INTEGRATED STRUCTURAL-AERODYNAMIC DESIGN OPTIMIZATION

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Abstract

This paper focuses on the processes of simultaneous aerodynamic and structural wing design as a prototype for design integration. We concentrate on the major difficulty associated with multidisciplinary design optimization processes, their enormous computational costs. Methods are presented for reducing this computational burden through the development of efficient methods for cross-sensitivity calculations and the implementation of approximate optimization procedures. Utilizing a *modular sensitivity analysis* approach, we show that the sensitivities can be computed without the expensive calculation of the derivatives of the aerodynamic influence coefficient matrix, and the derivatives of the structural flexibility matrix. The same process is used to efficiently evaluate the sensitivities of the wing divergence constraint, which should be particularly useful, not only in problems of complete integrated aircraft design, but also in aeroelastic tailoring applications.

Introduction

The introduction of composite materials is having a profound effect on aircraft design. Since these materials permit the designer to tailor material properties to improve structural, aerodynamic and acoustic performance, they require an integrated multidisciplinary design process. Furthermore, because of the complexity of the design process numerical optimization methods are required.

The utilization of integrated multidisciplinary design procedures for problems in aircraft design is not currently feasible because of the enormous computational burden. Even with the expected rapid growth of supercomputers and parallel architectures, these tasks will not be practical without the development of efficient methods for cross-disciplinary sensitivities and efficient optimization procedures.

The present research is part of an on-going effort which is focused on the processes of simultaneous aerodynamic and structural wing design as a prototype for design integration. A sequence of integrated wing design procedures have been developed in order to investigate various aspects of the design process.

In their initial efforts, the authors considered the integrated design of a high aspect-ratio sailplane wing. The sailplane mission was used to illustrate the advantages of including aerodynamic and structural interactions in the design process, by optimizing for circling flight in a thermal current followed by cross-country cruise. Furthermore, the simplicity of the sailplane wing planform and structural design allowed for the use of rudimentary analysis methods, (lifting-line and beam theory). The simplicity of these analyses made it feasible to calculate all the sensitivity derivatives of the aerodynamic shape and structural sizes, along with all the cross-sensitivity derivatives, directly, without any further approximation, at each step of the numerically optimized design process. The results, reported in Ref. 1, demonstrated that integrating the structural and aerodynamic design processes leads to wing designs superior to those obtained by the traditional sequential approach.

The next step of the integrated wing design procedure study again involved the sailplane wing design, but with analysis methods which are representative of methods used for low-speed aircraft wing designs. The utilization of a vortex-lattice method and a structural finite-element method, while providing for a more exact analysis and allowing for more general wing shapes, introduced the need for more design variables and constraints, and were significantly more expensive to use in the design process. In Ref. 2, it was shown that by incorporating perturbation methods for cross-sensitivity calculations and approximate optimization procedures, an estimated 10 hours of IBM 3084 CPU time for a complete integrated design, was reduced to less than ten minutes.

The present paper represents the third step of this study. The objective here is to develop an integrated wing design procedure for a subsonic transport aircraft. We still consider the use of the vortex-lattice method for the aerodynamics (so that we are restricted to subsonic speeds) and a finite-element analysis of the wing structure. Even with the elementary aerodynamic analysis and basic aerodynamic design variables, (planform shape and twist distribution), the increased complexity of an integrated transport design over the previous sailplane wing design requires further computational reductions. We consider two approaches for reducing the computational burden of multidisciplinary optimization:

i. the development of efficient methods for cross-sensitivity calculation; and

ii. the use of approximate optimization procedures.

