated.

I. Introduction

Fiber-reinforced composites have gained increasing attention in recent years. This attention is due to their widespread use in the design of primary and secondary load bearing structural members, where the requirement of high strength/stiffness to weight ratio is of a vital importance. Such applications include, e.g., the high-speed aircraft and aerospace structures, rocket engines, turbine blades, etc. Due to the high temperature gradients experienced by these structures, their constituent materials exhibit time-dependent properties which could be modelled by a linear (or nonlinear) constitutive law.

In addition, the composite material structures exhibit a weak rigidity in transverse shear which requires the incorporation of transverse shear deformation effects.

The stability of transversely-isotropic viscoelastic panels undergoing cylindrical bending was analyzed by Malmeister et al. [1] for the case when the composite exhibits viscoelastic properties in transverse shear direction only. Wilson and Vinson [2] analyse the stability of rectangular, viscoelastic, orthotropic plates subjected to biaxial compression. In their analysis, the equations governing the stability are obtained by using the quasi-elastic approximation, which overlooks the hereditary material behavior. Sims [3] performs a similar quasi-elastic analysis of the problem thereby implying an instantaneous time-dependent material behavior as opposed to the actual hereditary constitutive law.

In this study a method of analyzing the linearized dynamic stability of viscoelastic transversely-isotropic plates subjected to biaxially compressive load systems is developed. As is well known, due to their exotic properties, the transversely-isotropic materials (as e.g., the pyrolitic-graphite one) are used in components of various space vehicles for thermal protection purposes. An exact dynamic approach is used throughout the treatment of the stability problem. The material behavior is modelled through a 3-D linearly viscoelastic, hereditary, constitutive law (i.e., the Boltzmann hereditary law). Effects of transverse shear deformations (which are highly pronounced for materials exhibiting high degrees of anisotropy and/or for non-thin plates) have been incorporated into the analysis. Emphasis is also given to the effect of the transverse normal stress, σ_{33}, which was overlooked by the previous investigators.

By using the elastic-viscoelastic correspondence principle, the equations governing the stability of viscoelastic transversely-isotropic flat panels are derived starting with their elastic counterparts (considered in [4]). As in the elastic case [4], the equations governing the dynamic stability of viscoelastic transversely-isotropic plates may be recast into two independent ones, i.e., one defining the basic state of stress, and the other one describing the boundary-layer solution. The integro-differential equations governing the stability of plates is used in the Laplace transformed space, this yielding the characteristic equation for the time dependent part of the transverse displacement function. The asymptotic stability behavior of the plate is studied here as an eigenvalue problem.

The analysis is performed in the framework of a third order transverse shear deformation theory (TSDT) and of its first order counterpart (FSDT). Then, employing the single equation (representing the interior solution discussed above) it has been shown that for an isotropic plate, the results obtained by solving the exact system of three coupled equations fully agree with the solution obtained via the single equation. This result constitutes an extension for the viscoelastic case of its elastic counterpart obtained in [4]. Comparison studies between the TSDT, FSDT and the classical Kirchhoff theory of plates are also presented.

II. Preliminaries

The case of a flat plate of uniform thickness h is considered. By $S_{\pm}$ we denote the upper and lower bounding planes of the plate, symmetri-

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cally located with respect to its mid-plane σ.

The points of the 3-D space of the plate will be referred to a rectangular Cartesian system of coordinates x_i, where $x_\alpha (\alpha = 1,2)$ denote the in-plane coordinates, x_3 being the coordinate normal to the plane $x_3 = 0$. Throughout the analysis (unless otherwise stated) the Einsteinian summation convention is employed where the Greek indices range from 1 to 2, while the Latin indices range from 1 to 3.

III. Basic Equations

Geometric Equations

The higher-order theory of plates is developed by using the following representation of the displacement field across the thickness of the plate [4]:

$$V_\alpha[x_\omega, x_3, t] = \sum_{n=0}^{N} (x_3)^n \overset{(n)}{V}_\alpha[x_\omega, t] \tag{1}$$

$$V_3[x_\alpha, x_3, t] = \sum_{n=0}^{R} (x_3)^n \overset{(n)}{V}_\alpha[x_\omega, t] \tag{2}$$

The numbers N and R denote two natural numbers identifying the truncation levels in the displacement expansion. At this point it is worthwhile to note that in Eqs. (1) and (2) the terms corresponding to the stretching state of stress are $\overset{(2r)}{V}_\alpha$, $\overset{(2r+1)}{V}_3$ and those corresponding to the bending state are $\overset{(2r+1)}{V}_\alpha$ and $\overset{(2r)}{V}_3$.

For a third-order bending theory which retains the assumption of the inextensibility of the transverse normal elements, the following representation for the 3-D displacement components may be postulated

$$V_\alpha = x_3 \overset{(1)}{V}_\alpha + (x_3)^3 \overset{(3)}{V}_\alpha \tag{3}$$

$$V_3 = \overset{(0)}{V}_3 \tag{4}$$

where

$$\overset{(n)}{V}_i = \overset{(n)}{V}_i [x_\alpha, t] \tag{5}$$

It is worthwhile to note at this stage, that the above representation of V_α allows one to fulfill the static boundary conditions on the external bounding planes. The linear strain tensor is written as,

$$e_{ij}[x_\alpha, x_3, t] = \frac{1}{2} (v_{i,j} + V_{j,i}) \tag{6}$$

Introducing the displacement expansions (3) and (4) into (6) we obtain for the strain components the expressions

$$2e_{\alpha\beta} = x_3(\overset{(1)}{V}_{\alpha,\beta} + \overset{(1)}{V}_{\beta,\alpha}) + (x_3)^3(\overset{(3)}{V}_{\alpha,\beta} + \overset{(3)}{V}_{\beta,\alpha})$$

$$2e_{\alpha 3} = \overset{(1)}{V}_\alpha + 3(x_3)^2 \overset{(3)}{V}_\alpha + \overset{(0)}{V}_{3,\alpha} \tag{7}$$

$$e_{33} = 0$$

Constitutive Equations

Employment of the superposition principle for a linear viscoelastic material results in the constitutive law involving the Stieltjes convolution. This relates the time dependent stresses to the time varying input strains (or vice-versa). The derivation considers the input strains (or stresses) to be a superposition of Heaviside functions with various step sizes and in addition that the behavior is causal.

The resulting constitutive law for a linearly viscoelastic anisotropic material in a 3-D state of stress is (see Malmeister [1]):

$$\sigma_{ij}[t] = e_{mn}[t]E_{ijmn}[0] + \int_0^t \dot{E}_{ijmn}[s]e_{mn}[t - s]ds \tag{8}$$

where the first term in (8) represents the elastic part of the constitutive law. Taking the Laplace transform with respect to time of Eq. (8) we obtain

$$\bar{\sigma}_{ij} = s\bar{E}_{ijmn}\bar{e}_{mn} \tag{9}$$

where an overbar affecting a quantity denotes its Laplace transform (L.T.) with s as LT variable. The elastic counterpart of the constitutive equations (8) may be expressed in the following convenient form:

$$\sigma_{\alpha\beta} = \tilde{E}_{\alpha\beta\omega\pi}e_{\omega\pi} + \delta_A \hat{E}_{\alpha\beta 33}\sigma_{33}$$
$$\sigma_{\omega 3} = 2E_{\omega 3\lambda 3}e_{\lambda 3} \tag{10}$$

where, (see e.g., Librescu [4]):

$$\tilde{E}_{\alpha\beta\omega\pi} = E_{\alpha\beta\omega\pi} - \frac{E_{\alpha\beta 33}E_{33\omega\pi}}{E_{3333}}$$
$$\hat{E}_{\alpha\beta 33} = \frac{E_{\alpha\beta 33}}{E_{3333}} \tag{11}$$

In Eq. (10) δ_A is a tracer identifying the presence of σ_{33}. It takes the values 0 or 1, according to whether this influence is ignored or included.

