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THE USE OF STATIC ANALYSIS AND THE STRESS MODES APPROACH AS AN ENGINEERING ORIENTED PROCEDURE FOR CALCULATING THE RESPONSE OF AERONAUTICAL STRUCTURES TO RANDOM EXCITATION

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Abstract

Solution to the dynamic behavior of aeronautical structures subjected to random excitation is obtained by using characteristic results from static analysis. The dynamic response problem is solved by using the concept of "stress modes", which is also presented in this paper. Static results are also used in the solution of geometrically non-linear problems. It is believed that the use of this approach will contribute to a better intuitive "feel" of the design engineer, and thus to a better physical understanding of the structure's behavior. Schematic procedure for the application of the outlined approach is presented.

I. Introduction

The response of aeronautical structures to noise excitation is a major cause of fatigue failure. The pressure fluctuations in the flow that surrounds the structure due to turbulence and jet noise, cause it to vibrate continuously at high frequencies, as a result of which new cracks may be generated in the material or existing cracks may propagate. The advent of fiber-reinforced materials has increased the incidence of these types of failure, owing to the sestitivity of the materials to strains in particular directions.

Research efforts toward solution of the dynamic response of structure elements to wideband random excitation began in the 1950's. Since then, numerous conclusions, methods and procedures were developed. A feature common to a large part of the existing literature is emphasis on the mean square values of the response amplitudes. However, a designer has to predict the structure's life by comparing its existing stresses to known allowables. Moreover, in some cases it is the strains rather than the stresses which govern the failure envelope of a given structure or material. In these circumstances it is of major importance to be able to predict the statistical characteristics of both the stresses and strains throughout the structure.

To the design engineer, the problem of response to acoustic noise is not academic in nature. What he needs is a "design routine", adaptable to his needs and to the facilities (software and hardware) to which he has access. Past experience has shown that he usually has a good intuitive "feel" for static loads, and is able to identify points of weakness in a design by looking at a statical analysis. This "feel" is less reliable for dynamic stress states where the meaning of mode shapes may be understood, but it is difficult to visualize a physical interpretation of a weighted combination of several modes; it is even more difficult to "feel" random vibration, mainly when the excitation comprises many components, each with a different frequency. Hence the need for a rapid design-oriented procedure capable of displaying the dynamic response effects during design iterations.

In the last few decades, there was a tremendous progress in the analytical tools with which structural analysis experts can analyse the structure under consideration. Large computer codes were developed, and are in use by most of the establishments which deals with structural design. There is, although, a danger of losing the physical understanding of the structure's behavior by using these computer codes automatically, without pausing to think and interpret interim results. The author believes that by examination of the statical analysis results, more understanding of the dynamic behavior can be obtained. By using the methods outlined in this paper, or similar conceptual approach, the better understanding obtained will contribute to better designs.

The introduction of the concept of "stress modes" by the author (1), enable to analyse the dynamic behavior of aeronautical structures via statical techniques, thereby contributing to intuitive understanding of the results by design engineers. In this paper, the definition of stress modes and the statical methods of obtaining them are presented, and their use in the solution of a structural dynamic problem is outlined. Also, considerations of geometrical non-linearities in the analysis of a structural dynamic problem are described. These considerations are also based on results of statical, intuitively understood solutions, which are available to most designers. A schematic procedure for the application of the approach outlined in the paper is also presented.

II. Displacement response to random exciation

Les us denote the displacement of a typical structure (say a plate) by w(x,y,t), which can be expanded in the following series:

$$w(x,y,t) = \sum_{r=1}^{N} \phi_r(x,y) \cdot h \cdot \eta_r(t) \tag{1}$$

$\phi_r(x,y)$ is mode shape, $\eta_r(t)$ is a generalized coordinate, h is the thickness and N is the number of terms taken into account.