The sensitivity calculation is based on a recent development (Ref. 3) which shows how sensitivity derivatives of a system may be computed via partial sensitivity derivatives of the output with respect to the input and to the design variables of each component of the system. This approach, that may be termed *modular sensitivity analysis*, corresponds to the abstraction of a system as an assembly of interacting *black boxes*. This system is known to be a useful tool for constructing efficient computational sequences and data flow patterns for the purposes of the system solution, Ref. 4. It allows for the calculation of sensitivity derivatives of a system with a higher accuracy and, in most cases, at a lower cost than with conventional finite differencing. The system sensitivity derivatives may be used to guide a formal optimization and a Newton's method solution of the coupled interdisciplinary equations describing the system behavior. Within this framework, we show that the sensitivities can be computed without the expensive calculation of the derivatives of the aerodynamic influence coefficient matrix, and the derivatives of the structural flexibility matrix. In Ref. 2, these derivatives represented a substantial portion of the computational cost of an integrated design.

Furthermore, the same process, in application to the wing divergence constraint, enables the determination of the sensitivity of the divergence dynamic pressure with respect to a design parameter without the determination of the derivatives of the aerodynamic influence coefficient matrix and flexibility matrix. This feature should be particularly useful, not only in problems of complete integrated aircraft design, but also in aeroelastic tailoring applications.

Integrated Design Problem

We consider the optimum design of an aircraft wing. The objective function can be the structural weight of the wing, an aerodynamic performance index such as the lift-to-drag ratio, L/D or a combination thereof. In the present study we minimize the structural weight of the wing. The design variables associated with the aerodynamic design include the planform shape parameters defined on Figure 1, and the twist schedule along the span.

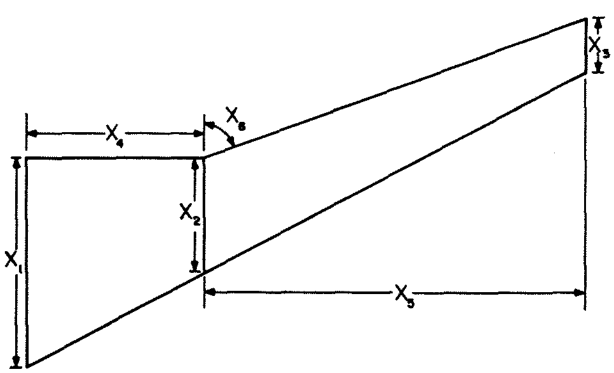

Figure 1. Planform Design Variables

For the present, preliminary study of integrated structural-aerodynamic design, we assume the airfoil shape to be supplied, along with known section characteristics. The design variables associated with the structural design are the structural sizes including panel thicknesses and spar-cap cross-sectional areas. The finite-element model of the wing is shown schematically in Figure 2. Additionally, composite material ply orientations in the cover panels are used as design variables.

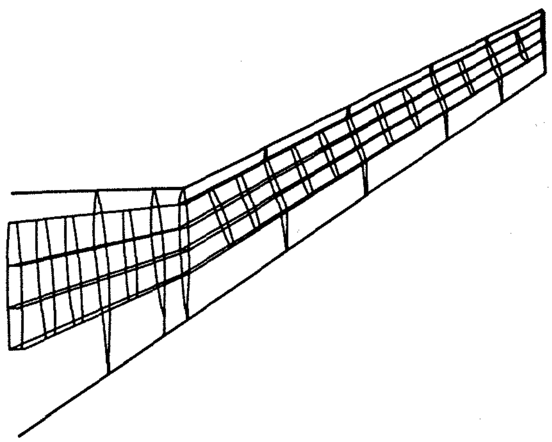

Figure 2. Finite-Element Model of Wing Structure

Constraints are placed on the magnitudes of stresses and strains in the structure, on the aeroelastic divergence speed, and on aerodynamic performance measures and stall conditions. Additional geometric constraints are imposed on the planform shape design variables to prevent unreasonable geometries.

The aerodynamic and structural response is calculated from a coupled set of equations discussed below. Aerodynamic performance is calculated at the cruise condition, while the limits on stresses and strain are applied for a high-g pull-up maneuver.