Equations of Motion

The equations of motion for a 3-D linear elastic continuum are as follows:

$$\sigma_{\alpha\beta,\beta} + \sigma_{\alpha 3,3} + = \rho\ddot{V}_\alpha$$
$$\sigma 13,1 + \sigma 23,2 + \sigma 33,3 + = \rho\ddot{V}3 \tag{12}$$

where ρ is the mass density of the medium while the overdots denote time derivatives.

The stability problem when formulated in terms of displacement may be reduced to a system of equations in three unknown quantities, $\overset{(1)}{V}_\alpha$, and $\overset{(0)}{V}_3$. In order to obtain the governing equations in terms of these displacement quantities three macroscopic equations of motion are needed. To this end, following [4], we consider the moment of order one of the first two equations of motion $(12)_1$ and the moment of order zero of the third equation $(12)_2$.

The moment of order one of $(12)_1$ yields:

$$\overset{(1)}{L}_{\alpha\beta,\beta} - \overset{(0)}{L}_{\alpha 3} = \delta_C \overset{(1)}{f}_\alpha \tag{13}$$

where the general definitions of moment resultants and force resultants are given by:

$$\overset{(n)}{L}_{\alpha\beta}[x_1,x_2,t] = \int_{-h/2}^{+h/2} \sigma_{\alpha\beta}(x_3)^n dx_3$$

$$\overset{(n)}{L}_{\alpha 3}[x_1,x_2,t] = \int_{-h/2}^{+h/2} \sigma_{\alpha 3}(x_3)^n dx_3$$

$$\overset{(n)}{f}_\alpha[x_1,x_2,t] = \int_{-h/2}^{+h/2} \rho \ddot{V}_\alpha (x_3)^n dx_3 \tag{14}$$

In Eq. (13) δ_C is a tracer which identifies the presence or absence of rotary inertia terms (i.e., $\overset{(n)}{f}_\alpha$) by taking values 1 and 0, respectively. In order to represent $\overset{(1)}{L}_{\alpha\beta}$ and $\overset{(0)}{L}_{\alpha 3}$ in terms of $\overset{(1)}{V}_\alpha$ and $\overset{(0)}{V}_3$ we use the constitutive law (i.e., Eqs. (10)) and strain-displacement relations (given by Eqs. (7)) into the above expressions for the stress couples, $\overset{(1)}{L}_{\alpha\beta}$, and stress resultants, $\overset{(0)}{L}_{\alpha 3}$. In addition, having in view the expression of σ_{33} obtained through the integration over the segment $[0,x_3)$ of the third equation of motion $(12)_2$ (see [4]) and by determining the expression of $\overset{(3)}{V}_\alpha$ in terms of $\overset{(1)}{V}_\alpha$ and $\overset{(0)}{V}_3$, i.e., $\overset{(3)}{V}_\alpha = -\frac{4}{3h^2}(\overset{(0)}{V}_{3,\alpha} + \overset{(1)}{V}_\alpha)$ (obtained through the fulfillment of static conditions on the bounding planes $x_3 = \pm h/2$ (see [7])), we derive the governing equations in terms of the basic variables $\overset{(1)}{V}_\alpha$ and $\overset{(0)}{V}_3$. For the case of transversely-isotropic plates (the plane of isotropy at each point being assumed parallel to the mid-plane of the plate) the pertinent governing equations may be obtained by converting the elastic coefficients for the anisotropic plate (appearing in Eqs. (10)) to the case of a transversely-isotropic one. This convertion may be achieved by using the relations presented in [4], and thus, we obtain:

$$- \frac{E}{1-\mu^2} \overset{(0)}{V}_{3,\alpha\alpha\beta} + 2\frac{E(\mu-1)}{(1-\mu^2)}[\overset{(1)}{V}_{\omega,\omega\beta} - \overset{(1)}{V}_{\beta,\rho\rho}]$$
$$+ \frac{4E}{1-\mu^2}\overset{(1)}{V}_{\omega,\omega\beta} - \frac{40}{h^2}G'\overset{(1)}{V}_\beta - 4\delta_A \frac{\mu' EG'}{E'(1-\mu)}$$
$$\times [\overset{(0)}{V}_{3,\lambda\lambda\beta} + \overset{(1)}{V}_{\lambda,\beta\lambda}] - \frac{40}{h^2}G'\overset{(0)}{V}_{3,\beta}$$
$$+ \delta_B \rho \frac{5E\mu'}{E'(1-\mu)} \overset{(\ddot{0})}{V}_{3,\beta} - \overset{(\ddot{0})}{V}_{3,\beta} + \frac{1}{hG'}\overset{(\ddot{1})}{p}_\beta] = 0 \tag{15}$$

In Eq. (15), E, μ, G ($\equiv E/2(1+\mu)$) and E', μ' G', denote the Young's modulus, Poisson's ratio and shear modulus, corresponding to the plane of isotropy and to the plane normal to the isotropy plane, respectively, while, the tracer δ_B identifies the dynamic effect of σ_{33}. Upon considering the zeroth order moment of the third equation of motion (i.e., Eq. $(12)_2$), we obtain:

$$\overset{(0)}{L}_{\alpha 3,\alpha} + \overset{(0)}{p}_3 - \delta_D \rho h \overset{(\ddot{0})}{V}_3 = 0 \tag{16}$$

where the general definition of the transverse force resultant $\overset{(n)}{p}_3$ is

$$\overset{(n)}{p}_3[x_1,x_2,t] = (\sigma_{33}(x_3)^n)\Big|_{-h/2}^{+h/2}$$

while the tracer δ_D identifies the effect of the transverse inertia term. Employment of the constitutive law and strain-displacement equations for the force resultants $\overset{(0)}{L}_{\alpha 3}$ and convertion of the resulting governing equation to the case of a transversely-isotropic medium, yields:

$$\overset{(1)}{V}_{\omega,\omega} + \overset{(0)}{V}_{3,\omega\omega} + \frac{3}{2hG'}\overset{(0)}{p}_3 - \delta_D \frac{3}{2}\frac{\rho}{G'}\overset{(\ddot{0})}{V}_3 = 0 \tag{17}$$

Equations (15) and (17) are the requested three governing equations of motion in bending of an elastic, transversely-isotropic, flat plate (TSDT).

