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VARIATION OF ANISOTROPIC AXES DUE TO MULTIPLE CONSTRAINTS IN STRUCTURAL OPTIMIZATION

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Abstract

In structural optimization great advances have been made in the past few years. Especially the mathematical methods developed rapidly due to their reliability and accuracy. The automated design of a given structure under statical loads might be done for a variety of different constraints and side constraints. The goal is to get a minimum weight design satisfying all restrictions. It is obvious that optimization methods are used with best benefits in the field of designing compound structures.

The problem is to find out the minimum weight design fulfilling all contraints by variation of a large number of design parameters, i.e. thicknesses of the elements or even layers, cross-sectional areas etc.

In this paper the influence of the parametric variation of the anisotropic axes is evaluated. The structural optimization is performed using the finite element model of a wing box. A minimum weight design shall be obtained which satisfies constraints in a displacement an a distorsion and constraints against failure of the compound elements as well. The design variables are the thicknesses of the layers and the angle of the anisotropic axes for the cover elements. The variation of the angle has been done by hand using a quadratic parabola for interpolation.

I. Introduction

Several years ago Dornier started to develop an optimization program which is based on finite element analysis and on sequential linearization of the nonlinear optimization problem. The combination of these two methods has proved quite adaptable to structural design problems. Solving structural optimization problems this way results in a number of considerable advantages

- exact mathematical formulation of the optimization problem
- adaptability to a lot of different types of constraints
- reliability and accuracy of the iterative computation
- good convergence
- extensions of the features can easily be done, i.e.
  - addition of new finite elements
  - addition of more constraints
  - consideration of additional design variables

The program system DYNOPT has a modular structure with respect to the different routine packages. This modularity has been kept through all extensions and alterations, thus it is possible to easily add further more features.

Constraints may be formulated with respect to

- allowable stresses
- safety against failure
- maximum deformations
- slope (relative displacements)
- buckling (under development)

The design variables are the thicknesses and the cross-sectional areas of the finite elements. For compound structures single-layer optimization is possible. Balanced and unbalanced compounds are permitted. The angle of the anisotropic axes of the compounds is expected to be an up to standard design variable very shortly.

Setting up design variable linking groups is encouraged. One design variable will be designated the representative, the modification of the others of the group will be proportional to the initial size. In case of proportion 1.0 and equal initial sizes all design variables within one group have the same size at any iteration. It is assumed that only finite elements of the same type may be linked.

II. The Optimization Problem

For a given structure under statical loads a minimum weight design is desired. Constraints with respect to failure of the composite elements and stiffness of the whole structure have to be satisfied. Furthermore side constraints for the design variables may be considered.

Starting with an initial design represented by the sizes of the finite elements the dimensioning of the structure will be improved iteratively. This process is an automated procedure. As a result of a sequence of structural redesigns the minimum weight design is obtained satisfying all constraints and side constraints. The computation can be stoped by

- a given number of iterations
- satisfaction of all constraints (feasible design) and the convergency criteria.

The structural weight W, the deformations U and the reserve factors RF are functions of the design variables t.

$$W = W(t_1, t_2, \ldots\ldots, t_m) = \text{Min.} \quad (1)$$

$$\left.\begin{aligned} U &= U(t_1, t_2, \ldots\ldots, t_m) \le U_{adm} \\ RF &= RF(t_1, t_2, \ldots\ldots, t_m) \ge RF_{adm} \end{aligned}\right\}(2)$$

The RF-values according to a simplified failure hypothesis by Tsai and Hill should be greater or equal 1. For values less than 1. a failure in one of the layers may occur.

Considering a Taylor series expansion of i.e. the structural weight neglecting higher order terms results in

$$W + \Delta W = W(t + \Delta t) = W(t) + \frac{\partial W}{\partial t}\Delta t \quad (3)$$

Using the ∇-operator the demand for minimum structural weight can be written as follows

$$\{\nabla W\}^T \{\Delta t\}^{(\nu+1)} = \Delta W = \text{Min.} \quad (4)$$

Equation (4) is the objective function of the optimization problem. The index ν herein is the indicator of the actual iteration step.