Aeroelastic Formulation

The aeroelastic analysis of the wing is simplified by making several assumptions. We assume that the effect of the aerodynamics on structural deformations can be approximated by lumping the aerodynamic forces at n_l structural grid points (called here the load set), and including only the vertical components of the loads. The vector of vertical aerodynamic loads is denoted as F_a. We assume that the overall aircraft response affects the wing only through the root angle of attack α. Finally, we assume that the effect of structural deformations on the aerodynamic response can be approximated in terms of the vector of vertical displacements θ at the load set.

The vertical aerodynamic loads at the load set, F_a, are determined from an aerodynamic analysis procedure. For low speed wing designs, we utilize a vortex lattice method (e.g., Ref. 5) to compute the lift and the induced drag. The wing is discretized into panels, with each panel containing an element of a horseshoe vortex of strength γ_j. By enforcing flow tangency at each panel, a vector of circulation strengths Γ may be computed from

$$V(p,\theta)\Gamma = C(p,\alpha,\theta) \tag{1}$$

where p is a vector of design parameters and V is a matrix of influence coefficients. The aerodynamic forces are computed from a local application of the Kutta-Joukowski theorem, and compressibility effects are included through a Göthert transformation. The profile drag for each wing section is calculated from the measured airfoil drag polar. The load vector F_a is then obtained as

$$F_a = F_a(p,\alpha,\theta,\Gamma) \tag{2}$$

Altogether we combine equations (1) and (2) as

$$F_a = f_1(p,\alpha,\theta) \tag{3}$$

The angle of attack is obtained from the overall vertical equilibrium of the aircraft as

$$f_2(p,F_a) = \frac{1}{2}nW - N^T F_a = 0 \tag{4}$$

where N is a summation vector, n is the load factor and W is the weight of the aircraft.

The vertical displacements at the load set are calculated by finite-element analysis using a modification of the WIDOWAC program (Ref. 6). First the nodal displacement vector U is calculated by solving

$$K(p)U = TF_a + nF_I(p) \tag{5}$$

where K is the stiffness matrix, T is a Boolean matrix which expands F_a to the full set of structural degrees of freedom, and F_I is the gravitational and inertia load vector. Strains and stresses are then calculated from the displacement vector U. The vertical displacements at the load set θ are extracted from U as

$$\theta = T^T U \tag{6}$$

Equations (5) and (6) can be combined as

$$\theta = f_3(p,F_a) \tag{7}$$

Solution Procedure

Equations (3), (4) and (7) are a set of nonlinear coupled equations for the vector of vertical aerodynamic loads, F_a, the wing root angle of attack, α and the vector of vertical displacements, θ. For the analysis problem, the vector of design parameters, p, is given. Reference 3 presented a modular sensitivity analysis of such coupled interdisciplinary equations. The modular approach permits treating the individual discipline analysis procedures as ***black boxes*** that do not need to be changed in the integration procedure. Here we employ a similar approach for the sensitivity analysis below, with f_1 representing an ***aerodynamic black box*** and f_3 a ***structural black box.*** We also use the same approach for the solution of the system via Newton's method.

Given an initial estimate for the solution F_a^0, α^0, θ^0 we use Newton's method to improve that estimate. The iterative process may be written as

$$J\Delta Y = \Delta f \tag{8}$$

where

$$\Delta Y = \begin{Bmatrix} \Delta F_a \\ \Delta\alpha \\ \Delta\theta \end{Bmatrix} \tag{9}$$

and

$$\Delta f = \begin{Bmatrix} f_1(p,\alpha^0,\theta^0) - F_a^0 \\ f_2(p,F_a^0) \\ f_3(p,F_a^0) - \theta^0 \end{Bmatrix} \tag{10}$$

and the Jacobian J is given as

$$J = \begin{bmatrix} I & -\partial f_1/\partial\alpha & -\partial f_1/\partial\theta \\ -\partial f_2/\partial F_a & 0 & 0 \\ -\partial f_3/\partial F_a & 0 & I \end{bmatrix} = \begin{bmatrix} I & -qR & -qA \\ N^T & 0 & 0 \\ -S & 0 & I \end{bmatrix} \tag{11}$$

