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IN-FLIGHT INERTIAL GUIDANCE ALIGNMENT

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Abstract

This work reports on an extension of a recent work presented by these authors. That work dealt with the observability analysis of inertial navigation systems during in flight alignment. The work was confined to the analysis of altitude damped systems. hence the error model considered was a two-channel model. The present work deals with an undamped three-channel error model. An analysis similar to the previous one is carried out. It is shown that it is possible to fully observe. and thus estimate. all the states of the system. This is in contrast to the previous two-channel system in which it is impossible to fully observe and estimate all the states of the system. The conclusions of the analysis presented in this paper are verified through covariance simulation which yields identical results.

Introduction

Inertial navigation systems (INS) belong to the family of dead-reckoning systems and as such have to be initialized before they can start navigating. The initialization is a crucial phase in the operation of the system since an INS can be only as good as its initial condition. In modern INS. initialization is accomplished using a Kalman-filter (KF) which estimates the INS velocity error and. most importantly. its misalignment angles. In the process. the INS sensor errors may also be estimated provided certain observability conditions are met.

In the early days. INS alignment was performed when the system was at rest. but later when INS was installed in moder air and sea launched vehicles. it became necessary to execute the alignment in-flight. While in-flight alignment (IFA) may seem to be less accurate and more complicated than alignment at rest. it turns out that the ability of the carrier to maneuver during the IFA phase is a blessing in disguise since it enables the excitation of certain latent modes and thus. enhance their observability and the observability of the whole system as well.

The observability issue is crucial since normally only velocity measurements are available while the number of parameters which need to be estimated is relatively large. The ability to properly estimate those parameters is then a function of the observability measure of the system.

The importance of observability in INS initial alignment has been recognized since the early days of state estimation [1]. A better understanding of INS behavior is gained from such analysis. This. in turn. enables us to improve the performance of the system. Such analysis. for example. indicates the alternative combinations of minimal measurements which can be used in order to turn the INS into a completely observable system [2].

The classical approach to observability analysis of a time-invariant system requires. as a first stage. the determination of the rank of the Observability Matrix. In the second stage. the state space is partitioned. in a somewhat arbitrary way. into observable and unobservable subspaces. then a transformation matrix can be defined which decomposes the dynamic system into observable and unobservable subsystems. This procedure can be applied to an INS at rest since its dynamics matrix is constant. In [2] an observability analysis of such a case was performed. The analysis made use of a special transformation which transformed the states of the system into the so-called Rate-State space.

The maneuvers which are usually performed during IFA cause the dynamics matrix of the error model to be time-varying. The observability analysis of time-varying systems is well known. but is complicated to perform and is not too informative. Consequently. a direct observability analysis during IFA involves tedious mathematical work with little constructive conclusions. Therefore in [3] a special IFA trajectory was defined in order to create a piece-wise constant dynamics model. Although this trajectory is never performed by the carrier of the INS. it is a very good approximation of a typical IFA maneuver and yields similar results. For this trajectory which yields a piece-wise constant dynamics model. a Stepped Observability Matrix (SOM) was defined in

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[3] where it was proven than when a general piece-wise constant system satisfies certain conditions concerning the null space of each constant stage, the null space spanned by its SOM is equal to the null space of the piece-wise constant system. As it turned out, the piece-wise constant error model of the INS during the special IFA trajectory defined in [3], satisfied these conditions. Consequently, an SOM could be used for the observability analysis of the two-channel INS during IFA. The use of that SOM enabled a simple analysis of the change of observability due to change in trajectory. This yielded a clear understanding of the physical phenomena which occur during IFA.

The conclusions drawn in [3] for the two-channel INS model were as follows:

- The observability of the two-channel INS can be improved by a stepped IFA trajectory which consists of only two segments in which constant horizontal specific force, which differs from segment to segment, is experienced by the system.
- The order in which the segments are flown is immaterial.
- The number of unobservable states is 2 out of the total 10 which are used to describe the error propagation of INS during IFA.
- To estimate the entire state vector, 2 additional states have to be measured. They have to be measured out of 2 groups of states, each chosen from a different group.

Description of the Present Work

The present work describes the observability analysis of a three channel INS during IFA. The model consists of the following 12 states:

V_N - north component of the velocity error
V_E - east component of the velocity error
V_D - down component of the velocity error
P_N - north component of the misalignment
P_E - east component of the misalignment
P_D - down component of the misalignment
B_N - north accelerometer bias
B_E - east accelerometer bias
B_D - down accelerometer bias
D_N - north gyro constant drift
D_E - east gyro constant drift
D_D - down gyro constant drift

The observability of the system is analyzed for a piece-wise constant trajectory which consists of 5 segments. The segments are characterized by their constant specific forces. They are described in Table I below in which F denotes the specific force along the segment and the subscripts N,E and D denote the north, east and down components respectively.

After developing the necessary mathematical tools this work examines the observability of the 12-state INS model along this trajectory.

Results

The conclusion of the analysis is summarized as follows:

> For any single segment of this trajectory, the rank of the observability matrix is 9. Consequently 9 states or linear combinations of states can be estimated at any single segment.

Table I Trajectory Definition

Number of Segment	Maneuver Characteristics	F_N	F_E	F_D
1	straight and level flight (SLF)	0	0	-g
2	SLF with north acceleration	F_N	0	-g
3	SLF with east acceleration	0	F_E	-g
4	SLF with horizontal acceleration	F_N	F_E	-g
5	pull up or dive	0	0	F_D

- The conclusion as to which state or combination of states are observable is determined using a transformation to the Canonical Observable Form.
- An IFA trajectory which consists of 2 segments will render 11 observable states or linear combinations of states.
- An IFA trajectory which consists of 3 segments will render a completely observable system, consequently all the states of the original (untransformed) system are observable.

A covariance simulation was carried out for the 12 state INS models described above. The results of the simulation were identical to the results of the analytic investigation.

Conclusions

A systematic theoretical analysis of an INS during IFA can be carried out in a way which shades light on the observability condition of the system. The importance of such analysis is manifested in particular in the ability to determine whether the system can be made completely observable and hence completely estimable, in the ability to determine which maneuvers are essential and, in case the system cannot be made completely observable, in the ability to determine which are the possible sets of measurements needed for turning the system into a completely observable one. The analysis is made possible by describing the IFA maneuvers as a combination of piece-wise constant specific force maneuvers. In order to determine which are the observable states and combination of states, the original INS error model is transformed into a Canonical Observable Form. The main analysis tool is the Stepped Observability matrix.

It was found that for the 12-state 3-channel INS, three segments of the IFA trajectory are sufficient to turn the system into a completely observable one. This is in contrast to the case of IFA of 10-state 2-channel INS model of altitude damped system where only 8 out of the 10 states can be made observable by IFA maneuvers.

Recommendations for further work are the addition of accelerometer and gyro scale factor errors to the INS error model and the repeated analysis of 2 and 3-channel INS for this case, and the extension of the analysis to the case of laboratory INS calibration.

References

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