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SENSITIVITY OF REDUCED FLIGHT DYNAMIC MODEL DEPENDING ON ELASTICITY OF AIRCRAFT STRUCTURE

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Abstract

In this paper a procedure of synthesis of dynamic sensitivity model of elastic structure aircraft depending on generalised parameter system reduction, coresponding to the each combination of reducing, is presented. By using this procedure we can define the criterion of system reduction as infinum of norm of sensitivity vector function depending on coresponding parameters of system reduction. Generalised discrete parameters of system reduction is simultaneosly presented in coupled form of complete and reduced dynamics models. The measure of elastic structure influence on flight dynamic behaviour of aircraft is defined as the supremum of norm of sensitivity functions depending on the coreposnding parameters of system reduction. Calculation of sensitivity functions norm is realised on the start of complete flight dynamic model synthesis of elastic structure aircraft, without determining of reduced models. By using a procedure of system reduction the synthesis of aproximate reduced sensitivity dynamic model of the mentioned system is also presented in the paper. It means that it is possible to determine the measure of influence of the structural dynamics on flight dynamic behaviour of aircraft and the best reduced aproximation of the system model by knowing a starting version only.

Introduction

In flight dynamic analysis of aircraft motion the influence of aircraft structure dynamics is of interest. This influence produces the diference between it's system state and state coresponding to the approximation of flight dynamics as a rigid structure aircraft. In a further text of this paper the mentioned diference of states is called as a system error, caused by elasticity of aircraft structure.

In the paper |6| a procedure of synthesis of flight dynamic model of elastic structure aircraft is presented. Given reduced flight dynamic model is presented in a general form in which the procedure of system reduction is a main problem. In order to complete procedure of system reduction the synthesis of sensitivity flight dynamic model of elastic structure aircraft is presented in this paper. Sensitivity dynamic model can be used for solving the following specified problems:

- determination of criterion of system reduction;
- determination of optimal system reduction;
- determination of measure of elastic structure influence or the measure of uper limitts of system error;
- determination of local stability of the complete system.

In this paper the determination of criterion of system reduction is presented.

Consider the initial complete flight dynamic model of elastic structure aircraft form. If system model includes n generalised coordinates of elastic displacements there exists n possible variants of system reduction. Each of them can be terminated with corresponding parameter of system reduction. If m ($m < n$) generalised coordinates of elastic displacement are included in a reduced form of the system we say that rang system reduction is equal m. It means that only one reduced form of system model with rang m exists, or the reduced system form of rang m is uniquelly defined.

The critterion of system reduction is defined as minimum value of natural number m which corresponds to the uper limit of the system error. The system error is defined as a norm of generalised vector of errors of aech generalised coordinates of the system state.

In order to define the critterion of system reduction the procedure of synthesis of sensitivity flight dynamic model of elastic structure aircraft depending on generalised parameter of system reduction is presented. The generalised parameter of system reduction is asumed as a discrete one, with corresponding values equal 0 or 1. If the parameter of system reduction is equal 0 a mathematical model of system dynamics is given in a reduced form, or if it is equal 1 system model is presented in a complete nonreduced form. If seams that unit perturbation of system parameter which value is equal 1 produce the changes in a system states as a transformation from nonreduced flight dynamic model of elastic structure aircraft to the reduced one. The measure of influence of system reduction is proportional to the norm of system sensitivity vector. Finally, the system error between system states is proportional to the norm of system sensitivity vector. This conclusion is a basic statement in procedure of determination of critterion of system reduction.