If the structure is excited by distributed pressure q(x,y,t), the governing equation of motion for $\eta_r(t)$ reads:

$$\ddot{\eta}_r + 2\varsigma_r\omega_r\dot{\eta}_r + \omega_r^2\eta_r = \frac{\iint q(x,y,t)\phi_r(x,y)dxdy}{M_r h} \quad (2)$$

where $M_r = \iint m\phi_r^2(x,y)dxdy$ the generalized mass

m = mass per unit area

ς_r = damping ratio of the r-th mode

ω_r = resonance frequency of the r-th mode

For distributed load q(x,y,t) which is a separable function

$$q(x,y,t) = p(x,y)f(t) \quad (3)$$

with p(x,y) a deterministic function, the equations of motion become

$$\ddot{\eta}_r + 2\varsigma_r\omega_r\dot{\eta}_r + \omega_r^2\eta_r = \frac{\iint p(x,y)\phi_r(x,y)dxdy}{M_r h}f(t) \quad (4)$$

A classical procedure (e.g. reference 2) yields the following equation for the mean square of the displacement response

$$\frac{\overline{w^2(x,y,t)}}{h^2} = \sum_{r=1}^{N}\sum_{s=1}^{N}\phi_r(x,y)\phi_s(x,y)\frac{\iint \phi_r(x,y)dxdy \iint \phi_s(x,y)dxdy}{h^2\omega_r^2\omega_s^2 M_r M_s}\times \frac{1}{2\pi}\int_0^\infty |H_r(\Omega)||H_s(\Omega)|F(\Omega)d\Omega \quad (5)$$

The generalized forces in the equations are in the form of double integrals of the mode shapes. In the evaluation of eq. 5 it was assumed that p(x,y) is deterministic, and that the random process is represented by f(t) in eq. 3. The double-integral form is due to absence of correlation between the random pressure fields acting at points of the structure. This is not always the case especially where large structures are concerned, and the excitation originates at a source in such a way that parts of the examined structure are downstream relative to other parts. In such cases, the pressure field downstream is correlated with the field upstream, and the calculation of the generalized forces involves quadruple integrals (for details, see e.g. reference 3, section 7.3) This fact, however, does not affect the general approach as described here.

III. Stress modes and stress response to random excitation

Stresses in elastic materials are uniquely defined by the deflection, through the material's constitutive relations and the compatibility equations. It dose not matter how the deflection was obtained-statically or dynamically. The effect of the dynamic load factor, well known in the theory of vibration of structures is included when the deflections are calculated.

For a specific type of structure, there always exists the following relationship

$$\sigma_j = CL_j[w(x,y,t)] \quad (6)$$

where L_j is a differential operator in the spatial coordinates, and C is constant that depends on the elastic constants of the material, and the geometry of the structure. The subscript j indicates which stress at which location is being calculated, not necessarily one of the components of the stress tensor. For instance

$\sigma_1 = \sigma_x$

$\sigma_2 = \sigma_y$

σ_7 = shear stress between two plies in a laminated structure

σ_8 = stress at the edge of a hole, etc.

In view of eq.1 we have

$$L_j[w(x,y,t)] = \sum_{r=1}^{N} hL_j[\phi_r(x,y)]\eta_r(t) \quad (7)$$

and

$$\sigma_j = Ch\sum_{r=1}^{N} L_j[\phi_r(x,y)]\eta_r(t) \quad (8)$$

Equation 8 can be rewritten as

$$\sigma_j = \sum_{r=1}^{N} \psi_r^j(x,y)\eta_r(t) \quad (9)$$

where

$$\psi_r^j = ChL_j[\phi_r(x,y)] \quad (10)$$

Equation (9) resembles eq. 1, and accordingly the functions $\psi_r^j(x,y)$ are called hereinafter "Stress modes". The latter permits an evaluation analogous to that of that mean-square value of the amplitudes, and therefore

$$\overline{\sigma_j^2(x,y,t)} = \sum_{r=1}^{N}\sum_{s=1}^{N}\psi_r^j(x,y)\psi_s^j(x,y)\frac{\iint \phi_r(x,y)dxdy \iint \phi_s(x,y)dxdy}{h^2\omega_r^2\omega_s^2 M_r M_s}\times \frac{1}{2\pi}\int_0^\infty |H_r(\Omega)||H_s(\Omega)|F(\Omega)d\Omega \quad (11)$$

Examination of eq. 10 reveals that the stress mode is a characteristic of the structure. Once stress modes for a given structure have been computed, a complete stress distribution can be found, by use of eq.9.