The behaviour constraints (2) may be evaluated the same way.

The displacement constraints

$$\{U\}^{(\nu)} + [\nabla U^{(\nu+1)}]^T \{\Delta t\}^{(\nu+1)} \le U_{adm} \quad (5)$$

and the failure constraints as well

$$\{RF\}^{(\nu)} + [\nabla RF^{(\nu+1)}]^T \{\Delta t\}^{(\nu+1)} \ge RF_{adm} \quad (6)$$

are linear dependent on the design variables t.

In addition side constraints may be considered such as

$$t^{(\nu)} + \Delta t^{(\nu+1)} \ge t_{min.} \quad (7)$$

The reserve factor RF in (6) is calculated from the three in-plane stresses σ_1, σ_2, τ_{12} related to the corresponding ultimate stresses (index u).

$$\left(\frac{\sigma_1}{\sigma_{1u}}\right)^2 + \left(\frac{\sigma_2}{\sigma_{2u}}\right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}}\right)^2 = H \quad (8)$$

The reserve factor is

$$RF = \frac{1}{\sqrt{H}} \quad (9)$$

The reserve factor is determined for each layer of an element. To avoid failure the minimum reserve factor must be greater than 1.

III. Evaluation of the Gradients

For statically loaded structures the basic equations are

$$[K]\,[U] = [F] \quad (10)$$

wherein [K] is the global stiffness matrix, [U] is the matrix of the unknown deformations and [F] is the load matrix. It is assumed that the loads are not dependent on the deformations of the structure.

Looking more closely at the inequality constraints (5) and (6) it is obvious that the gradients of the deformations with respect to the design variables have to be calculated.

Derivation of equation (10) with respect to the design variables results in

$$[\nabla U] = -[K]^{-1}\,[\nabla K]\,[U] \quad (11)$$

The reserve factors RF in (6) are a function of the stresses as is pointed out in (8) and (9) and these on the other hand are a function of the deformations either.

For compound membrane elements the relation between stresses and strains is

$$[\sigma] = [T_\sigma]\,[S]\,[T_\sigma]^T\,[\varepsilon] \quad (12)$$

For each layer of the compound the in-plane stresses σ_1, σ_2, τ_{12} have to be determined. Matrix [S] in (12) is the elasticity matrix of the layer and $[\varepsilon]$ is the matrix of the strains in the special coordinate system of the element.

The strains are dependent on the nodal displacements U in the global coordinate system.

$$[\varepsilon] = [\varepsilon_U]\,[T]\,[U] \quad (13)$$

The stress gradients are obtainable by deriving (12) under consideration of (13) with respect to the design variables.

$$[\nabla\sigma] = [T_\sigma]\,[S]\,[T_\sigma]^T\,[\varepsilon_U]\,[T]\,[\nabla U] \quad (14)$$

The slopes of the function RF = RF (σ_1, σ_2, τ_{12}) are the results of the derivation of (9) with respect to the design variables t under consideration of (8) and (14).

This procedure is known as sequential linearization of the optimization problem.

All constraints (5), (6) and (7) and the objective function (4) as well are linear in the unknown modifications Δt of the design variables.

In this paper two kinds of design variables will be considered

- thicknesses t of the elements
- angle of orientation of the anisotropic axes

At the moment the second one is not yet an up to standard design variable.

Just the thicknesses t will be modified automatically. The orientation of the anisotropic axes will be done stepwise with a follow-on interpolation. The variation of this angle is assumed to be a powerful means for great benefits in the structural weight.

Altogether the objective function and the behaviour and side constraints are a linear programming problem. This will be solved by using the Simplex algorithm. However, solutions for the unknown values Δt can only be obtained in the positive design space, therefore a transformation is necessary.

As the whole optimization problem is linearized some move limits should be selected for the changes of the design variables. The bandwidth of the modifications of the design variables is a good means to control the convergency.

IV. The Optimization Program DYNOPT

The optimization program DYNOPT consists of four major blocks. All these blocks communicate with common datasets. The input data describing the finite element model, the initial values for the design variables, element data and constraints and some post processing control values pass the first module. The purpose of this subroutine package is data processing and determining control values and setting flags according to the type of input data. Furthermore the input data are checked and cross-checked to eliminate incorrect data. Most of the error messages are printed in this block.