The Jacobian is given in terms of the dynamic pressure q, the incremental aerodynamic force vector, qR, the aerodynamic influence coefficient matrix, qA and the flexibility matrix S. The incremental aerodynamic force vector is defined such that its component qr_i represents the change in F_{ai} due to a unit change in α, and the aerodynamic influence coefficient matrix, is defined such that its component qa_{ij} represents the change in F_{ai} due to unit change in θ_j. Similarly, the flexibility matrix, is such that s_{ij} is the change in θ_i due to a unit change in F_{aj}.

Partial solution of equation (8) yields the following three equations for the increments $\Delta\theta$, $\Delta\alpha$ and ΔF_a:

$$(I - qSA^x)\Delta\theta = SB\Delta f_1 + \frac{SR}{N^T R}\Delta f_2 + \Delta f_3 \tag{12}$$

$$\Delta\alpha = \frac{\Delta f_2 - N^T\Delta f_1 - qN^T A\Delta\theta}{qN^T R} \tag{13}$$

$$\Delta F_a = \Delta f_1 + qR\Delta\alpha + qA\Delta\theta \tag{14}$$

where we define

$$B \equiv I - \frac{RN^T}{N^T R} \tag{15}$$

and

$$A^x \equiv AB \tag{16}$$

In our case we start with a rigid wing approximation $F_a^0 = F_{ar}$, $\alpha^0 = \alpha_r$, $\theta^0 = 0$, where

$$F_{ar} = f_1(p,0,0) + q\alpha_r R \tag{17}$$

$$\alpha_r = \frac{\frac{1}{2}nW - N^T f_1(p,0,0)}{qN^T R} \tag{18}$$

and execute a single Newton iteration to approximate the flexible wing response.

The aeroelastic divergence instability is calculated at a fixed angle of attack, because it is assumed that the pilot does not react fast enough to change the angle of attack as the wing diverges. The instability is characterized by a homogeneous solution to Eq. (8), that is

$$\begin{bmatrix} I & -qA \\ -S & I \end{bmatrix} \begin{Bmatrix} \Delta F_a \\ \Delta \theta \end{Bmatrix} = 0 \tag{19}$$

Equation (19) is an eigenvalue problem for q. The lowest eigenvalue is the divergence dynamic pressure q_D. We denote the corresponding eigenvector as $[F_{aD}, \theta_D]^T$. Equation (19) can be reduced to a standard linear eigenproblem by substituting for $\Delta\theta$ in terms of ΔF_a to obtain

$$(AS - \frac{1}{q}I)\Delta F_a = 0 \tag{20}$$

Sensitivity Calculation

As stated above, it is common practice to follow the above procedure and use a single Newton's iteration in the analysis of a flexible wing. Then for a design problem, where derivatives with respect to a design parameter p are required, equations (12), (13) and (14) are differentiated with respect to p (e.g., Ref. 2). This approach requires the calculation of derivatives of the matrices A and S which can be very costly. Here, instead, we follow Ref. 3 and differentiate equations (3), (4) and (7) with respect to p to obtain

$$JY' = f' \tag{21}$$

where a prime denotes differentiation with respect to p and where

$$Y' = [\, F_a' \quad \alpha' \quad \theta' \,]^T \tag{22}$$

and

$$f' = [\, f_1' \quad f_2' \quad f_3' \,]^T \tag{23}$$

along with the definition $f_i' = \partial f_i / \partial p$ for $i = 1,2,3$. The Jacobian J appearing in equation (21) is the identical matrix utilized in the analysis in equation (11). Equation (21) can be partially solved to yield

$$(I - qSA^x)\theta' = SBf_1' + \frac{SR}{N^T R} f_2' + f_3' \tag{24}$$

$$\alpha' = \frac{f_2' - N^T f_1' - qN^T A\theta'}{qN^T R} \tag{25}$$

$$F_a' = f_1' + qR\alpha' + qA\theta' \tag{26}$$

This approach does not require any derivatives of A and S but only partial derivatives of f_1, f_2 and f_3. For example, f_1' denotes the derivative of F_a with respect to a design variable when α and θ are fixed.