Basic equations

Flight dynamic model of elastic structure aircraft in complete nonreduced form can be written in the following mathematical formualtion:

$$\begin{aligned} \dot{X}_1 &= A_{11}(X_1,X_3)\,X_1 + A_{12}(X_1,X_3)\,X_2 + \\ &+ A_{13}(X_1,X_3)\,X_3 + B_{1\delta}(X_1,X_3)\delta + \\ &+ B_{1\delta}(X_1,X_3)\delta + B_{1\delta}(X_1,X_3)\delta + Q_1 \quad (1) \\ \dot{X}_2 &= A_{21}\,X_1 + A_{22}\,X_2 + A_{23}\,X_{23} + B_{2\delta}\,\delta + \\ &+ B_{2\delta}\,\delta + Q_2 \\ \dot{X}_3 &= A_{31}\,X_1 + A_{32}\,X_2 + A_{33}\,X_3 + B_{3\delta} + \\ &+ B_{3\delta}\,\delta + B_{3\delta}\,\delta + Q_3 \end{aligned}$$

where subvector X_1, X_2 and X_3 are given in the following subvector forms

$$\begin{aligned} X_1 &= \{\, V \quad \omega \,\}^T \\ X_2 &= \{\, q \quad K \}^T \qquad (2) \\ X_3 &= \{\, \xi \quad S \}^T \end{aligned}$$

in which the V is a vector of generalised coordinates of the linear velocity of aircraft motion and ω is a vector of generalised coordinates of angular velocity of aircraft motion, both of them coresponding to the fixed aircraft coordinate system in a point of center of gravity of its undeformable structure. Vector k is the first derivation of the vector q which represents the vector of nodal values of potential of airflow around the aircraft deformable structure. Vector S is the first derivation of the nodal elastic displacements of elastic aircraft structure presented with vector, corresponding to the fixed coordinate system.

Explanation of procedure of synthesis of given mathematical model is presented in the paper | 6|.

Reduced form of the flight dynamic model explained by equations (1) given in the following form:

$$\dot{X}_1 = A_{11}(X_1, X_2^1, X_3^1)\cdot X_1 + A_{12}(X_1, X_2^1 X_3^1)\cdot X_2^1 + A_{13}(X_1, X_2^1, X_3^1)\cdot X_3^1 + B_{1\delta}(X_1, X_2^1, X_3^1)\cdot\delta + B_{1\dot{\delta}}(X_1, X_2^1, X_3^1)\cdot\dot{\delta} + B_{1\ddot{\delta}}(X_1, X_2^1, X_3^1)\cdot\ddot{\delta} + Q_1 \quad (3)$$

$$\dot{X}_2 = A_{21}\cdot X_1 + A_{22}\cdot X_2^1 + A_{23}\cdot X_3^1 + B_{2\ddot{\delta}}\delta + B_{2\dot{\delta}}\dot{\delta} + Q_2$$

$$\dot{X}_3 = A_{31}\cdot X_1 + A_{32}\cdot X_2^1 + A_{33}\cdot X_3^1 + B_{3\ddot{\delta}}\dot{\delta} + B_{3\dot{\delta}}\delta + Q_3$$

where subvectors $\{X_2\}$ and $\{X_3\}$ are

$$\{X_2\} = \{X_2^1 \quad X_2^2\}^T \quad \{X_3\} = \{X_3^1 \quad X_3^2\}^T \quad (4)$$

Consider the following vectors in a forms

$$Y_2 = \{X_2^1 \quad X_3^1\}^T \qquad Y_3 = \{X_2^2 \quad X_3^2\}^T \quad (5)$$

which must satisfied matrix relation

$$Y_3 = - L\, Y_2 \quad (6)$$

where L is matrix of reducing tramsformation of elastic generalised coordinates.

Transformation matrix L is a solution of the Ricatti matrix algebraic equation

$$a_{32} - a_{33}L + La_{22} - La_{23}\, L = 0 \quad (7)$$

where matrices of coeficients are given in the next matrix forms:

$$[a_{22}] = \begin{bmatrix} A_{22}^{11} & A_{23}^{11} \\ A_{32}^{11} & A_{33}^{11} \end{bmatrix} \quad [a_{23}] = \begin{bmatrix} A_{22}^{12} & A_{23}^{12} \\ A_{32}^{12} & A_{33}^{12} \end{bmatrix}$$

$$[a_{32}] = \begin{bmatrix} A_{22}^{21} & A_{23}^{21} \\ A_{32}^{21} & A_{33}^{21} \end{bmatrix} \quad [a_{33}] = \begin{bmatrix} A_{22}^{22} & A_{23}^{22} \\ A_{32}^{22} & A_{33}^{22} \end{bmatrix} \quad (8)$$

where are the uper index corresponds to the subvector transformations presented with relations (4) and (5). Subvectors $\{X_2^2\}$ and $\{X_3^2\}$ are neglected.