The second block deals with the structural analysis. Herein the global matrices, such as stiffness- and/or mass matrix, are calculated followed by the structural analysis. As it can be seen from Fig. 1 the upper part of this block is outside the optimization loop. All variables and arrays which are independent of the iteration steps are prepared and calculated here, i.e. stiffness and mass gradients for most of the element types. The lower part of this block is responsible for the determination of the structural analysis using the actual sizes of the design variables. This is inside the optimization loop and will be repeated in each iteration.

The third block is the optimizer. Calculation of the gradients of the constrained deformations, stresses and reserve factors is done here. The inequality constraints are formed as well as the move limits. When the complete linear programming problem is established the Simplex algorithm routines are started which results in the modifications of the element sizes Δt. Afterwards the improved structural design will be determined by superposition of the actual thicknesses with the modifications

$$\{t\}^{(\nu+1)} = \{t\}^{(\nu)} + \{\Delta t\}^{(\nu+1)} \qquad (15)$$

with $\Delta t \geq 0$.

The thus obtained design will be checked for feasibility whether satisfying the constraints or not and for convergency. Provided a feasible design meets the convergency criteria a flag will be set from zero to one and a final analysis will be made followed by the fourth block of the optimization program DYNOPT. This block is responsible for the post-processing of all the data, the constrained variables as well as the design variables. Furthermore the data will be prepared for graphical display.

If the improved design is not feasible and/or the convergency criteria are not yet met the analysis part of block two and all the block three will be repeated again and again (see Fig. 1) until either the maximum number of iterations is exceeded or the optimum is obtained.

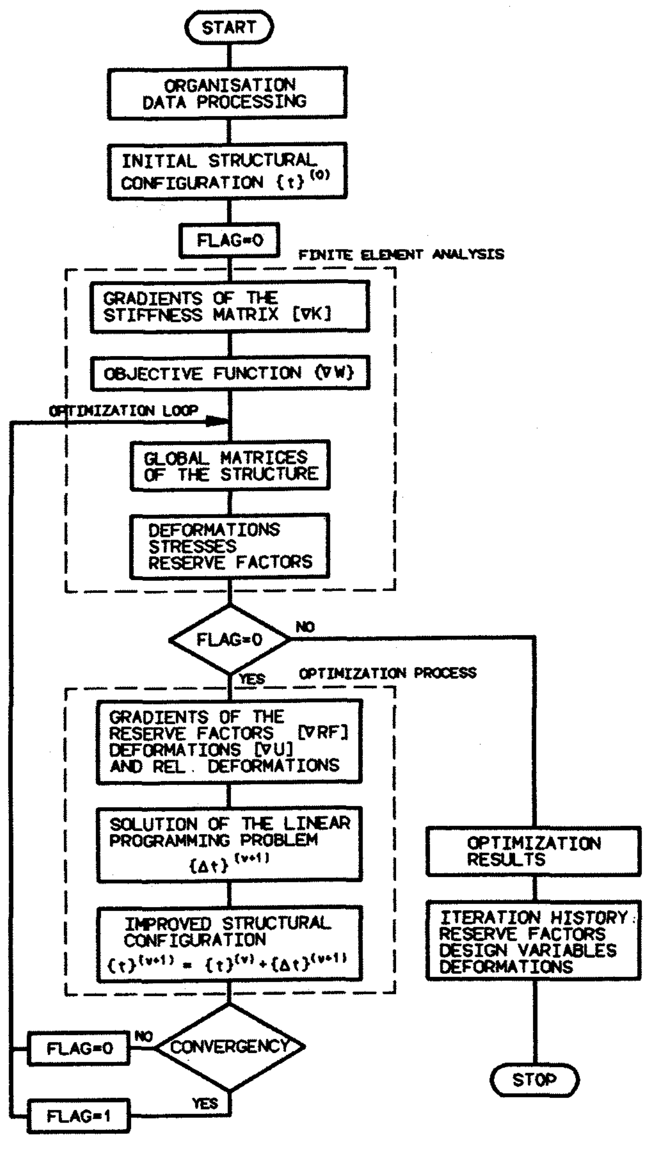

FIG. 1: FLOW CHART OF THE OPTIMIZATION PROGRAM DYNOPT

V. Optimization of a Wing Box

In this section of the paper results will be presented of the optimization of a swept compound wing box.