By contrast, the more traditional approach (e.g., Ref. 2) to the derivative calculation is obtained by differentiating the aeroelastic analysis equations, such as Eqs. (12) to (14) with respect to p. For example, consider the derivative of Eq. (12) with respect to p

$$\begin{aligned}(I - qSA^x)\Delta\theta' = {} & qS'A^x\Delta\theta + qSA'B\Delta\theta + qSAB'\Delta\theta \\ & + S'B\Delta f_1 + SB'\Delta f_1 + SB\Delta f_1' \\ & + \frac{S'R}{N^T R}\Delta f_2 + S(\frac{R}{N^T R})'\Delta f_2 + \frac{SR}{N^T R}\Delta f_2' \\ & + \Delta f_3' \end{aligned} \tag{27}$$

This complicated expression can be shown to be equivalent to Eq. (24). However, the traditional approach which employs Eq. (27) requires the expensive calculation of the derivatives of the aerodynamic influence coefficient matrix, A' and the derivatives of the flexibility matrix S'.

To find the derivative of the divergence dynamic pressure q_D with respect to a design parameter p, we differentiate Eq. (19) at $q = q_D$ with respect to p

$$\begin{bmatrix} I & -q_D A \\ -S & I \end{bmatrix} \begin{Bmatrix} F_{aD}' \\ \theta_D' \end{Bmatrix} + \begin{bmatrix} 0 & -(q_D A)' \\ -S' & 0 \end{bmatrix} \begin{Bmatrix} F_{aD} \\ \theta_D \end{Bmatrix} = 0 \tag{28}$$

We premultiply Eq. (28) by the left eigenvector of Eq. (19), $[F_{aL}^T, \theta_L^T]$, defined by

$$[F_{aL}^T, \theta_L^T] \begin{bmatrix} I & -q_D A \\ -S & I \end{bmatrix} = 0 \tag{29}$$

and obtain

$$[F_{aL}^T, \theta_L^T] \begin{bmatrix} 0 & -(q_D A)' \\ -S' & 0 \end{bmatrix} \begin{Bmatrix} F_{aD} \\ \theta_D \end{Bmatrix} = 0 \tag{30}$$

or

$$q_D' = -\frac{q_D F_{aL}^T A' \theta_D + \theta_L^T S' F_{aD}}{F_{aL}^T A \theta_D} \tag{31}$$

Equation (31) contains derivatives of A and S with respect to p which we have managed to avoid before. However, the corresponding terms can be simplified. Using the definition of S, Eq. (11), we note that

$$S'F_{aD} = \frac{\partial}{\partial p}\left(\frac{\partial f_3}{\partial F_a}\right) F_{aD} \tag{32}$$

To see how $S'F_{aD}$ can be calculated without obtaining S' consider a more generic case. Let f be a function of a vector X, and let D be another vector. Let X_0 be a particular choice for X, then

$$\begin{aligned}\frac{\partial f}{\partial X}(X_0)D &= \lim_{\epsilon \to 0} \frac{1}{\epsilon}[f(X_0 + \epsilon D) - f(X_0)] \\ &= \frac{d}{d\epsilon} f(X_0 + \epsilon D)\end{aligned} \tag{33}$$

Equation (33) provides us with a way of calculating the product $\partial f / \partial X(X_0)$ times D without calculating the individual components of $\partial f / \partial X$. Therefore, to calculate $S'F_{aD}$ we start by calculating the derivative of f_3 to a perturbation in F_a in the form of F_{aD} (because we use linear structural analysis this is the response of the structure to F_{aD}). Then we calculate the derivative of this response with respect to p assuming that F_{aD} is fixed. The term $A'\theta_D$ in Eq. (31) is treated in a similar way.