The counterparts of Eqs. (15) and (17) for the FSDT may be obtained by postulating the following representation of the displacement field:

$$\overset{}{V}_\alpha = x_3 \overset{(1)}{V}_\alpha \; ; \; V_3 = \overset{(0)}{V}_3 \tag{18}$$

In addition, disregarding the influence of σ_{33} in the constitutive equations (10) and making use of the general procedure outlined for the TSDT, we obtain the following governing equations for the bending theory of elastic, transversely-isotropic flat-plates [4]:

$$\frac{h^3}{12} G[\overset{(1)}{V}_{\beta,\mu\mu} - \overset{(1)}{V}_{\mu,\mu\beta}] + \frac{h^3}{12} \frac{2G}{1 - \mu} \overset{(1)}{V}_{\mu,\mu\beta}$$

$$- hK^2G'[\overset{(1)}{V}_\beta + \overset{(0)}{V}_{3,\beta}] - \delta_c m_1 \overset{(\ddot{1})}{V}_\beta = 0 \tag{19}$$

$$\overset{(1)}{V}_{\beta,\beta} + \overset{(0)}{V}_{3,\beta\beta} - \delta_D \frac{\rho}{K^2G'} \overset{(\ddot{0})}{V}_3 + \frac{1}{K^2G'h} \overset{(0)}{p}_3 = 0 \tag{20}$$

In Eqs. (19) and (20), K^2 denotes the transverse shear correction factor which is associated with the FSDT while $m_1 = \rho h^3/12$; $m_o = \rho h$.

<u>Alternative Representation of the Governing Equations (19,20) for the FSDT Theory</u>

For the case of a transversely-isotropic medium, it may readily be shown that by introducing a suitable potential function $\phi[x_1,x_2,t]$, in which terms the displacement components $\overset{(1)}{V}_\alpha$ are expressed, the Eqs. (19,20) may be reduced exactly to the form in which the two states of stress (viz, the interior solution and the boundary layer effect), appear in a decoupled form (see [4-6] for details) follows.

$$- \frac{h^2}{12} \frac{G}{K^2G'} \phi_{,\lambda\lambda} + \phi + \delta_c \frac{m_1}{hK^2G'} \ddot{\phi} = 0 \tag{21}$$

$$- \frac{h^3}{12} \frac{E}{1 - \mu^2} \overset{(0)}{V}_{3,\mu\mu\beta\beta} + \frac{h^2}{12} \frac{E}{K^2G'(1 - \mu^2)}$$

$$\times (\delta_D m_o \overset{(\ddot{0})}{V}_{3,\beta\beta} - \overset{(0)}{p}_{3,\beta\beta}) + \delta_c m_1 \overset{(\ddot{0})}{V}_{3,\beta\beta} + \overset{(0)}{p}_3$$

$$+ \frac{\delta_c m_1}{hK^2G'} \overset{(\ddot{0})}{p}_3 - \delta_D m_o \overset{(\ddot{0})}{V}_3 - \frac{1}{hK^2G'} \delta_c \delta_D m_1 m_o \overset{(\ddddot{0})}{V}_3 = 0 \tag{22}$$

Thus, the equations governing the motion of elastic, transversely-isotropic plates can be recast into two independent equations i.e., one governing the basic state of stress, i.e., the <u>interior solution</u> (22) and the latter one <u>governing the boundary layer solution</u> (21). Similarly we may obtain the counterparts of (21) and (22) for the TSDT. Here it is worthwhile to note that the de-coupled system of equations for the FSDT coincides with that for the TSDT when the transverse shear correction factor $K^2 \to \frac{5}{6}$ in the FSDT and the transverse normal stress is neglected in the TSDT (i.e., $\delta_A = \delta_B = 0$).

IV. Derivation of Equations Governing the Stability of Viscoelastic Transversely-Isotropic Flat Plates

The equations governing the stability of flat plates may be derived by starting with the equations of motion of a continuum undergoing finite deformations (see Chandiramani [10]). However, as an alternative procedure the stability equations may be formally obtained by replacing in the previous governing equations $\overset{(0)}{p}_3$ with $\overset{(0)}{p}_3 + \overset{(0)}{L}_{11} \overset{(0)}{V}_{3,11} + \overset{(0)}{L}_{22} \overset{(0)}{V}_{3,22} + 2 \overset{(0)}{L}_{12} \overset{(0)}{V}_{3,12}$ (see [5]), where $\overset{(0)}{L}_{11}$, $\overset{(0)}{L}_{22}$ and $\overset{(0)}{L}_{12}$ play the role of uniform (in space but depending possibly on time) edge loads, considered positive in extension. Furthermore, the viscoelastic counterparts of the elastic stability equations can be obtained by employing the elastic-viscoelastic correspondence principle (C.P.) for linear viscoelasticity. This principle states that the L.T. of the governing equations for a viscoelastic continuum can be obtained by taking the Laplace transform of the corresponding governing equations of an elastic continuum and then replacing the moduli and compliances by their Carson transforms.

<u>Equations Governing the Stability of Viscoelastic Flat Plates Using a Third Order Refined Theory (TSDT)</u>

Following the procedure outlined above and making use of the C.P. in Eqs. (15) and (17) the L.T. of the equations governing the stability of viscoelastic, transversely-isotropic, flat plates write as

$$-2\bar{C}_3^* \overset{(\bar{0})}{V}_{3,\alpha\alpha\beta} - 4\bar{C}_1^*[\overset{(\bar{1})}{V}_{\omega,\omega\beta} - \overset{(\bar{1})}{V}_{\beta,\rho\rho}] + 8\bar{C}_3^* \overset{(\bar{1})}{V}_{\omega,\omega\beta}$$

$$- \frac{40}{h^2} \overset{(\bar{1})}{V}_\beta - 4\delta_A \bar{C}_4^*[\overset{(\bar{0})}{V}_{3,\lambda\lambda\beta} + \overset{(\bar{1})}{V}_{\lambda,\beta\lambda}] - \frac{40}{h^2} \overset{(\bar{0})}{V}_{3,\beta}$$

$$+ 5\delta_B \bar{C}_5^* \rho \overset{(\bar{\ddot{0}})}{V}_{3,\beta} - \rho\delta_c[4 \overset{(\bar{\ddot{1}})}{V}_\beta - \overset{(\bar{\ddot{0}})}{V}_{3,\beta}] = 0 \tag{23}$$

$$\overset{(\bar{1})}{V}_{\omega,\omega} + \overset{(\bar{0})}{V}_{3,\omega\omega} + \frac{3}{2h} \bar{C}_2^*[\overset{(\bar{0})}{p}_3 + L\{ \overset{(0)}{L}_{11} \overset{(0)}{V}_{3,11}$$

$$+ \overset{(0)}{L}_{22} \overset{(0)}{V}_{3,22} + 2 \overset{(0)}{L}_{12} \overset{(0)}{V}_{3,12}\}] - \frac{3}{2} \rho\bar{C}_2^* \overset{(\bar{\ddot{0}})}{V}_3 = 0 \tag{24}$$

where,

$$C_1[t] = L^{-1}\{\frac{1}{s}\frac{\overline{G}^*}{\overline{G}'^*}\} \quad ; \quad C_2[t] = L^{-1}\{\frac{1}{s}(\frac{1}{\overline{G}'^*})\}$$

$$C_3[t] = L^{-1}\{\frac{1}{2}\frac{1}{s}(\frac{\overline{E}^*}{\overline{G}'^*(1-\overline{\mu}^{*})^2})\} \quad (25)$$

$$C_4[t] = L^{-1}\{\frac{1}{s}\frac{\overline{\mu}'^*\overline{E}^*}{\overline{E}'^*(1-\overline{\mu}^*)}\}; \; C_5[t] = L^{-1}\{\frac{1}{s}\overline{C}_2^*\overline{C}_4^*\}$$

In Eqs. (23)-(24) an overbar (—) followed by a star (*) denotes Carson transform while τ denotes a dummy time variable. In the forthcoming developments these equations will be used in the stability analysis of viscoelastic, transversely-isotropic, flat plates in the framework of the TSDT. Henceforth the case of constant, inplane, edge loads is considered and so

$$\overset{(0)}{L}_{11}[t] \to \overset{(0)}{L}_{11}, \; \overset{(0)}{L}_{22}[t] \to \overset{(0)}{L}_{22}, \; \overset{(0)}{L}_{12}[t] \to \overset{(0)}{L}_{12} \quad (26)$$

where $\overset{(0)}{L}_{11}$, $\overset{(0)}{L}_{22}$, $\overset{(0)}{L}_{12}$ take on the meaning of constant applied edge loads.