This structure (Fig. 2) has three spars and four ribs. It is a clamped structure carrying three different air pressure distributions as statical load cases.

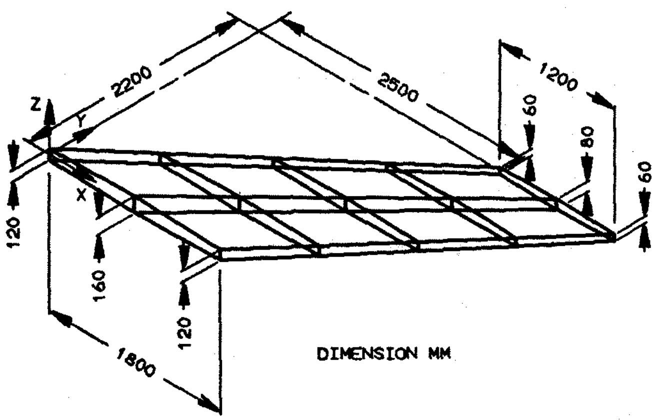

FIG. 2: WING BOX OF A SWEPT WING

A z-displacement in the leading edge of the far rib and a slope between leading and trailing edge of the same rib are constrained (Fig. 3).

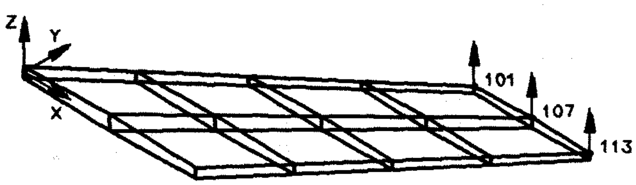

FIG. 3: FE-MODEL OF THE WING BOX WITH THE CONSTRAINED Z-DISPLACEMENTS

Furthermore the reserve factors according to a simplified failure hypothesis of Tsai/Hill in the quadrilateral compound elements are constrainted. Primary design variables are the layer thicknesses of the compound elements in the covers and the spars (Fig. 4). The thicknesses of the ribs are fixed.

A secondary design variable in the sense of not being fully automated is the orientation of the anisotropic axes in the cover elements. Optimizations have been done for three different angles and the corresponding minimum mass has been determined. In Fig. 5 these results have been plotted. The three angles of the anisotropic axes can be seen from Fig. 6. They are related to the direction of the y-axis. Changing from 15° to 30° results in a benefit of about 24 % in structural mass, whereas an increase of the angle to 45° adds 7.3 kg.

The approximation of these three points by a second order parabola leads to an optimum angle of the anisotropic axes of 31.6°.