To obtain the stress modes,one naturally tends to use eq. 10. i.e. apply the operator L_j on the mode shapes. There is, however, a disadvantage in doing so, since the modal shapes of the structure are not always avaliable in closed form and are then approximated by assumed functions which satisfy boundary conditions. While this ensures satisfactory accuracy in calculating frequencies and deflections, large errors may set in on differentiation (usually double in determining the stresses); the same is the case when the modal shapes are obtained by some numerical method (e.g. a finite-element computer code).

Examining equations 9 and 10 it is readily seen that the stress mode is also the stress distribution in the structure for the particular case of a deformation coincident with the modal shape, with the generalized coordinate equal to unity and the maximum deflection equal to one thickness. This observation points to a convenient method for calculating the stress modes:

> "Determine the stress distribution in the structure, when it is deformed to a deflection equal to $h\phi_r(x,y)$ ".

This is a case of static pre-described deflection loading, which most of the modern computer programs have the capability to accept, as input.

Further examination of the basic equations from which the generalized equations of motion were derived points to another method for obtaining the stress modes[1]:

> "For each required stress mode, solve the static state of stresses in the structure, when subjected to a static distributed load of $m\omega_r^2$ times the modal deflection $h\phi_r(x,y)$ ".

This method is readily applied both for structures solvable analytically, and for more complex ones solvable by any of the computer codes used for static solution of linear systems. A statical analysis technique thus yields information on the dynamic behavior of the structure.

Once the stress modes have been obtained, the points in the structure where maximum stresses occur can be found out for a given mode. If the resonance frequencies are well apart, the structure will usually respond with the basic (lower) mode, and examination of this mode, obtained by statical analysis, will show the points of expected maximum stresses in a dynamic response problem. Even when there are modes which lie close together, examination of their stress modes can provide a good prediction of the structure's most stressed points under random excitation. In such examination, one should bear in mind that the contribution of one mode to the total stress is inversly proportional to the square of the frequency.

IV Geometrically nonlinear response

The statical behavior of geometrically nonlinear elastic structure can be approximated by a cubic relation between the load and the deflection (e.g. reference 4). Due to the hardening or softening effect of the nonlinearity, the frequency response curve of such a structure is bent around a "backbone" curve(3)(4)(5) . Using the same formula as in reference (6) , it can be shown that the following relationships exist between the general coordinate, the loading and the stresses.

$$\omega_{0r}^2\eta_r + \beta_r\eta_r^3 = \frac{P_r}{hMr} \quad (12)$$

$$\sigma_j = D_1^j\eta + D_2^j\eta^2 \quad (13)$$

Equation (12) is a static degeneracy of the dynamic nonlinear equation of motion.

$$\ddot{\eta}_r + 2\zeta_r\omega_{0r}\dot{\eta}_r + \omega_{0r}^2\eta + \beta_r\eta_r^3 = \frac{P_r}{hM_r} \quad (14)$$