It is essential at this point to note that the transverse load $\overset{(0)}{p}_3[t]$ represents a forcing function which is required in a dynamic response analysis. However, in a linear stability analysis, $\overset{(0)}{p}_3$ is not required. Similarly, the initial conditions for the displacement field do not affect the stability which is a characteristic of the system itself and hence, together with $\overset{(0)}{p}_3$ they may be dropped (see [6]).

Boundary Conditions

The equations displayed before represent sixth order governing equation systems. Their solution must be determined in conjunction with the prescribed boundary conditions (which are in number of three at each edge). For a simply supported plate, (hinged-free in the normal direction), the following boundary conditions are to be satisfied

$$\overset{(1)}{V}_2 = \overset{(0)}{V}_3 = \overset{(1)}{L}_{11} = 0 \text{ at } x_1 = 0, L_1$$

$$\overset{(1)}{V}_1 = \overset{(0)}{V}_3 = \overset{(1)}{L}_{22} = 0 \text{ at } x_2 = 0, L_2 \quad (27)$$

Stability Analysis Using the Third Order Transverse Shear Deformation Theory (TSDT)

As was noted previously the solution of the equations governing the stability of simply supported plates requires the fulfillment of the boundary conditions (27). To this end, the following representation of the displacement field $\overset{(1)}{V}_\alpha[x_\omega,t]$ and $\overset{(0)}{V}_3[x_\omega,t]$ is postulated:

$$\overset{(1)}{V}_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\cos[\lambda_m x_1]\sin[\lambda_n x_2]f_{mn}[t]$$

$$\overset{(1)}{V}_2 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin[\lambda_m x_1]\cos[\lambda_n x_2]f_{mn}[t]$$

$$\overset{(0)}{V}_3 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sin[\lambda_m x_1]\sin[\lambda_n x_2]f_{mn}[t] \quad (28)$$

where $\lambda_m = m\pi/L_1$, $\lambda_n = n\pi/L_2$ and A_{mn}, B_{mn}, C_{mn} are constants representing the amplitudes of the displacement quantities. The stability will be analyzed in L.T. space. For this purpose Eqs. (23) and (24), will be considered. Introducing L.T. of (28) into Eq. (23) corresponding to the free index $\beta = 1$ yields the following equation

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(\overline{Y}_{mn}[s]\overline{f}_{mn} + \overline{I}_{mn}[s])\cos[\lambda_m x_1]\sin[\lambda_n x_2] = 0 \quad (29)$$

where,

$$\overline{Y}_{mn}[s] = A_{mn}[4\overline{C}_1^*\lambda_n^2 - 8\overline{C}_3^*\lambda_m^2 - \frac{40}{h^2} + 4\delta_A\overline{C}_4^*\lambda_m^2 - 4\delta_c\rho s^2] + B_{mn}[4\overline{C}_1^*\lambda_m\lambda_n - 8\overline{C}_3^*\lambda_m\lambda_n + 4\delta_A\overline{C}_4^*\lambda_m\lambda_n]$$
$$+ C_{mn}[2\overline{C}_3^*(\lambda_m^3 + \lambda_m\lambda_n^2) + 4\delta_A\overline{C}_4^*(\lambda_m^3 + \lambda_m\lambda_n^2) - \frac{40}{h^2}\lambda_m + 5\rho\delta_B\overline{C}_5^*s^2\lambda_m - 4\delta_c\rho s^2\lambda_m] \quad (30)$$

$$\overline{I}_{mn}[s] = \{A_{mn}[4\delta_c\rho] + C_{mn}[-\delta_c\rho\lambda_m - 5\rho\delta_B\overline{C}_5^*\lambda_m]\}\{sf_{mn}[0] + \dot{f}_{mn}[0]\}$$

The equation corresponding to the free index $\beta = 2$ can be obtained from Eq. (29) by replacing λ_m by λ_n and A_{mn} by B_{mn} (and vice versa). Examining (30) (and its counterpart for the index $\beta = 2$), we infer that due to the orthogonality of the sine and cosine functions, one may write:

$$\overline{Y}_{mn}[s]\overline{f}_{mn}[s] + \overline{I}_{mn}[s] = 0 \quad (31)$$

As was noted earlier, the stability of a linear dynamical system (of the type represented by (32)) does not depend on the initial conditons (i.e., $\overline{I}_{mn}[s]$) but is simply determined by the nature of its impulse response i.e., $L^{-1}\{(\overline{Y}_{mn}[s])^{-1}\}$). Thus one of the stability equations writes:

$$\overline{Y}_{mn}[s] = 0 \tag{32}$$

(*the second one being its counterpart for the* index $\beta = 2$). Now introducing (29) into (24) for the case of uniform bi-axial compression (i.e., $\overset{(0)}{L}_{11} = \sigma_{11}h$, $\overset{(0)}{L}_{22} = \sigma_{22}h$, $\overset{(0)}{L}_{12} = 0$) we obtain,

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(\overline{W}_{mn}[s]\overline{f}_{mn}[s] + \overline{J}_{mn}[s])\sin[\lambda_m x_1]\sin[\lambda_n x_2] = 0 \tag{33}$$

where

$$\overline{W}_{mn}[s] = A_{mn}\lambda_m + B_{mn}\lambda_n + C_{mn}[\lambda_m^2 + \lambda_n^2 + \frac{3}{2}\overline{C}_2^{\star}\{\sigma_{11}\lambda_m^2 + \sigma_{22}\lambda_n^2\} + \frac{3}{2}\rho\overline{C}_2^{\star}s^2]$$

$$\overline{J}_{mn}[s] = -C_{mn}[sf_{mn}[0] + \dot{f}_{mn}[0]]$$

A similar reasoning as above yields:

$$\overline{W}_{mn}[s] = 0 \tag{34}$$

Equations (33) (and its counterpart for the index $\beta = 2$) together with Eq. (35) are the equations governing the stability problem. This set of three equations represents a homogeneous system of equations in terms of the unknown amplitudes A_{mn}, B_{mn}, C_{mn} (which take the role of the eigenvector). Thus, using (32) and (34) in conjunction with (30) and (34), allows one to write this system of homogeneous equations in the following form,