Synthesis of sensitivity dynamic model of linear reduced system

Procedure of system reduction is presented in a paper |6|. By introduction of discrete system reducing parameter μ_i corresponding to the system reduction which rang is equal i, it is possible to writte the complete initial and reduced models in the integrated form as

$$\dot{X} = A(\mu_i)\cdot X + B(\mu_i)\cdot u \quad (9)$$

in which the value $\mu_i = 0$ coresponds to a reduced dynamic model of the system and $\mu_i = 1$ corresponds to the initial complete dynamic form of nonreduced system. In a case of linear system form equation (9) can be written by the following approximate matrix relation

$$\begin{vmatrix} \dot{X}_1 \\ \dot{X}_2 \end{vmatrix} = \left(\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} + (\mu-1)\cdot \begin{bmatrix} A_{12}L & A_{12} \\ A_{22}L & A_{22} \end{bmatrix} \right) \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \quad (10)$$

where matrix L is a solution of algebraic matrix Ricatti equation, given in a form

$$LA_{11} - LA_{12}L + A_{21} - A_{22}L = 0 \quad (11)$$

and matrix M is a solution of algebraic Ricatti matrix equation

$$M(LA_{12} + A_{22}) - (A_{11} - A_{12}L)M + A_{12} = 0 \quad (12)$$

Reduced model of the system is given in a two alternative forms which are approximatelly equal because the matrix M in approximation zero-matrix. In that case exists linear combination between generalised coordinates of the system given by the next matrix relation

$$X_2 = - LX_1 \quad (13)$$

We must note that exists a unique corespondation between dimension of reduced form of the system (which is equal to dimension of subvector X_1) and its parameter of system reduction .

Sensitivity dynamic model coresponding to the system form given by equation (9) can be presented in a following matrix form

$$u = A(\mu)\cdot u + \frac{\partial A}{\partial \mu}\cdot X + \frac{\partial B}{\partial \mu}\cdot u \qquad (14)$$

In a case of linear form of the mathematical model system, given by relation (10), sensitivity dynamic model can be presented by the next differential matrix equation

$$\begin{vmatrix} u_1 \\ u_2 \end{vmatrix} = \left(\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} + (\mu-1)\cdot \begin{bmatrix} A_{12}L & A_{12} \\ A_{22}L & A_{22} \end{bmatrix} \right) \begin{vmatrix} u_1 \\ u_2 \end{vmatrix} + \begin{bmatrix} A_{12}L & A_{12} \\ A_{22}L & A_{22} \end{bmatrix} \cdot \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} \qquad (15)$$

with initial conditions

$$u_1(to) = 0 \qquad u_2(to) = 0 \qquad (16)$$

If sensitivity dynamic model is presented in a complete nonreduced form parameter μ is equal 1. Corresponding sensitivity dynamic model is written in the following form

$$\begin{vmatrix} \dot{u}_1 \\ \dot{u}_2 \end{vmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{vmatrix} u_1 \\ u_2 \end{vmatrix} + \begin{bmatrix} A_{12}L & A_{12} \\ A_{22}L & A_{22} \end{bmatrix} \cdot \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} \qquad (17)$$

For the case of sensitivity dynamic model of reduced system form, vector of sensitivity functions is equal zero. If system is not reduced the second matrix form of relation (17) is a small vector, proportional to the vector δ, given in a form

$$\delta = LX_1 - X_2 \qquad (18)$$

Coeficients of proportionality are determined with submatrix A_{12} and A_{22}. Matrix A_{12} define the measure of influence of structure dynamics to a flight dynamics. Quadratic matrix A_{22} is a corresponding structural system matrix.