Where β is the coefficient of amplitude nonlinearity, D1 the linear coefficient between stress and deflection, D2 the coefficient of nonlinearity of stresses, and Pr the generalized force acting on the rth mode. If D1 and D2 are known for a specific structure, its response to stationary random excitation with Gaussian distribution will be given by the following expression for the expected value of the amplitude

$$E[(w/h)^2] = \phi^2(x,y)E[\eta^2] \quad (15)$$

where $E[\eta^2] = \frac{\iint[\phi(x,y)dxdy]^2}{\Omega^4h^2M^2}\frac{1}{2\pi}\int_0^\infty |H(\omega)|^2F(\omega)d\omega \quad (16)$

and $\Omega^2 = \omega_0^2 + 3\beta E[\eta^2] \quad (17)$

The corresponding expression for the expected value of the stress reads :

$$E[\sigma_j^2] = D_1^{j\,2}E[\eta^2] + 3D_2^{j\,2}\{E[\eta^2]\}^2 \quad (18)$$

Since β, D1, and D2 depend only upon the geometry and material of the structure but not on its loadings, they can be found by applying a static load to the system (numerically or experimentally), and tabulating load versus deflection and stress versus deflection. From the tables, the best cubic curve is fitted according to eq. 12 to yield β and the best quadratic line according to eq. 13 to yield D1 and D2. Thus a statical technique is again used to obtain coefficients for analysis of the dynamic behavior of the structure.

V Engineering Oriented Procedure

The approach described in the preceding chapters can be translated into an engineering oriented procedure which is demonstrated schematically in Figure 1. Data on the basic characteristics of the structure (enclosed in a dashed-line rectangle) must be gained one way or another. They can be calculated analytically for a certain (limited) number of cases, by formulating the stiffness and mass matrices, and solving the eigenvalue problem, the stiffness matrix serving also for analysis of the statical behavior of the structure; they can be calculated numerically by any available method (finite-element, finite-difference computer codes); finally, they can be determined experimentally, with the results of dynamic and static tests performed on the structure serving as input to the analysis. It should be noted that, in order to establish (analytically or numerically) the factor of geometrical nonlinearities of the structure, monlinear analysis should be used.

With the basic dynamic and statical properties of the structure known, its dynamic behavior can be calculated. The concept of stress modes, presented earlier, yields a general idea of the dynamic behavior via statical techniques. The examination of the stress modes reveals the points in which maximal stresses exist in the dynamic response of the structure. This process contributes to the intuitive understanding of the results by the design engineer. For a given loading, the generalized forces can be calculated, or, if the loading (deterministic or random) is uniformly distributed over the structure, certain integrals of the mode shapes are required.

Following these steps, the linear response of the structure (in terms of displacements and