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{Bmatrix} A_{mn} \\ B_{mn} \\ C_{mn} \end{Bmatrix} = 0 \tag{35}$$

where

$$Z_{11} = 4\overline{C}_1^{\star}\lambda_n^2 - 8\overline{C}_3^{\star}\lambda_m^2 - \frac{40}{h^2} + 4\delta_A\overline{C}_4^{\star}\lambda_m^2 - 4\delta_c\rho s^2$$

$$Z_{12} = 4\overline{C}_1^{\star}\lambda_m\lambda_n - 8\overline{C}_3^{\star}\lambda_m\lambda_n + 4\delta_A\overline{C}_4^{\star}\lambda_m\lambda_n$$

$$Z_{13} = 2\overline{C}_3^{\star}(\lambda_m^3 + \lambda_m\lambda_n^2) + 4\delta_A\overline{C}_4^{\star}(\lambda_m^3 + \lambda_m\lambda_n^2) - \frac{40}{h^2}\lambda_m + 5\rho\delta_B\overline{C}_5^{\star}s^2\lambda_m - 4\delta_c\rho s^2\lambda_m \tag{36}$$

$$Z_{22} = 4\overline{C}_1^{\star}\lambda_m^2 - 8\overline{C}_3^{\star}\lambda_n^2 - \frac{40}{h^2} + 4\delta_A\overline{C}_4^{\star}\lambda_n^2 - 4\delta_c\rho s^2$$

$$Z_{23} = 2\overline{C}_3^{\star}(\lambda_n^3 + \lambda_n\lambda_m^2) + 4\delta_A\overline{C}_4^{\star}(\lambda_n^3 + \lambda_n\lambda_m^2) - \frac{40}{h^2}\lambda_n + 5\rho\delta_B\overline{C}_5^{\star}s^2\lambda_n - 4\delta_c\rho s^2\lambda_n$$

$$Z_{31} = \lambda_m, \quad Z_{32} = \lambda_n, \quad Z_{21} = Z_{12}$$

$$Z_{33} = \lambda_m^2 + \lambda_n^2 + \frac{3}{2}\overline{C}_2^{\star}\{\sigma_{11}\lambda_m^2 + \sigma_{22}\lambda_n^2\} + \frac{3}{2}\rho\overline{C}_2^{\star}s^2$$

From (36) it is seen that for non-trivial solutions of A_{mn}, B_{mn}, C_{mn}, the following determinantal equation is to be fulfilled

$$\det[Z_{ij}] \equiv 0 \tag{37}$$

Equation (32) yields a characteristic equation of the form,

$$\frac{P_{mn}[s]}{Q_{mn}[s]} = 0$$

where $P_{mn}[s]$ and $Q_{mn}[s]$ are polynomials in s. Thus, the zeroes of the above equation are determined from:

$$P_{mn}[s] = 0 \tag{38}$$

Equation (38) is the characteristic equation of the system (represented by the plate subjected to uniform bi-axial compressive loads). The zeros of this equation, i.e., the roots s_i of $P_{mn}[s]$, are the eigenvalues of the system which in general are complex quantities.

They decide the nature of $f_{mn}[t]$ and hence the stability of the system. When $\mathrm{Re}[s_i] > 0$, $f_{mn}[t]$ becomes unbounded with time and the following cases of instability may arise due to the nature of s_i: (i) $\mathrm{Im}[s_i] = 0$: In this case $f_{mn}[t]$ grows exponentially with time, and we have instability by divergence, (ii) $\mathrm{Im}[s_i] \neq 0$: In this case $f_{mn}[t]$ has an oscillatory growth with time and the amplitude of oscillations is given by $e^{\alpha t}$. This leads to instability by flutter.

Therefore the stability problem is reduced to the examination of the nature of the zeros of the characteristic equation of the system (38). The coefficients of the characteristic polynomial, $P_{mn}[s]$, in Eq. (38) can be varied by suitably varying the inplane edge loads σ_{11} and σ_{22} in order to yield convenient stability boundaries of the system.

Stability Analysis Using a First Order Transverse Shear Deformation Theory (FSDT)

By paralleling the procedure already developed for the HSDT we obtain the characteristic equation of the system in exactly the same form as given by (38) but with different coefficients. These coefficients are determined by Eq. (37) in which, for the FSDT, we have,

$$Z_{11} = \frac{h^3}{12}\overline{C}_1^{\star}\lambda_n^2 + \frac{h^3}{6}\overline{C}_3^{\star}\lambda_m^2 + h + \delta_c m_1\overline{C}_2^{\star}s^2$$

$$Z_{12} = -\frac{h^3}{12}\bar{C}_1^*\lambda_m\lambda_n + \frac{h^3}{6}\bar{C}_3^*\lambda_m\lambda_n$$

$$Z_{13} = h\lambda_m \qquad (39)$$

$$Z_{22} = \frac{h^3}{12}\bar{C}_1^*\lambda_m^2 + \frac{h^3}{6}\bar{C}_3^*\lambda_n^2 + h + \delta_c m_1 \bar{C}_2^* s^2$$

$$Z_{23} = h\lambda_n\ ,\ Z_{31} = \lambda_m\ ,\ Z_{32} = \lambda_n\ ,\ Z_{21} = Z_{12}$$

$$Z_{33} = \lambda_m^2 + \lambda_n^2 + \delta_D \frac{\rho}{K^2}\bar{C}_2^* s^2 + \frac{\bar{C}_2^*}{K^2}(g_{11}\lambda_m^2 + g_{22}\lambda_n^2)$$

Proceeding in exactly the same manner as for the HSDT, we obtain the stability boundaries of the transversely-isotropic plate subjected to uniform inplane compressive edge loads.

Stability Analysis Using the Equations Representing the Interior Solution in the Framework of the FSDT

Towards the goal of revealing through numerical comparisons with the solution obtained via Eqs. (26) and (27) that Eq. (22), (governing the *interior solution*), is thus by itself sufficient to analyze the stability of transversely-isotropic plates, we will consider now Eq. (22). To this end, the following representation for the transverse displacement $\overset{(0)}{V}_3$

$$\overset{(0)}{V}_3[x_1,x_2;t] = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \sin[\lambda_m x_1]\sin[\lambda_n x_2]f_{mn}[t] \qquad (40)$$

which satisfies the boundary conditions is postulated. Replacement of (40) into the L.T. of Eq. (22), yields:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(\bar{R}_{mn}[s]\bar{f}_{mn}[s] + \bar{Q}_{mn}[s])\sin[\lambda_m x_1]\sin[\lambda_n x_2] = 0 \qquad (41)$$

where

$$\bar{R}_{mn}[s] = \bar{D}^*(\lambda_m^2 + \lambda_n^2)^2 + h(g_{11}\lambda_m^2 + g_{22}\lambda_n^2)$$
$$+ \frac{h^3}{6K^2}\bar{C}_3^*(g_{11}(\lambda_m^4 + \lambda_m^2\lambda_n^2) + g_{22}[\lambda_n^4 + \lambda_m^2\lambda_n^2])$$
$$+ \delta_D m_o s^2 + \delta_D m_o \frac{h^2}{6K^2}\bar{C}_3^* s^2(\lambda_m^2 + \lambda_n^2)$$
$$+ \delta_c \frac{m_1 m_o}{hK^2}\bar{C}_2^* s^4 + \delta_c \frac{m_1}{K^2}\bar{C}_2^* s^2(g_{11}\lambda_m^2 + g_{22}\lambda_n^2)$$
$$+ \delta_c m_1 s^2(\lambda_m^2 + \lambda_n^2) \qquad (42)_1$$

and,

$$\bar{Q}_{mn}[s] = \delta_D m_o f_{mn}[0]s + \delta_D m_o(\lambda_m^2 + \lambda_n^2)\frac{h^2}{6K^2}\bar{C}_3^*(sf_{mn}[0]$$
$$+ \dot{f}_{mn}[0]) + \delta_c \frac{m_1 m_o}{hK^2}\bar{C}_2^*[s](s^3 f_{mn}[0] + s^2\dot{f}_{mn}[0]$$
$$+ s\ddot{f}_{mn}[0] + \dddot{f}_{mn}[0]) + \delta_c m_1 f_{mn}[0](\lambda_m^2 + \lambda_n^2))s$$
$$+ (\delta_D m_o \dot{f}_{mn}[0] + \delta_c m_1 \dot{f}_{mn}[\lambda_m^2 + \lambda_n^2])$$
$$+ \delta_c \frac{m_1}{K^2}(g_{11}\lambda_m^2 + g_{22}\lambda_n^2)\bar{C}_2^*(sf_{mn}[0] + \dot{f}_{mn}[0]) \qquad (42)_2$$

where

$$D[t] = L^{-1}\{\frac{1}{s}(\frac{h^3}{12}\frac{\bar{E}^*}{1-(\bar{\mu}^*)^2})\}. \qquad (42)_3$$