Approximate Optimization Procedure

The optimization problem addressed in this paper is to minimize the structural weight W of the wing subject to aerodynamic, performance and structural constraints. It can be written as

$$\begin{aligned} &\textit{minimize} \quad W(p) \\ &\textit{such that} \quad g_1(\Gamma, p) \geq 0 \\ &\qquad\qquad\quad g_2(\Gamma, \alpha, p) \geq 0 \\ &\qquad\qquad\quad g_3(U, p) \geq 0 \end{aligned} \tag{34}$$

where g_1, g_2 and g_3 denote aerodynamic, performance, and structural constraints, respectively. The vector of circulation strengths, Γ is calculated from Eq. (1) and the nodal displacement vector, U, is calculated from Eq. (5).

Even with the more efficient sensitivity analysis, a fully coupled structural-aerodynamic analysis and sensitivity is quite expensive. Thus, it is not feasible to optimize the design problem by directly connecting an optimization algorithm with the analysis procedure. Instead, a sequential approximate optimization algorithm is considered to be the best approach (e.g., Ref. 7). This approach replaces the original objective function and constraints with approximations based upon nominal values and derivatives at an initial point. Move limits are used to prevent the design from moving outside the bound of validity of the approximations.

The approximate optimization problem is based on a linear approximation of the aerodynamic and structural constraints about a candidate design point p_0. That is, the approximate constraints g_{1a} and g_{3a} are given as

$$\begin{aligned} g_{1a}(p) &= g_1(p_0) + g_1'(p_0)\Delta p \\ g_{3a}(p) &= g_3(p_0) + g_3'(p_0)\Delta p \end{aligned} \tag{35}$$

where $\Delta p = p - p_0$. The performance constraints are typically quite nonlinear, and inexpensive to calculate, so they are calculated exactly from the linear approximation to the aerodynamic solution. The approximate optimization problem is given then as

$$\begin{aligned} &\textit{minimize} \quad W(p) \\ &\textit{such that} \quad g_{1a}(p) \geq 0 \\ &\qquad\qquad\quad g_2(\Gamma_a, \alpha_a, p) \geq 0 \\ &\qquad\qquad\quad g_{3a}(p) \geq 0 \\ &\qquad\qquad\quad \|\Delta p\| \leq E \end{aligned} \tag{36}$$

where E represents a vector of move limits imposed to guarantee the quality of the approximation.

The approximate optimization problem is solved sequentially as shown in Fig. 3, till the change in the design is smaller than a specified tolerance or the improvement in the objective function is small than another tolerance. After an optimum is found, a new approximation is constructed there, and the process is repeated until convergence is achieved.