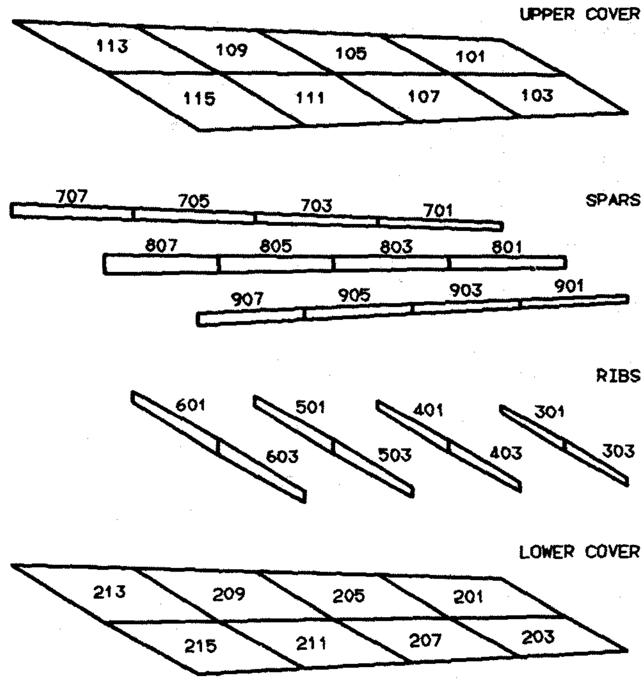

FIG. 4: FE-MODEL OF THE WING BOX WITH ANISOTROPIC QUADRILATERAL ELEMENTS

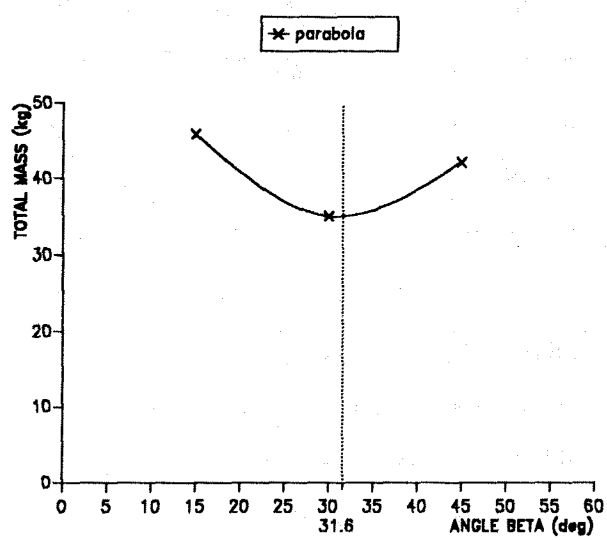

FIG. 5 : TOTAL MASS VERSUS ORIENTATION OF ANISOTROPIC AXES

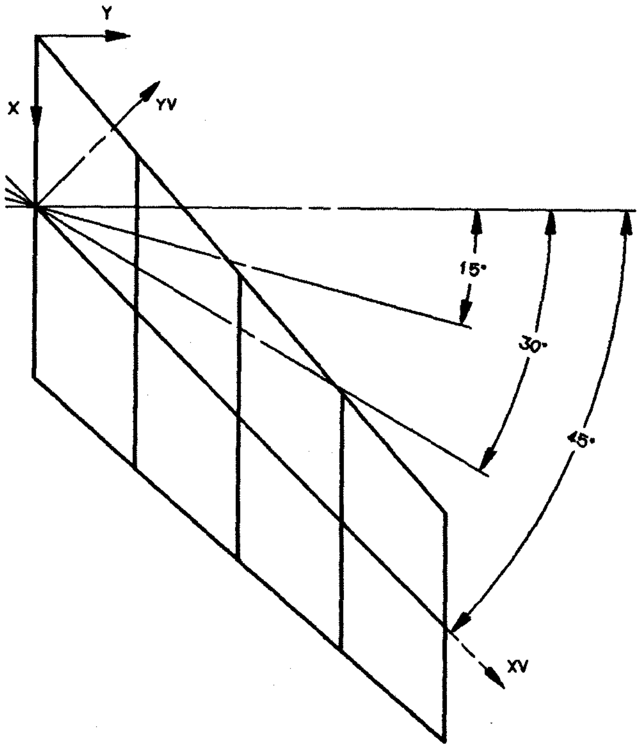

FIG. 6: VARIATION OF THE ANISOTROPIC AXES

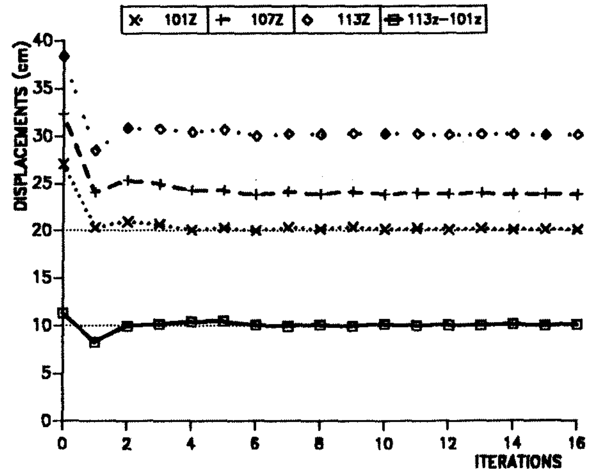

FIG. 7 : CONSTRAINED Z-DISPLACEMENTS

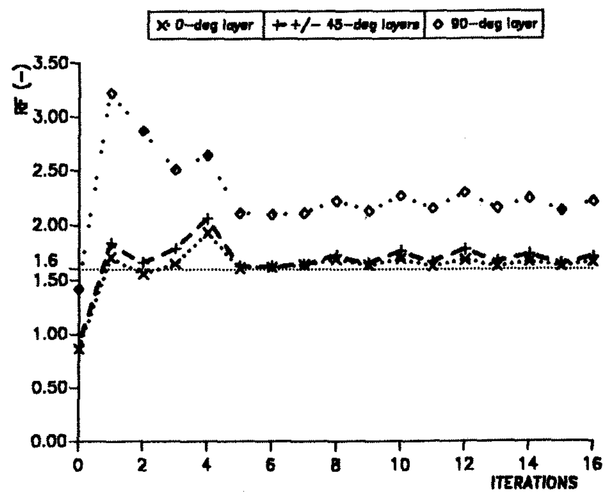

FIG. 8 : RESERVEFACTOR OF COVER ELEMENT 115

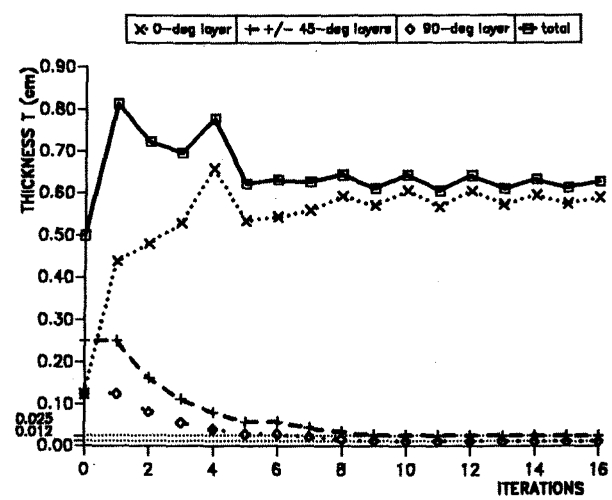

FIG. 9 : THICKNESS OF THE LAYERS OF COVER ELEMENT 115

As it can be seen from Fig. 5 the orientation of the anisotropic axes is of great importance for the structural design. The obtained optimum angle of 31.6° is parallel to the load path extending from the trailing edge of the supported rib to the leading edge of the far rib. Any different angle results in a considerable mass penalty of up to 0.7 kg/deg.

It is evident that the angle β always should be considered a design variable in optimization of compound structures. Expecially in aerospace design mass penalties as mentioned above are inacceptable.

The final run has been made with angle $\beta = 31.6°$ with the sizes of the layers as only design variables. In Fig. 7 the contrained z-displacements are plotted. Displacements and slope as well are approaching their limits rapidly in six iteration steps.

In Fig. 8 and 9 the reserve factors and the thicknesses of upper cover element 115 are plotted versus the iterations. A limit reserve factor of 1.6 has been selected against failure. The initial configuration is an infeasible design with reserve factors of 0.85 well below the limit.

In about six redesign loops the limits are approached by the 0°-layer and the ± 45°-layers as well. The 90°-layer is downsized to minimum gauge and due to angle optimization the share of loads is next to nothing. This results in a reserve factor of well above 2.0.

Most responsible for carrying the loads in this element is the 0°-layer as could be expected. Due to the normal stress in this element it is dominating the compound in thickness. The final reserve factor is 1.6 as required.

The same plots are done for element 215 (Fig. 10 and Fig. 11).