Using similar arguments as in the previous cases the characteristic equation reduces to:

$$\bar{R}_{mn}[s] = 0 \qquad (43)$$

Equation (43) may also be used to determine the stability boundaries of the transversely-isotropic plate subject to uniform inplane edge loads.

VI. Stability of a Transversely Isotropic Viscoelastic Plate Undergoing Cylindrical Bending

Consider a transversely-isotropic plate with an infinitely large aspect ratio (i.e., $L_2/L_1 \to \infty$) and simply supported along its edges $x_1 = 0, L_1$. The plate is subjected to a uniform compressive force system applied along its edges. Thus, the plate undergoes cylindrical bending and in this case the operator $\partial/\partial x_2 \to 0$. In the following developments, we analyze the stability of the plate in the framework of the FSDT by making use of the correspondence principle. To this end, we consider the uniaxial counterpart of Eq. (22) which yields:

$$D\overset{(0)}{V}_{3,1111}[t] - \overset{(0)}{L}_{11}\overset{(0)}{V}_{3,11}[t]$$
$$+ \frac{h^2}{6K^2}C_3 \overset{(0)}{L}_{11}\overset{(0)}{V}_{3,1111}[t] + \delta_D m_o \overset{(\ddot{0})}{V}_3[t]$$
$$- \frac{h^2}{6K^2}C_3\delta_D m_o \overset{(\ddot{0})}{V}_{3,11}[t] - \delta_c \frac{m_1}{hK^2}C_2 \overset{(0)}{L}_{11}\overset{(\ddot{0})}{V}_{3,11}[t]$$
$$+ \delta_c\delta_D \frac{m_1 m_o}{hK^2}C_2 \overset{(\ddot{\ddot{0}})}{V}_3[t] - \delta_c m_1 \overset{(\ddot{0})}{V}_{3,\beta\beta}[t] = 0 \quad (44)$$

In Eq. (44), D, C_2, C_3 are the elastic counterparts (defined at t = 0) of the previously defined quantities D[t], C_2[t], C_3[t].

Now making use of the relations presented e.g. in [4] we obtain the following results for a transversely isotropic body:

$$\tilde{E}_{1111} = \frac{E}{1-\mu^2}\,,\ E_{1313} = G' \qquad (45)$$

Replacement of Eqs. (45) into Eqs. $(25)_2$ and $(42)_3$, evaluated at t = 0, yields:

$$D = \tilde{E}_{1111}\frac{h^3}{12}\,,\ C_3 = \frac{1}{2}\frac{\tilde{E}_{1111}}{E_{1313}} \qquad (46)$$

Introducing (46) into (44), neglecting the effect of rotary and transverse inertias (i.e., $\delta_c = \delta_D = 0$), and postulating that $K^2 = 5/6$, we obtain:

$$\tilde{E}_{1111}\frac{h^3}{12}\overset{(0)}{V}_{3,1111} - \overset{(0)}{L}_{11}\overset{(0)}{V}_{3,11} + \frac{h^2}{10}\frac{\tilde{E}_{1111}}{E_{1313}}\overset{(0)}{L}_{11}\overset{(0)}{V}_{3,1111} = 0 \qquad (47)$$

Now we consider the constitutive equations for a transversely isotropic plate undergoing cylindrical bending and exhibiting viscoelastic properties in transverse shear only. These pertinent equations may be written as

$$\sigma_{11}[t] = \tilde{E}_{1111}e_{11}[t] \qquad (48)$$

$$\bar{\sigma}_{13}[t] = 2\int_0^t E_{1313}[t-\tau]\dot{e}_{13}[\tau]d\tau.$$

Making use of the C.P. in Eq. (47) in conjunction with (48) yields

$$-\frac{h^2}{10}\frac{\tilde{E}_{1111}}{s\bar{E}_{1313}}\overset{(0)}{L}_{11}\overset{(\bar{0})}{V}_{3,1111} - \tilde{E}_{1111}\frac{h^3}{12}\overset{(\bar{0})}{V}_{3,1111} + \overset{(0)}{L}_{11}\overset{(\bar{0})}{V}_{3,11} = 0 \qquad (49)$$

Introducing the orthogonality relation between the creep compliance and relaxation moduli given by $4s\bar{E}_{\omega3\lambda3}s\bar{F}_{\omega3\pi3} = \delta_{\pi\lambda}$ into (49) we obtain:

$$-\frac{h^2}{10}\tilde{E}_{1111}\overset{(0)}{L}_{11}(L[F_{1313}] + \dot{F}_{1313}[0])\overset{(\bar{0})}{V}_{3,1111} - \tilde{E}_{1111}\frac{h^3}{12}\overset{(\bar{0})}{V}_{3,1111} + \overset{(0)}{L}_{11}\overset{(\bar{0})}{V}_{3,11} = 0 \qquad (50)$$

Equation (50) is the equation in L.T. space governing the stability of viscoelastic transversely-isotropic infinite aspect ratio plates, exhibiting viscoelastic properties in transverse shear only.

<u>Solution of Stability Problem Based on Eq. (50)</u>:

The following representation of the creep compliance in transverse shear which corresponds to a 3-Parameter Solid is considered:

$$F_{1313}[t] = \overset{(1)}{F}_{1313} - \overset{(2)}{F}_{1313}e^{-\overset{(3)}{F}_{1313}t} \qquad (51)$$

We also assume the following representation of the transverse displacement (valid for simply supported boundary conditions at $x_1 = 0, L_1$):

$$\overset{(0)}{V}_3 = \sum_{m=1}^{\infty} f_m[t]\sin[\lambda_m x_1],\quad \lambda_m = m\pi/L_1 \qquad (52)$$

Insertion of (51) and (52) into Eq. (50) yields

$$\sum_{m=1}^{\infty}\Big[\frac{h^2}{10}\overset{(0)}{L}_{11}\tilde{E}_{1111}\Big[\overset{(1)}{F}_{1313} - \frac{s\overset{(2)}{F}_{1313}}{s+\overset{(3)}{F}_{1313}}\Big]\lambda_m^4 + \tilde{E}_{1111}\frac{h^3}{12}\lambda_m^4 + \overset{(0)}{L}_{11}\lambda_m^2\Big]\bar{f}_m\sin[\lambda_m x_1] = 0 \qquad (53)$$

on which basis, by invoking the same arguments in the previous analysis one obtains:

$$\frac{h^2}{10}\overset{(0)}{L}_{11}\tilde{E}_{1111}[(\overset{(1)}{F}_{1313} - \overset{(2)}{F}_{1313})s]\lambda_m^4 + \frac{h^2}{10}\overset{(0)}{L}_{11}\tilde{E}_{1111}[\overset{(1)}{F}_{1313}\overset{(3)}{F}_{1313}]\lambda_m^4 + [s + \overset{(3)}{F}_{1313}][\tilde{E}_{1111}\frac{h^3}{12}\lambda_m^4 + \overset{(0)}{L}_{11}\lambda_m^2] = 0 \qquad (54)$$