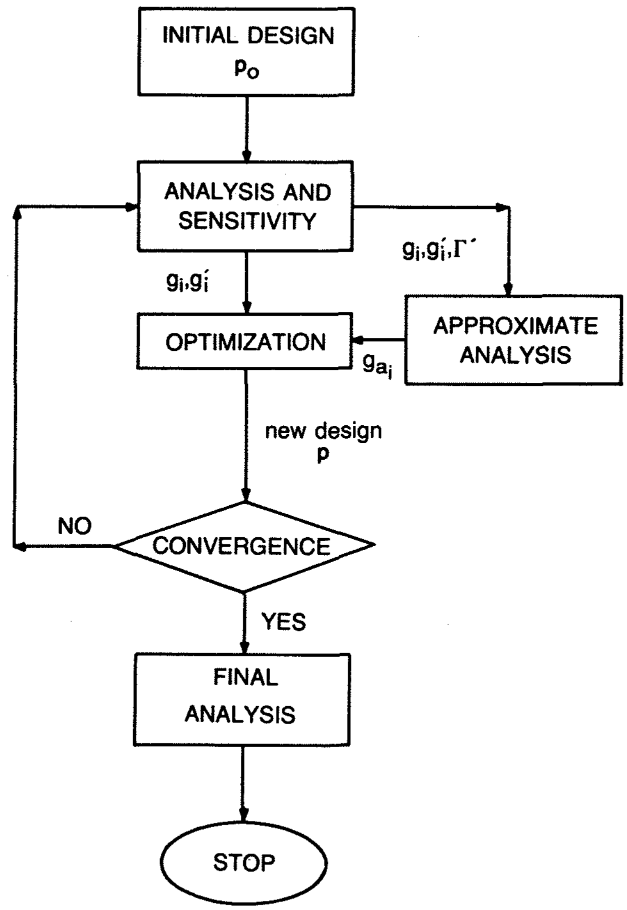

Figure 3. Flowchart of Optimization Procedure

The optimizer used is the NEWSUMT-A program, Ref. 8, which is based on an extended interior penalty function procedure, and allows for various levels of constraint and objective function approximations.

Concluding Remarks

This paper focused on the processes of simultaneous aerodynamic and structural wing design as a prototype for design integration. The research concentrated on the major difficulty associated with multidisciplinary design optimization processes, their enormous computational costs. Methods were presented for reducing this computational burden through the development of efficient methods for cross-sensitivity calculations and the implementation of approximate optimization procedures. Utilizing a *modular sensitivity analysis* approach, we showed that the sensitivities can be computed without the expensive calculation of the derivatives of the aerodynamic influence coefficient matrix, and the derivatives of the structural flexibility matrix. The same process was used to efficiently evaluate the sensitivities of the wing divergence constraint, which should be particularly useful, not only in problems of complete integrated aircraft design, but also in aeroelastic tailoring applications. Computational results for the integrated aerodynamic–structural design of a foward swept wing for a transport aircraft will be given in Ref. 9.

Acknowledgment

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References

1. Grossman, B., Strauch, G. J., Eppard, W. M., Gurdal, Z. and Haftka, R. T., "Integrated Aerodynamic – Structural Design of a Sailplane Wing", *AIAA Paper No. 86-2623*, Oct. 1986. (To appear in *J. Aircraft*).
2. Haftka, R. T., Grossman, B., Eppard, W. M. and Kao, P. J., "Efficient Optimization of Integrated Aerodynamic – Structural Design", *Proceedings of 2nd Intl. Conf. on Inverse Design Concepts and Optimization in Engineering Sciences*, Oct. 1987, pp. 369–386.
3. Sobieszczanski-Sobieski, J., "On the Sensitivity of Complex, Internally Coupled Systems", *AIAA Paper No. 88-2378*, presented at the *AIAA/ ASME/ ASCE/ AHS 29th Structures, Structural Dynamics and Materials Conference*, Apr. 1988.
4. Steward, D. V., *Systems Analysis and Management*, P. B. I., publishers, 1981.
5. Bertin, J. J. and Smith, M. L., *Aerodynamics for Engineers*, Prentice Hall Inc., 1979.
6. Haftka, R. T. and Starnes, J. H. Jr., "WIDOWAC: Wing Design Optimization with Aeroelastic Constraints-Program Manual", *NASA TM X-3071*, 1974.
7. Schmit, L. A. and Farshi, B., "Some Approximation Concepts for Structural Synthesis", *AIAA J.*, **12**, 1974, pp. 692–699.
8. Grandhi, R. V., Thareja, R. and Haftka, R. T., "NEWSUMT-A: A General Purpose Program for Constrained Optimization Using Constraint Approximations", *ASME J. Mech., Trans. & Automation in Design*, **107**, 1985, pp. 94–99.
9. Haftka, R. T., Grossman, B., Kao, P. J., Polen, D. and Sobieszczanski-Sobieski, J., "Integrated Aerodynamic–Structural Design of a Forward-Swept Transport Wing", to appear, *Proc. Second NASA / Air Force Symposium on Recent Experiences in Multidisciplinary Analysis and Optimization*, Sept. 28-30, 1988.