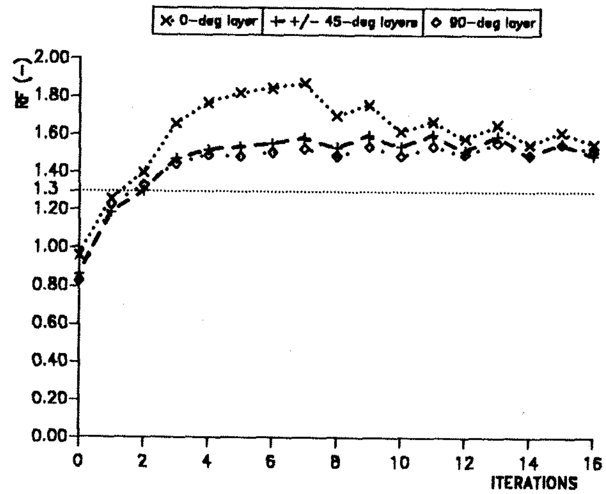

FIG. 10 : RESERVEFACTOR OF COVER ELEMENT 215

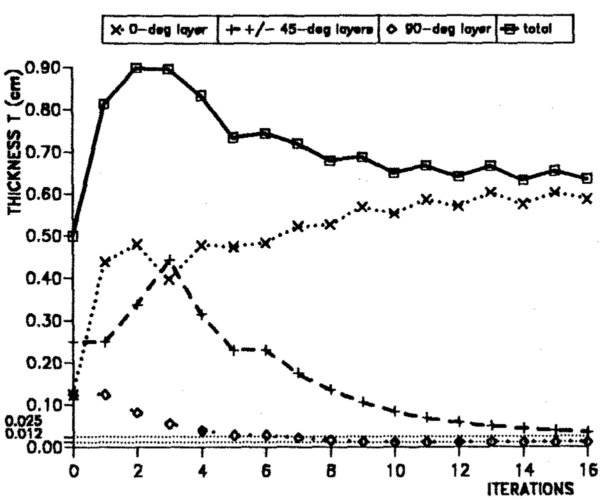

FIG. 11 : THICKNESS OF THE LAYERS OF COVER ELEMENT 215

The limit reserve factor is 1.3 in all lower cover elements due to tensile stresses only. None of the layers is approaching this limit. Again the 0°-layer is carrying the biggest share of the loads and the thicknesses of the 90°- and ± 45°-layers are approaching the lower limits. The iteration procedure is starting with an infeasible design for this element, the lowest reserve factor is 0.83. At the end of the redesign procedure the reserve factor is 1.48, that is well above the limit of 1.3

Furthermore the iteration history of the innermost rear spar element is presented. The element sizes of this quadrilateral element have been kept fixed. Nevertheless the reserve factor has been constrained and getting it up to the limit has to be done by variation of the adjacent cover elements. This process had been successful as it is obvious from Fig. 12. The reserve factors of all layers are in the feasible region.

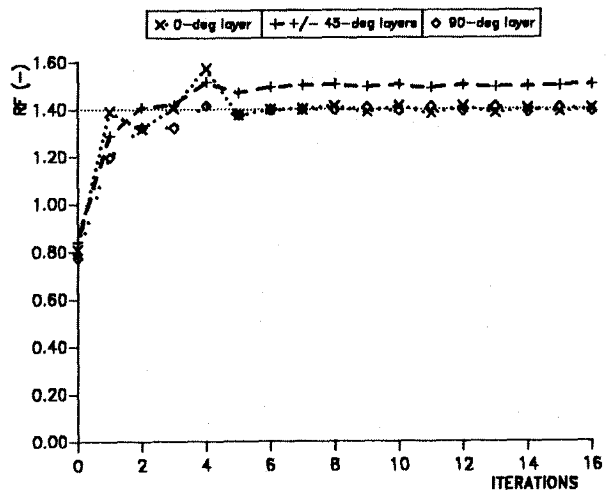

FIG. 12 : RESERVEFACTOR OF SPAR ELEMENT 907

In Fig. 13 the values of the objective function are plotted against the iterations. A rapid convergency can be observed, however it is not decreasing monotonously. The resizing procedure seems to go wrong way. The reason is that some elements in the outer part of the load path are oversized and others close to the support are undersized.

In the first iteration step a stiffness rearrangement is done to obtain an adequate stiffness distribution to the loads. This procedure causes a penalty in structural mass as can be seen from Fig. 13. The following redesigns result in a nearly monotonous decrease in structural mass.