Criterion of system reduction

Ricatti matrix algebraic equation (11) has a real solution if exist a next condition of matrix norms:

$$||A_{22}^{-1}|| < \frac{1}{3}(||A_o|| + ||A_{12}||\cdot||L_o||)^{-1} \qquad (19)$$

where matrices A_o and L_o are given in the following fomrs

$$A_o = A_{11} - A_{12} L_o \qquad L_o = A_{22}^{-1} A_{21} \qquad (20)$$

Solution of the mentioned matrix algebraic Riccati equation must satisfy relation

$$0<||L-L_o|| < \frac{2||A_o||\cdot||L_o||}{||A_o|| + ||A_{12}||\cdot||L_o||} \qquad (21)$$

Presented statements are explained in a paper |3|.

The following norm can be written in a next form

$$||X_2+LX_1||<||X_2||+ \left(||L_o|| + \frac{2||A_o||\cdot||L_o||}{||A_o||+||A_{12}||\cdot||L_o||} \right) \cdot ||X_1|| \qquad (22)$$

In a case of homogenous initial conditions vector of sensitivity functions can be written in the next form

$$\begin{vmatrix} u_1 \\ u_2 \end{vmatrix} = \int_0^t \begin{bmatrix} \Phi_{11}(t-\tau) & \Phi_{12}(t-\tau) \\ \Phi_{21}(t-\tau) & \Phi_{22}(t-\tau) \end{bmatrix} \cdot \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} \cdot$$

$$\bullet(X_2 + LX_1)d\tau \qquad (23)$$

If the system matrix A is stable it is possible to calculate the finite value of norm in a form

$$\theta_{jk} = ||\int_o^t \theta_{jk}(t-\tau)d\tau|| \quad j,k=1,2 \quad (24)$$

Finally the norms of the vectors of sensitivity functions can be limited in a fomrs as

$$||u_1|| < (\theta_{11}\cdot||A_{12}|| + \theta_{12}\cdot||A_{22}||)$$
$$|||X_2||+(||L_o||+\frac{2||A_o||\cdot||L_o||}{||A_o||+||A_{12}||\cdot||L_o||})||X_1|| \qquad (25)$$

$$||u_2|| < (\theta_{21}\cdot||A_{12}|| + \theta_{12}\cdot||A_{22}||)$$
$$|||X_2||+(||L_o||+\frac{2||A_o||\cdot||L_o||}{||A_o|| + ||A_{12}|| \cdot ||L_o||}) \; ||X_1|||$$

By using relations (25), it is possible to determine the norm of sensitivity vector without solving given matrix Ricatti equation. It means that we can define the uper limit of the system error without calculation of system transformation matrix. Only one term what we must know is the asumed rang of the reduced form of the system.

Sensitivity dynamic model of elastic structure aircraft

Complete and reduced dynamic models of elastic structure aircraft can be written in a following integrated form

$$\dot{X}_1 = \bar{A}_{11}(X_1,Z_2,Z_3)X_1 + \left[\bar{A}_{12}(X_1,Z_2,Z_3) + (\mu-1)\bar{A}_{13}(X_1,Z_2,Z_3)\right]Y_2 +$$
$$+ \mu\bar{A}_{13}(X_1,Z_2,Z_3)Y_3 + \bar{B}_{1\delta}(X_1,Z_2,Z_3)\delta +$$
$$+ \bar{B}_{1\delta}(X_1,Z_2,Z_3)\delta + \bar{B}_{1\delta}(X_1,Z_2,Z_3)\delta + \bar{Q}_1$$

$$\dot{Y}_2 = \bar{A}_{21} X_1 + \left[\bar{A}_{22} + (\mu-1)\bar{A}_{23} L\right] Y_2 + \bar{A}_{23}Y_3 +$$
$$+ \bar{B}_{2\delta}\delta + \bar{B}_{2\dot{\delta}}\dot{\delta} + \bar{B}_{2\ddot{\delta}}\ddot{\delta} + \bar{Q}_2 \qquad (26)$$