A cursory inspection of Eq. (54) reveals that it posses real roots only (since it is linear in s). This allows us to conclude that the instability could occur by divergence only. Having in view the fact that for divergence instability, s = 0 in (54), we obtain,

$$\overset{(0)}{L}_{11} = \frac{-\tilde{E}_{1111}\frac{h^3}{12}\lambda_m^2}{[\frac{h^2}{10}\tilde{E}_{1111}\overset{(1)}{F}_{1313}\lambda_m^2 + 1]} \qquad (55)$$

It is easily seen that the lowest value of this load corresponds to m = 1. Considering (51) with $\overset{(3)}{F}_{1313} = 0$ (i.e., the elastic case), it is easy to verify that the static buckling load for the case of elastic transversely-isotropic plates reads

$$L_{11}^{(0)} = \frac{-\tilde{E}_{1111}\frac{h^3}{12}(\frac{\pi}{L_1})^2}{[\frac{h^2}{10}\tilde{E}_{1111}(F_{1313}^{(1)} - F_{1313}^{(2)})(\frac{\pi}{L_1})^2 + 1]} \quad (56)$$

Since $F_{1313}^{(1)}$, $F_{1313}^{(2)}$ and $F_{1313}^{(3)}$ in (51) are all greater than zero, we may infer, upon comparing (56) and (55) (considered for m = 1), that the (divergence) instability loads for the viscoelastic case are lower than for its elastic counterpart.

VII. Material Property Determination

In the previous section, by considering the case of cylindrical bending, a closed form solution for the instability problem was determined. However, for the more general case of the plate instability a numerical procedure is necessary to be developed. It is why, the material properties are to be expressed explicitly as functions of time.

The effective moduli E, μ, E', μ', G' for an elastic, transversely-isotropic material may be obtained by using suitable micromechanical equations which expresse the effective composite-material properties in terms of its constituent counterparts (i.e., the fiber and matrix). Then, by using the correspondence principle we may obtain the relevant micromechanical relations for a viscoelastic material. Due to the unavailability of suitable micromechanical relations for a transversely-isotropic material of this type we will restrict ourselves, for computational purposes, to the case of an isotropic material. In such a case $\mu' = \mu$, $E' = E$, $G' = G$.

Using the properties for the isotropic, viscoelastic epoxy matrix considered by Schapery [7] the following relations were obtained when the matrix behavior is modelled as a 3-parameter solid:

$$E[t] = 0.8 \times 10^5 + 0.18 \times 10^6 e^{-0.4115 \times 10^{-3} t}$$

for $0 < t < 2000$ hrs

$$\mu[t] = 0.372 - 0.007 e^{-0.2403 \times 10^{-2} t}$$

for $0 < t < 2000$ hrs (63)

where the time t is in minutes.

Introducing (63) into (27) and $(32)_2$ we obtain the material properties defined by $\bar{C}_1^*$, $\bar{C}_2^*$, $\bar{C}_3^*$, $\bar{C}_4^*$, $\bar{C}_5^*$ and $\bar{D}^*$.

VIII. NUMERICAL RESULTS AND CONCLUSIONS

The stability boundaries were obtained by solving the characteristic polynomials associated with TSDT (Eqn. 38), its FSDT counterpart and FSDT single equation (43). This was done by using the IMSL subroutine ZPOLR. The numerical applications were considered for an isotropic, viscoelastic plate. By considering the initial value theorem for the Laplace transformed material properties appearing in Eqs. (25) and $(42)_3$, the numerical applications include also their elastic counterparts.

In all these cases the full dynamic solution was considered in the sense that throughout the applications $\delta_A = \delta_B = \delta_C = \delta_D = 1$ where δ_B, δ_C, δ_D are tracers identifying the dynamic effect of σ_{33}, rotary and transverse inertias, respectively, while δ_A is a tracer identifying the overall (i.e., static and dynamic) effect of σ_{33}. It was observed that the inclusion or exclusion of the inertia terms does not affect the results.

The results associated with the classical Kirchhoff theory were obtained as a special case of the FSDT by considering therein $K^2 \to \infty$ which is equivalent to consider infinite transverse shear rigidities. The results obtained in this study are not universal since a non-dimensional analysis was not possible due to the inherent complexity of the problem.

The stability boundaries are displayed in Figs. 1, 2, 5 and 7 for the case of isotropic, viscoelastic, flat plates, while in Figs. 3, 4, 6 and 8 for their elastic, counterpart. Results for thick ($L_1/h = 4.8$) as well as thin plates ($L_1/h = 24$) are considered in Figs. 1-4, and in Figs. 5-8, respectively. In Figs. 1, 5 and 6 the case of biaxial compression was investigated. For this case, the aspect ratio (A.R. = L_1/L_2) of the plate was taken as unity. The values of the inplane, normal edge loads $\underset{\sim}{\sigma}_{11}$ versus $\underset{\sim}{\sigma}_{22}$ are plotted to obtain the stability boundaries. In Figs. 2-4 and 7-8 the case of uniaxial compression was examined. In this case, the aspect ratio, A.R., was varied and the corresponding value of $\underset{\sim}{\sigma}_{11}$ was plotted in order to obtain the stability boundaries. For all plots shown, M and N denote the mode numbers in the x_1 and x_2 direction, respectively. It was observed that for biaxial compression, the stability boundaries corresponding to M=1 were the lowest ones, whereas for uniaxial compression, those corresponding to N = 1 were the lowest ones. Therefore, in each of these two sub-cases, only the lowest stability boundaries were displayed. For all the cases envisioned herein, instability occurs by divergence only.

Conclusions

In this study, a stability analysis of transversely-isotropic, viscoelastic rectangular plates has been undertaken. The equations governing the stability were derived by using the correspondence principle.

In the modeling of the problem, the Boltzmann hereditary constitutive law for a 3-D viscoelastic medium has been used. In order to determine the asymptotic stability behavior the stability problem was analyzed in the Laplace transformed space.

The special cases considered in the numerical applications allow one to conclude the followings:

1. The stability boundary determined for a viscoelastic plate are lower than those pertaining

to its elastic counterpart. This conclusion appears evident when comparing Figs. 2 with Figs. 3, 4; Figs. 5 and 6 and Figs. 7 and 8.

2. Incorporation of transverse shear deformation effects results in stability boundaries which are lower than those of their transversely-rigid (classical) counterpart. In this sense Figs. 2-4 are relevant. This property appears more prominent for low aspect ratios panels.

3. The results displayed in Figs. 1 show that in the case of thick panels σ_{33} may influence the viscoelastic stability boundary in a strong and beneficial way. However, from the cases considered herein the critical stability boundary, corresponding to M = N = 1, is not influenced by this effect. In addition, for thin panels (see Fig. 5) the effect on the instability boundaries is insignificant.