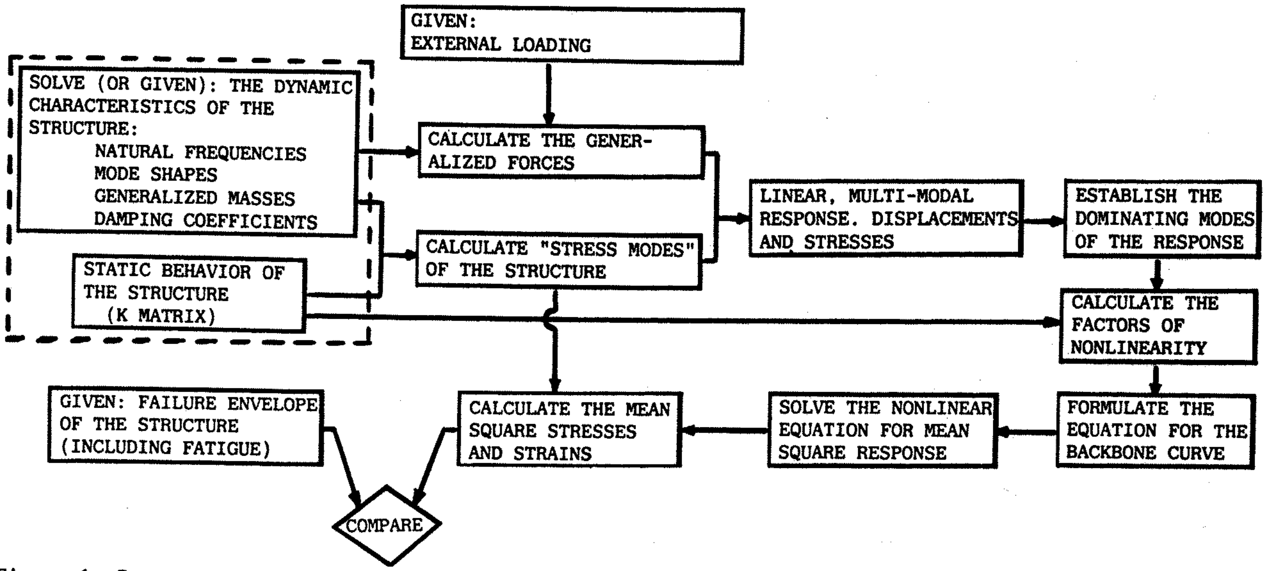

Figure 1: Steps in the Engineering Oriented Procedure.

stresses) to random excitation forces can be calculated. Usually, the response has one dominating mode, and the preceding analysis identifies it. It also indicates according to the magnitude of the response amplitude relative to a characteristic dimension of the structure (e.g. thickness of a plate or a shell), whether the nonlinearity factor of the structure should be determind. The methods to be used in the latter case are described in chapter IV.

After the mean-square displacement reponse has been calculated, the mean square of the stresses and the strains in the structure can be found with the aid of the stress modes. These can be compared to allowable dynamic stresses, taking into account the fatigue characteristics of the structure and its failure envelope.

Based on the theoretical background presented in the preceding chapters and on the block diagram presented in Figure 1, a computational procedure is proposed. The procedure is composed of three major parts:

1. A computer program for the dynamic response, as formulated above.

2. Data files, which include characheristic data of the structure and the excitations. The data may be obtained analytically, numerically or experimentally.

3. A numerical computer code for the analysis of structures, which has at last the following capabilities:

   (a) Solution of the eigenvalues problem of a given structure.
   (b) Linear statical analysis of an elastic structure.
   (c) Nonlinear static analysis.

By calculating the modal shapes and the resonance frequencies, the user can then calculate the stress modes by mean of the static analysis program. In cases where nonlinear analysis is required, data on the nonlinear characteristics of the structure is obtained by using the static nonlinear capability.

The process is described schematically in Figure 2. More details can be found in reference (7).

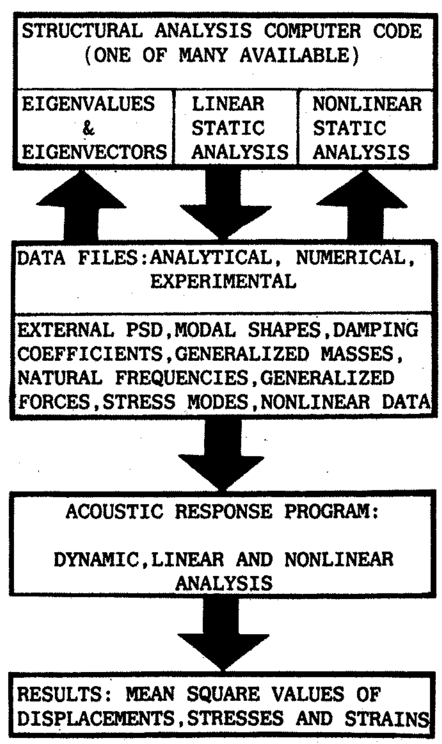

Figure 2: Basic Scheme of the Computational Process.

VI Conclusion

The main purpose of the suggested approach is to provide the user, the design engineer, with a systematic procedure which will give him insight into the physical meaning of the solutions he obtains when he solves a random dynamic problem. Statical analysis, more accessible to most users, serves as basic tool while a new concept, stress modes, facilitates understanding of the dynamic behavior. The procedure was developed by the author into a practical working method at the Lockheed-Georgia Company, and numerous test cases were run successfully for multilayered composite structural elements.

The procedure proved simple, quick, convenient in use and particularly suitable to design engineers.

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