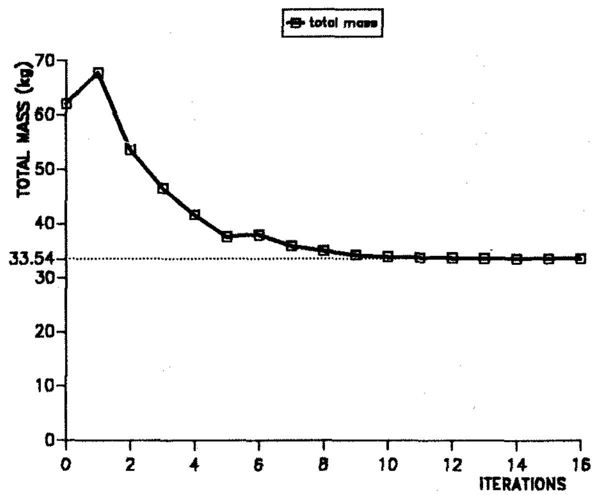

FIG. 13 : ITERATION HISTORY OF THE STRUCTURAL MASS

The final design is a minimum mass design satisfying all constraints. In Fig. 14 and 15 the element thicknesses and the percentages of the layers related to the overall thickness are presented.

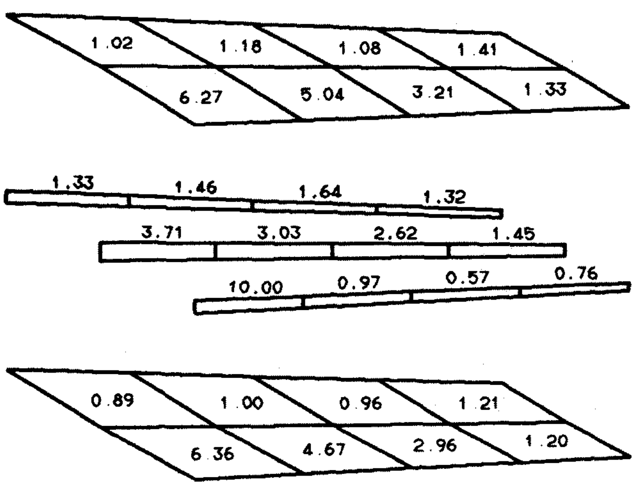

FIG. 14: THICKNESSES (MM) OF THE OPTIMIZED STRUCTURE

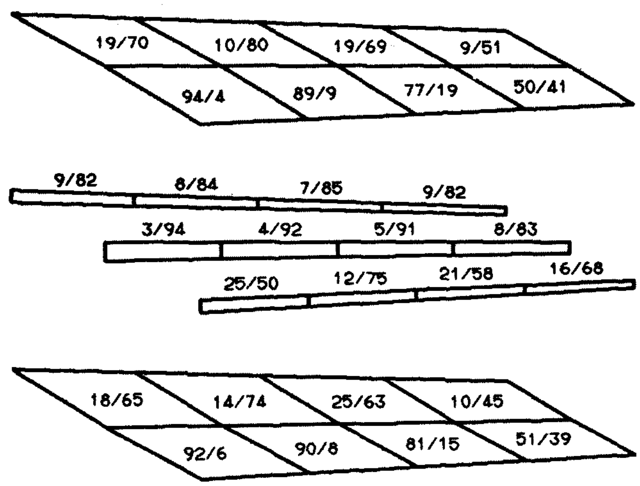

FIG. 15: PERCENTAGES OF THE 0°/±45°-LAYERS

VI. Conclusion

An optimization has been presented to minimize the structural mass of a wing box made from carbon fiber reinforced plastic. The influence of the variation of the anisotropic axes is evaluated combined with the variation of the thicknesses of the element layers. Different types of constraints have to be satisfied such as

- displacement constraints
- slope constraints
- constraints against failure
- side constraints for the layers

The finite element method in conjunction with the sequential linearization of the optimization problem worked well. The optimum design is determined within the margins of the convergency criteria passing 16 iterations.

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