4. The transverse shear deformation effects appear to be more pronounced in the case of the viscoelastic plates than in their elastic counterpart. *Figures 2-4; 5-6 and 7-8 are* relevant in this sense. It may also be remarked that for the special case of the viscoelasticity experienced in the transverse shear direction only, the discard of transverse shear deformations results in identical solutions for the viscoelastic and the elastic cases.

5. The analysis performed here allows one to obtain the nature of loss of stability, i.e., the one by divergence or by flutter. However, as it was observed for an isotropic, viscoelastic plate the instability occurs by divergence only.

6. In light of the results obtained from Figs. 1-8 it may be concluded that the boundary layer solution associated with an isotropic viscoelastic plate has no effect on the stability boundary. This means that consideration of the interior solution equation yields the same results as the full system of equations.

7. It is observed that for large aspect ratios (L_1/L_2) the stability boundaries come closer to the ones based on the classical Kirchhoff theory of plates. This conclusion holds valid for both the viscoelastic and elastic cases.

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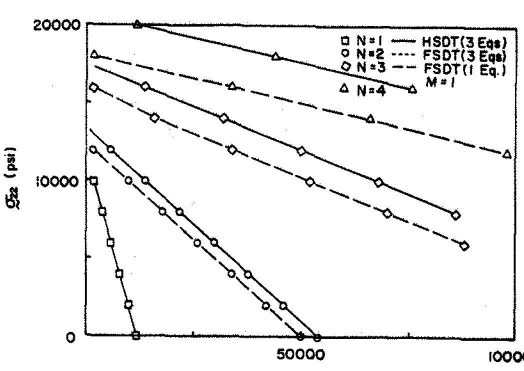

Figure 1. Stability boundary for isotropic viscoelastic plate; $L_1/h = 4.8$; biaxial compression; $\delta_A = 1$.

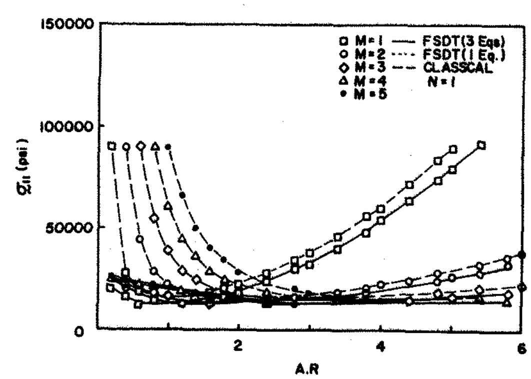

Figure 2. Comparison of stability boundaries for isotropic viscoelastic plate; $L_1/h = 4.8$; uniaxial compression.

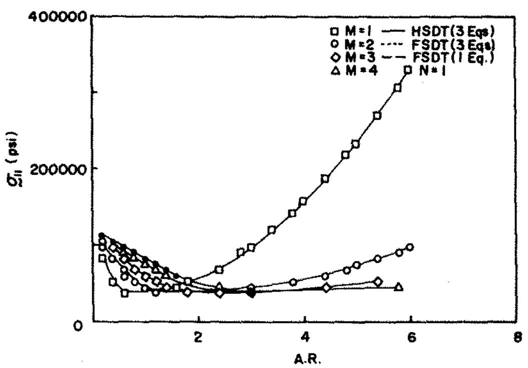

Figure 3. Stability boundary for isotropic elastic plate; L_1/h = 4.8; uniaxial compression; δ_A = 1 or δ_A = 0.

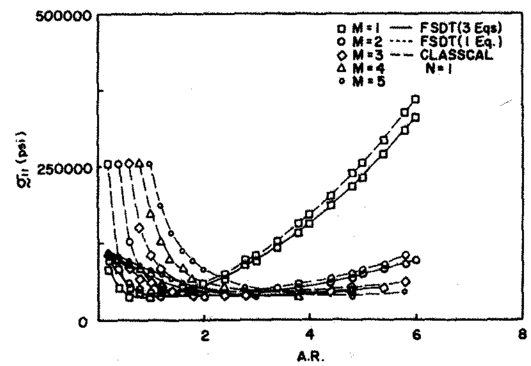

Figure 4. Comparison of stability boundaries for isotropic elastic plate; L_1/h = 4.8; uniaxial compression.

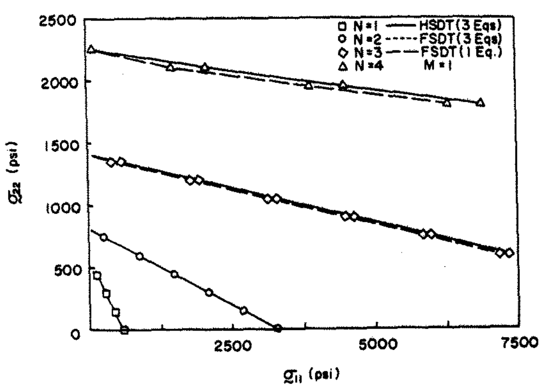

Figure 5. Stability boundary for isotropic viscoelastic plate L_1/h = 24; biaxial compression; δ_A = 1 or δ_A = 0.

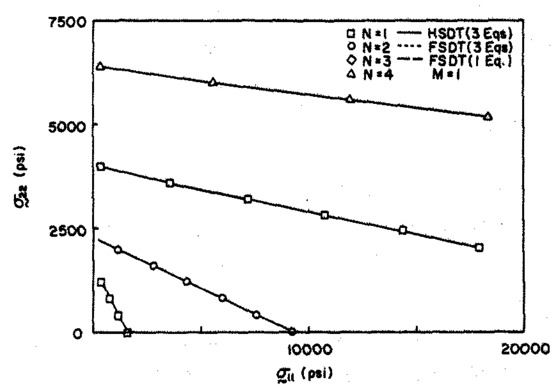

Figure 6. Stability boundary for isotropic elastic plate; L_1/h = 24; biaxial compression; δ_A = 1 or δ_A = 0.

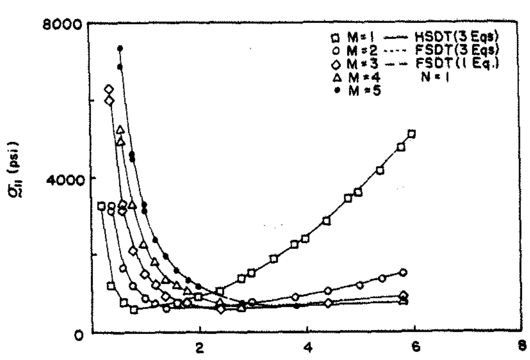

Figure 7. Stability boundary for isotropic viscoelastic plate; L_1/h = 24; uniaxial compression; δ_A = 1 or δ_A = 0.

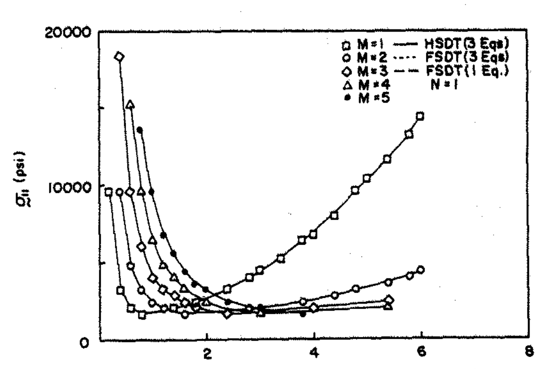

Figure 8. Stability boundary for isotropic elastic plate; L_1/h = 24; uniaxial compression; δ_A = 1 or δ_A = 0.