$$\dot{Y}_3 = \bar{A}_{31} X_1 + \left[\bar{A}_{32} + (\mu-1)\bar{A}_{33}L\right] Y_2 + \bar{A}_{33} Y_3 +$$
$$+ \bar{B}_{3\delta}\delta + \bar{B}_{3\dot{\delta}}\dot{\delta} + \bar{B}_{3\ddot{\delta}}\ddot{\delta} + \bar{Q}_3$$

where subvectors Z_1 and Z_2 are given by

$$Z_2 = Y_2 + (\mu - 1)\cdot L\cdot Y_3$$

$$Z_3 = \mu\cdot Y_3$$

Sensitivity dynamic model of elastic structure aircraft can be written in a next form:

$$\dot{u}_1 = \bar{A}_{11} u_1 + [\bar{A}_{12} + (\mu-1)\bar{A}_{13}]\nu_2 + \mu\,\bar{A}_{13}\nu_3 +$$
$$+ \frac{\partial\bar{A}_{11}}{\partial\mu} X_1 + \left[\frac{\partial\bar{A}_{12}}{\partial\mu} + (\mu-1)\frac{\partial\bar{A}_{13}}{\partial\mu} + \bar{A}_{13}\right]\cdot Y_2 +$$
$$+ \left[\mu\frac{\partial\bar{A}_{13}}{\partial\mu} + \bar{A}_{13}\right]\cdot Y_3 + \frac{\partial\bar{B}_{1\delta}}{\partial\mu}\cdot\delta + \frac{\partial\bar{B}_{1\dot{\delta}}}{\partial\mu}\cdot\dot{\delta} + \frac{\partial\bar{B}_{1\ddot{\delta}}}{\partial\mu}\cdot\ddot{\delta}$$

$$\dot{\nu}_2 = \bar{A}_{21} u_1 + [A_{22} + (\mu-1)\bar{A}_{23}L]\;\nu_2 + \mu\,\bar{A}_{23}\,\nu_3 +$$
$$+ \bar{A}_{23}\,L\,Y_2 + \bar{A}_{23}\,Y_3 \qquad (27)$$

$$\dot{\nu}_3 = \bar{A}_{31} u_1 + [\bar{A}_{32} + (\mu-1)\bar{A}_{33}\,L]\;\nu_2 + \mu\,\bar{A}_{33}\nu_3 +$$
$$+ \bar{A}_{33}\,L\,Y_2 + \bar{A}_{33}\,Y_3$$

Determination of norm of sensitivity functions u_1, ν_2 and ν_3 can be done on a same way as for the linear system form. Norms of the vector ν_2 and ν_3 can be calculate by their denotation with vectors u_1 and u_2 in relations (25) correspondelly.

Conclusion

Presented procedure of system reducting included determination of system error as the result of system reduction is powerfull method for simulation of flight dynamic of elastic structure aircraft. Determination of system error is very simple because it needs not expection of norm of nonlinear part of the system. Starting point is defined with initial rang of subvector Y_2 equal 1 and corresponding rang of subvector Y_3 equal n-1 where n is a total number of adittional "elastic" and "flow" coordinates of the system state. By increasing the rang of subvector Y_2 from 1 to 2,3,... ...,m we can define the infinum of m which satisfied the condition that the system error, equal to the norm $\|u_1\|$ is less than assumed value. If the elastic dynamic influence on a total space motion of the aircraft is small we can change the terms $\|X_1\|$ and $\|X_2\|$ form equation (25) in approximation with the maximmum harmonic free deflection of elastic structure, if its value is known. In oposite case, we must simulate the vectors $\{X_1\}$ and $\{X_2\}$ of starting complete dynamic model and determine its norms.

Calculations of system eror in some hypotetical examples shows that it is practicaly small if we cut off system modes with corresponding frequences, greater several times than frequences of rigid body aircraft motion modes. In that cases the system error monotonically falls down with increasing the rang of subvector $\{Y_2\}$.

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